

Spectra of Antineutrinos from Nuclear Reactors

Anna Hayes, with input from colleagues at LANL, BNL, and TUNL

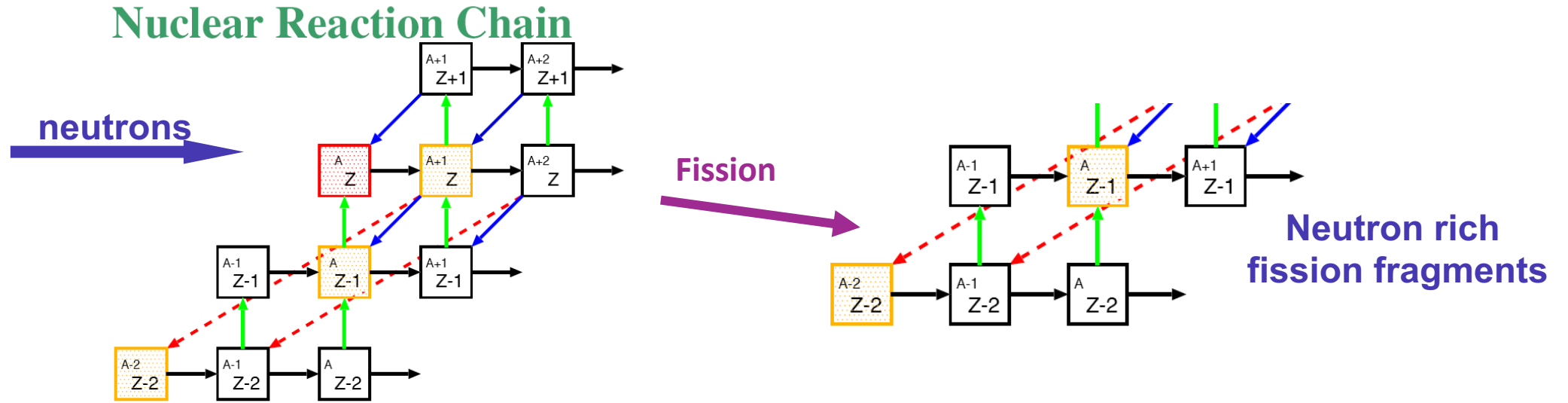
2021 Nuclear Data for Reactor Antineutrino Measurements Workshop

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Abstract

Knowledge of reactor anti-neutrino spectra are needed at the few percent level, but our understanding has yet to reach this level of accuracy. This is mostly because the nuclear data that go into anti-neutrino spectra, fission fragment yields and their beta decay spectra, are not known for all the nuclei that contribute to the aggregate spectra. In this talk I will present the two different methods for determining reactor anti-neutrino spectra and explain why each method involves uncertainties at the few percent level.

Beta Decay of fission fragments are the source of reactor anti-neutrinos, with $\sim 6 \nu_e$ emitted per fission.

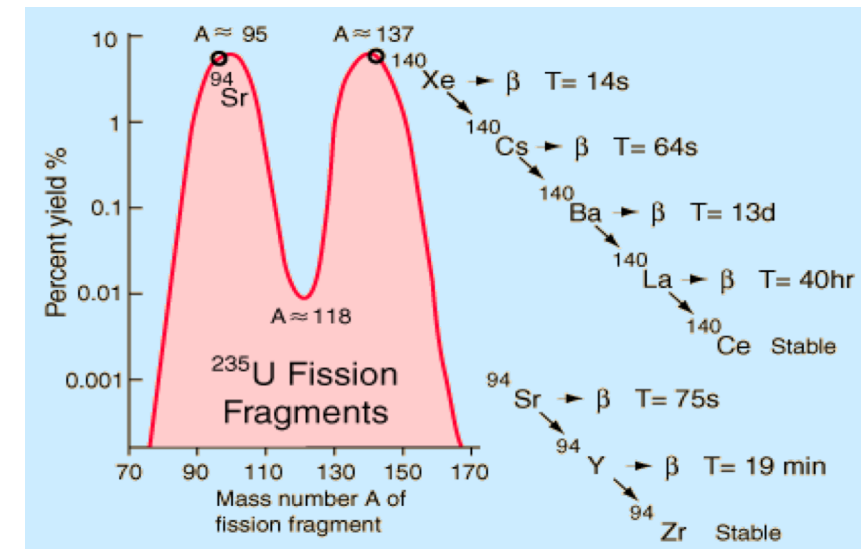


- Hundreds of fission fragments – **most all are neutron rich**

- Most fragments β -decays with several branches

\Rightarrow About $6 \nu_e$ per fission

\Rightarrow Aggregate spectrum made up of thousands of end-point energies



Two ways to determine the antineutrino spectra

- The summation method – Sum up all the beta decays, weighted by their fission yields.

$$\frac{dN_{\nu}}{dE_{\nu}} = \sum Y(Z, A) \sum B(E_0) P(E, E_0, Z, A)$$

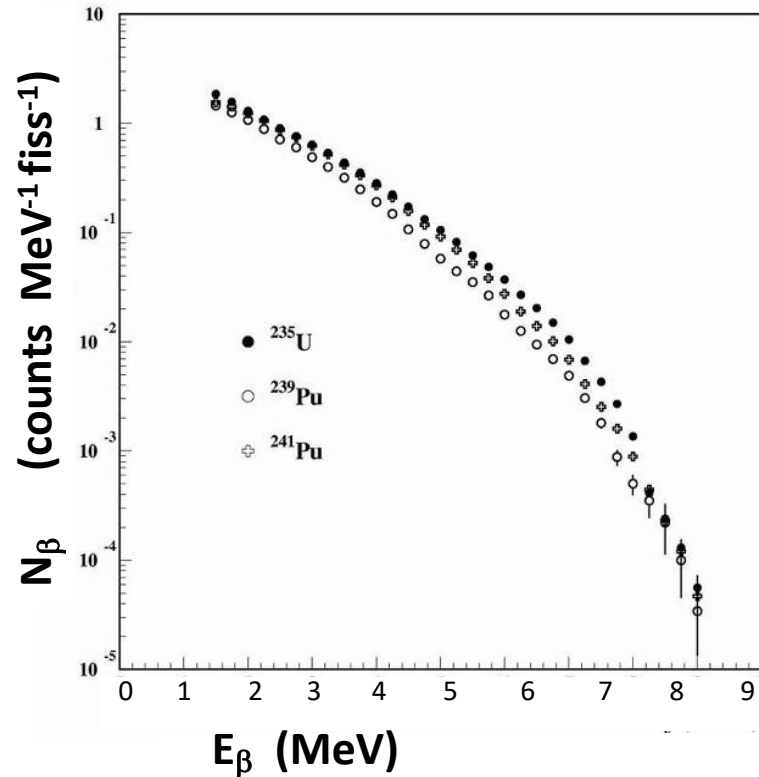
Cumulative Fission yields Beta branching ratios Individual spectra

- The conversion method - Measure the aggregate beta electron spectrum and convert it into an anti-neutrino spectrum:

$$\frac{dN_e}{dE_e} \longrightarrow \frac{dN_{\nu}}{dE_{\nu}}$$

Both methods introduce uncertainties

The Original Expected Fluxes were determined via the Conversion Method using β -Spectra (electrons) made at the ILL Reactor in the 1980s



- The thermal fission beta spectra for ^{235}U , ^{239}Pu , ^{241}Pu were measured at ILL.
- These β -spectra were converted to antineutrino spectra by fitting to 30 end-point energies
- Vogel *et al.* used the ENDF-5 nuclear database to estimate ^{238}U , which requires fast neutron fission

Vogel, et al., Phys. Rev. C24, 1543 (1981).

K. Schreckenbach et al. PLB118, 162 (1985)

A.A. Hahn et al. PLB160, 325 (1989)

$$S_{\beta}(E) = \sum_{i=1,30} (a_i) S^i(E, E_0^i)$$

FIT ↙

$$S^i(E, E_0^i) = E_{\beta} p_{\beta} (E_0^i - E_{\beta})^2 F(E, Z_{\text{eff}}) (1 + \delta_{\text{corrections}})$$

Parameterized
↙ ↘

Two inputs are needed to convert β -spectra to antineutrino spectra:
(1) Z of the fission fragments for the Fermi function, (2) sub-dominant corrections

$$S^i(E, E_0^i) = E_\beta p_\beta (E_0^i - E_\beta)^2 F(E, Z)(1 + \delta_{corrections})$$

The Fermi function:

Z_{eff} used for Fermi function

The corrections:

$$\delta_{correction}(E_e, Z, A) = \delta_{FS} + \delta_{WM} + \delta_R + \delta_{rad}$$

δ_{FS} = Finite size correction to Fermi function

δ_{WM} = Weak magnetism

δ_R = Recoil correction

δ_{rad} = Radiative correction

A change to the approximations used for these effects led to the anomaly

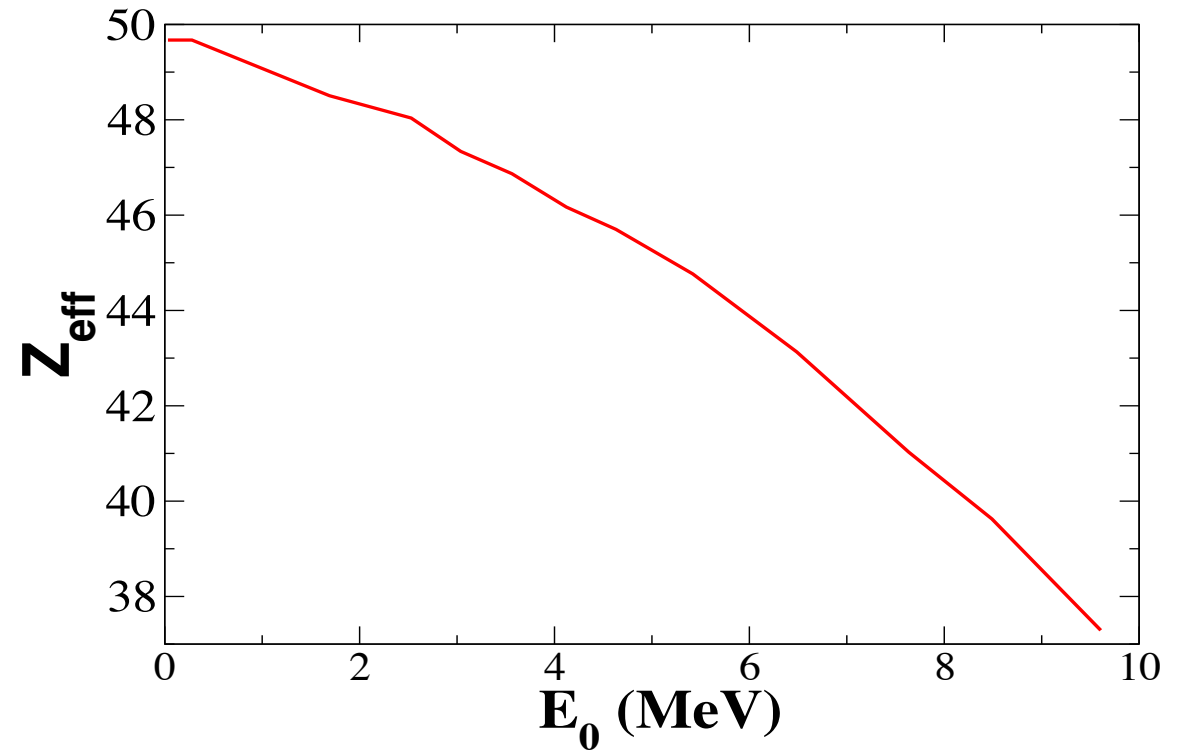
An energy-dependent Z_{eff} representing the fission fragments is needed to determine the Fermi function

- On average, higher end-point energy means lower Z .
- Comes from nuclear binding energy differences

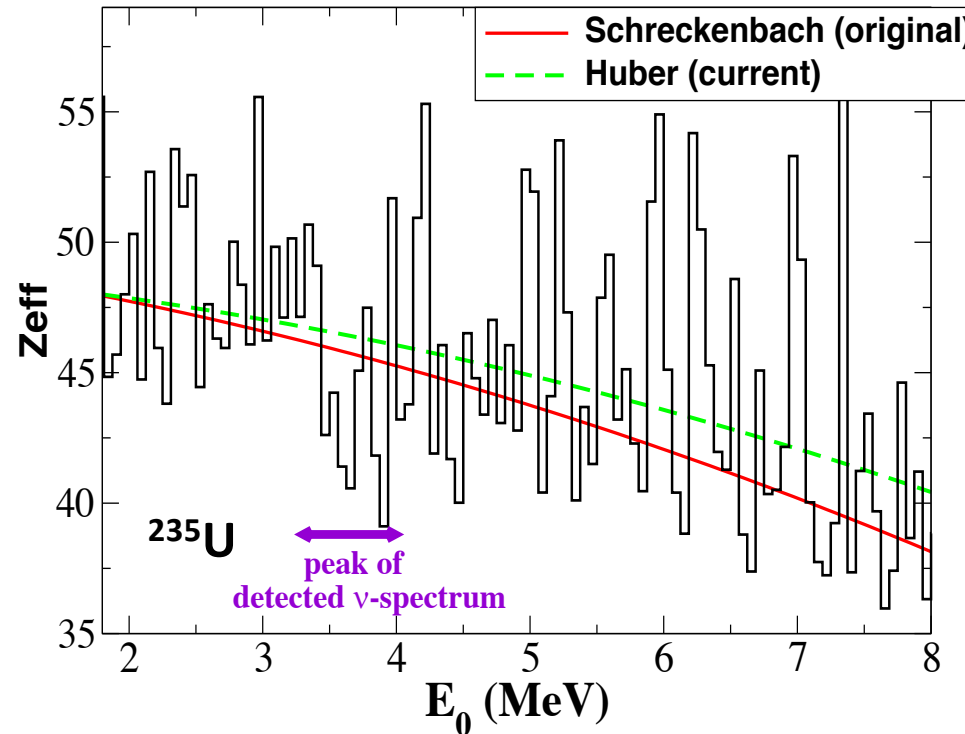
Parameterized used by both Schreckenbach and Huber involved a quadratic function:

$$Z_{eff} \sim a + b E_0 + c E_0^2$$

But the difference in their parameterizations is a large part of the problem anomaly.

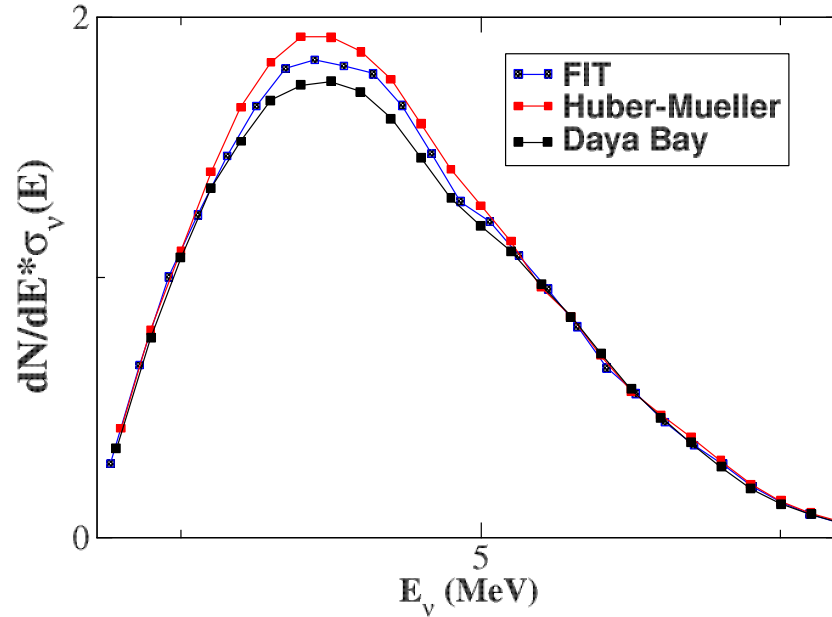
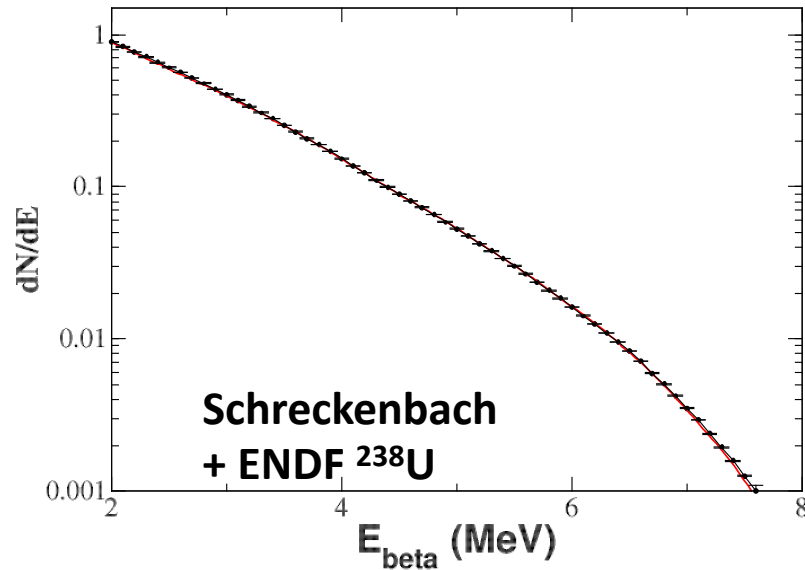


The newer fit to Z_{eff} used in the Fermi function for the conversion of the aggregate β -spectrum, led to a higher ν -spectrum



- Huber's new parameterization of Z_{eff} with end-point energy E_0 changes the Fermi function, accounting for 50% of the current anomaly.
- But the data do not follow a simple quadratic form.

Simultaneous fit of the Daya Bay antineutrino spectrum and the equivalent aggregate β -spectrum with improved (1) Z_{eff} and (2) descriptions of forbidden transitions reduces the anomaly from 5% to 2.5%



The magnitude of the IBD cross sections change, depending on assumptions, but not the ratio of one isotope to another

	all allowed $Z_{\text{eff}}^{\text{Huber}}$	all allowed Z_{eff}	allow.+forbid. Z_{eff}	allow.+forbid. $(Z_{\text{eff}}^2)^{1/2}$
^{235}U	6.69	6.58	6.47	6.48
^{239}Pu	4.36	4.3	4.22	4.23
ratio	1.534	1.530	1.533	1.532

The sub-dominant corrections

$$S^i(E, E_0^i) = E_\beta p_\beta (E_0^i - E_\beta)^2 F(E, Z) (1 + \delta_{\text{corrections}})$$

$$\delta_{\text{correction}}(E_e, Z, A) = \delta_{FS} + \delta_{WM} + \delta_R + \delta_{rad}$$

δ_{FS} = Finite size correction to Fermi function

δ_{WM} = Weak magnetism

δ_R = Recoil correction

δ_{rad} = Radiative correction

- Recoil and radiative corrections are well-known and nucleus independent.
- The finite size and weak magnetism corrections are nucleus dependent and should be applied to each β -decay transition, which is a problem for the conversion method.

Corrections for GT Transitions

1. Finite size of the nucleus

Friar, Holstein

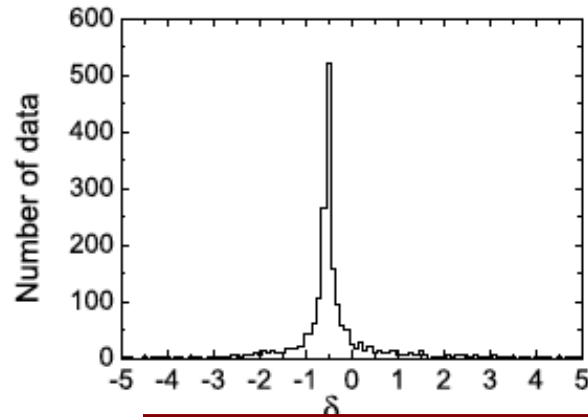
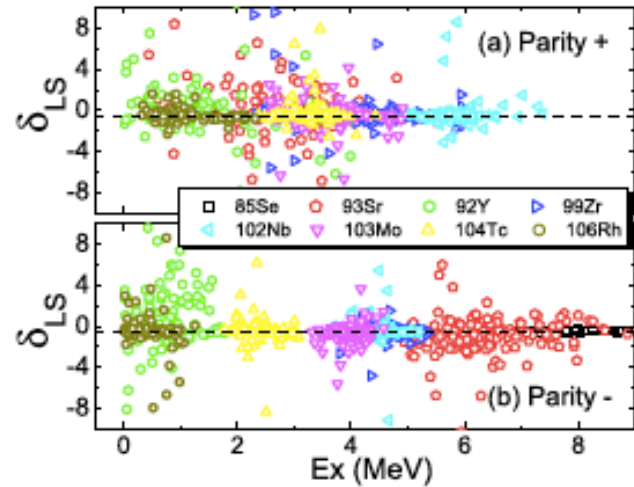
$$\delta_{\text{FS}} = A_c = -\frac{3Z\alpha R}{2\hbar c} \left(E_\beta - \frac{E_\nu}{27} + \frac{m_e^2}{E_\beta} \right); \quad R = \frac{36}{35} (1.2A^{1/3})$$

2. Weak magnetism

Friar, Holstein

$$\delta_{\text{WM}} = A_w = \frac{4(\mu_\nu - 1/2)}{6M_n} (E_\beta \beta^2 - E_\nu)$$

Weak Magnetism has an uncertainty arising from the approximation used for the orbital contribution and from omitted 2-body currents.
But, dominant $0^+ \rightarrow 0^-$ transitions have zero δ_{WM} , with no uncertainty



$$\delta_{WM}^{GT} = \frac{4(\mu_N - 1/2)}{6M_N g_A} (E_e \beta^2 - E_\nu)$$

$$\delta_{LS}^{j_f j_i} \equiv \frac{\langle J_f || \vec{\Lambda} || J_i \rangle}{\langle J_f || \vec{\Sigma} || J_i \rangle} \simeq -\frac{1}{2}$$

- Checked for a subset of fission fragments.
- A check for all fission fragments, including 2-body terms, requires a large super-computing effort.

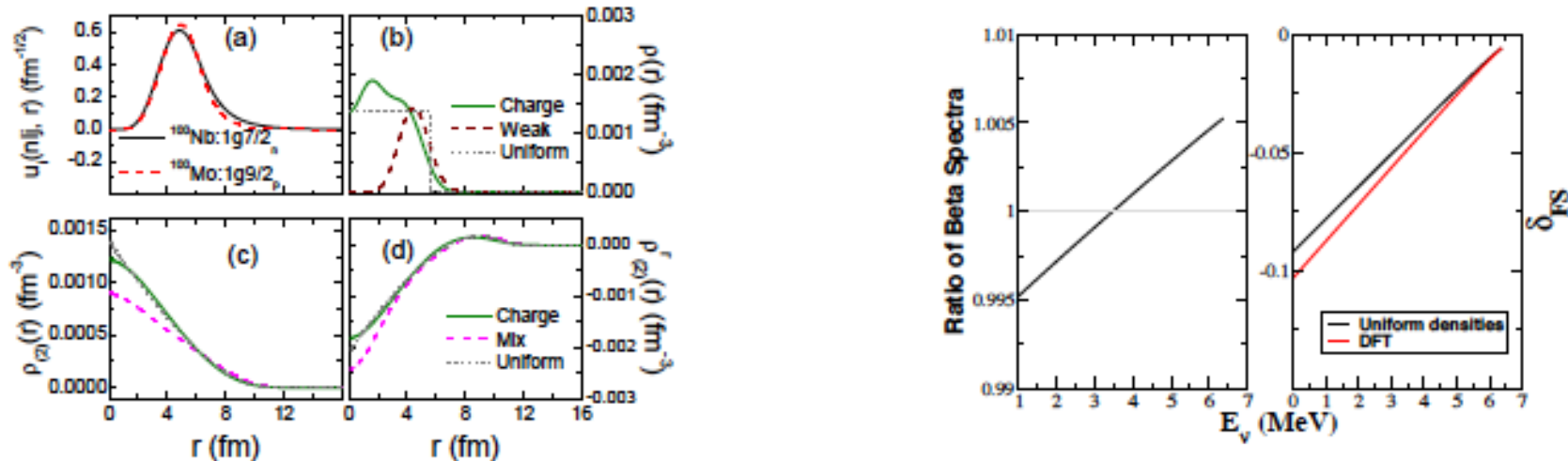
Estimated uncertainty $\sim 30\%$ for this 4% correction to the spectra

The Finite Size Correction can be expressed in terms of Zemach moments

$$\delta_{FS} = \Delta F_{REL}/F_{REL} = -\frac{Z\alpha}{3\hbar c} \left(4E \langle r \rangle_{(2)} + E \langle r \rangle_{(2)}^r - \frac{E_\nu \langle r \rangle_{(2)}^r}{3} + \frac{m^2 c^4}{E} (2 \langle r \rangle_{(2)} - \langle r \rangle_{(2)}^r) \right)$$

Approximated as :

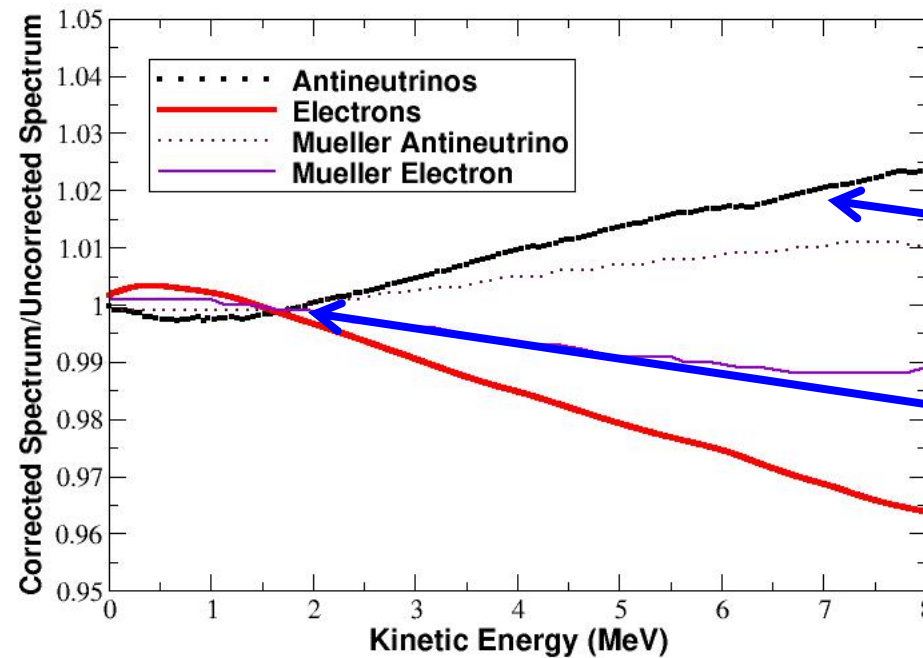
$$\delta_{FS} = -\frac{3Z\alpha}{2\hbar c} \langle r \rangle_{(2)} \left(E_e - \frac{E_\nu}{27} + \frac{m^2 c^4}{3E_e} \right)$$



- Found to be a good approximation for allowed transitions.
- Not checked for forbidden transitions.

Estimated uncertainty ~ 20% for this 5% correction to the spectra

Effect of FS and MW Corrections to Spectrum using ENDF/B-VII, and assuming that all Transitions are allowed



$$\approx \frac{1}{2} \left[\frac{4(\mu_\nu - \frac{1}{2})}{3} - \frac{3Z\alpha R}{2\hbar c} \right]$$

$$\bar{E}_0/2$$

Originally approximated by a parameterization: $\delta_{FS} + \delta_{WM} = 0.0065(E_\nu - 4MeV)$

30% of the beta-decay transitions involved are so-called forbidden

Allowed transitions $\Delta L=0$; Forbidden transitions $\Delta L \neq 0$

Forbidden transitions introduce a shape factor $C(E)$:

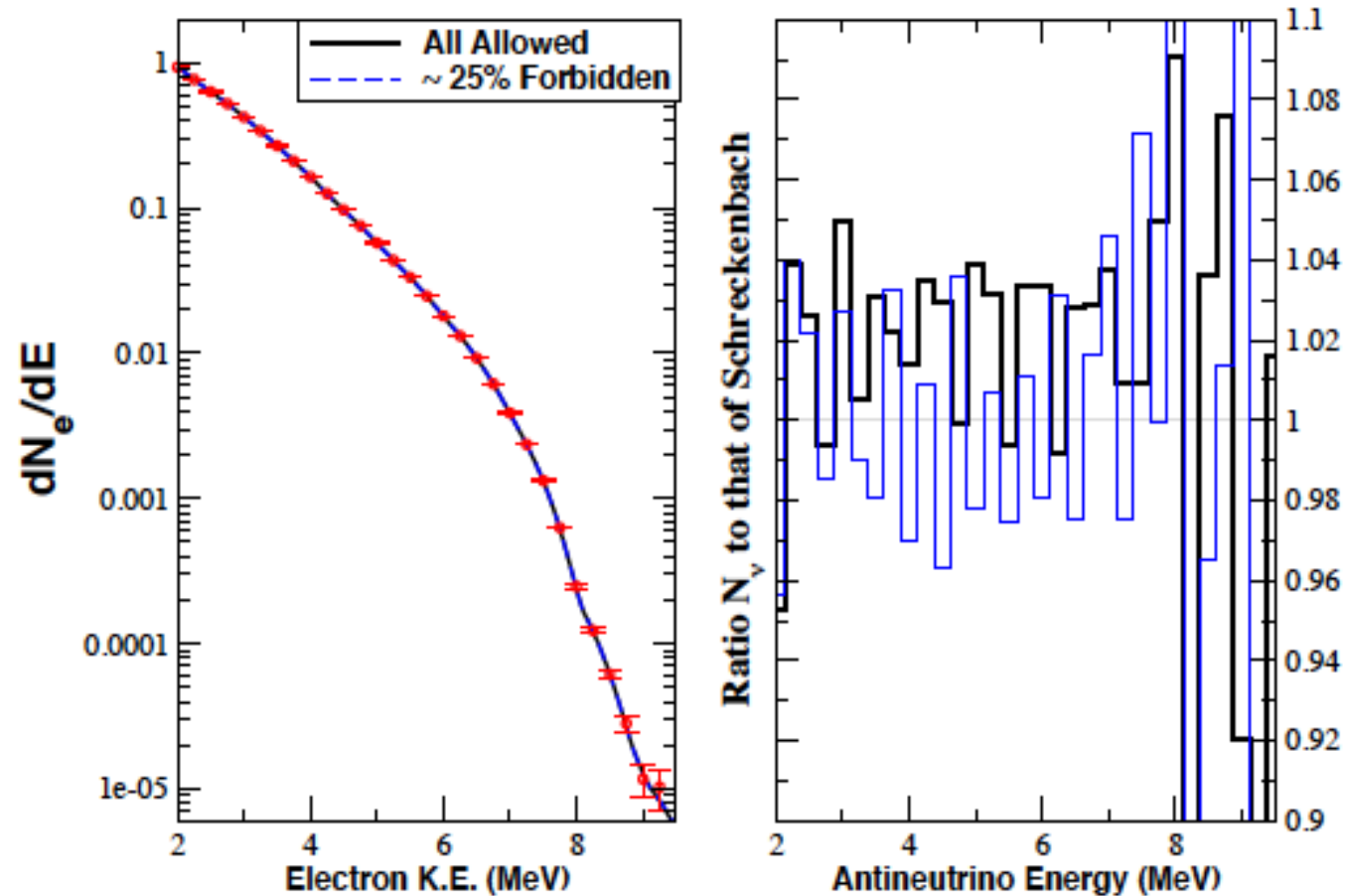
$$S(E_e, Z, A) = \frac{G_F^2}{2\pi^3} p_e E_e (E_0 - E_e)^2 \underline{C(E)} F(E_e, Z, A) (1 + \delta_{corr}(E_e, Z, A))$$

The corrections for forbidden transitions are also different and sometimes unknown :

Classification	ΔJ^π	Operator	Shape Factor $C(E)$	Fractional Weak Magnetism Correction $\delta_{WM}(E)$
Allowed GT	1^+	$\Sigma \equiv \sigma\tau$	1	$\frac{2}{3} \left[\frac{\mu_N - 1/2}{M_N g_A} \right] (E_e \beta^2 - E_\nu)$
Non-unique 1 st Forbidden GT	0^-	$[\Sigma, r]^{0-}$	$p_e^2 + E_\nu^2 + 2\beta^2 E_\nu E_e$	0
Non-unique 1 st Forbidden ρ_A	0^-	$[\Sigma, r]^{0-}$	λE_0^2	0
Non-unique 1 st Forbidden GT	1^-	$[\Sigma, r]^{1-}$	$p_e^2 + E_\nu^2 - \frac{4}{3}\beta^2 E_\nu E_e$	$\left[\frac{\mu_N - 1/2}{M_N g_A} \right] \left[\frac{(p_e^2 + E_\nu^2)(\beta^2 E_e - E_\nu) + 2\beta^2 E_e E_\nu (E_\nu - E_e)/3}{(p_e^2 + E_\nu^2 - 4\beta^2 E_\nu E_e/3)} \right]$
Unique 1 st Forbidden GT	2^-	$[\Sigma, r]^{2-}$	$p_e^2 + E_\nu^2$	$\frac{3}{5} \left[\frac{\mu_N - 1/2}{M_N g_A} \right] \left[\frac{(p_e^2 + E_\nu^2)(\beta^2 E_e - E_\nu) + 2\beta^2 E_e E_\nu (E_\nu - E_e)/3}{(p_e^2 + E_\nu^2)} \right]$
Allowed F	0^+	τ	1	0
Non-unique 1 st Forbidden F	1^-	$r\tau$	$p_e^2 + E_\nu^2 + \frac{2}{3}\beta^2 E_\nu E_e$	0
Non-unique 1 st Forbidden \vec{J}_V	1^-	$r\tau$	E_0^2	-

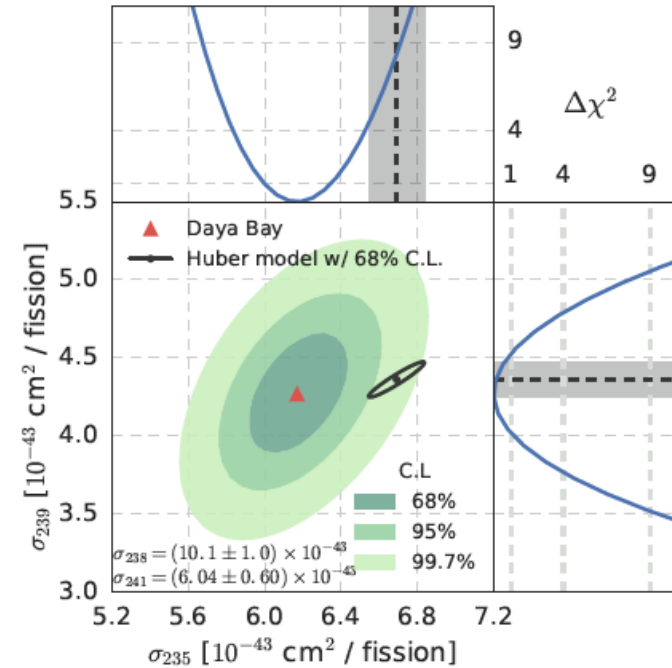
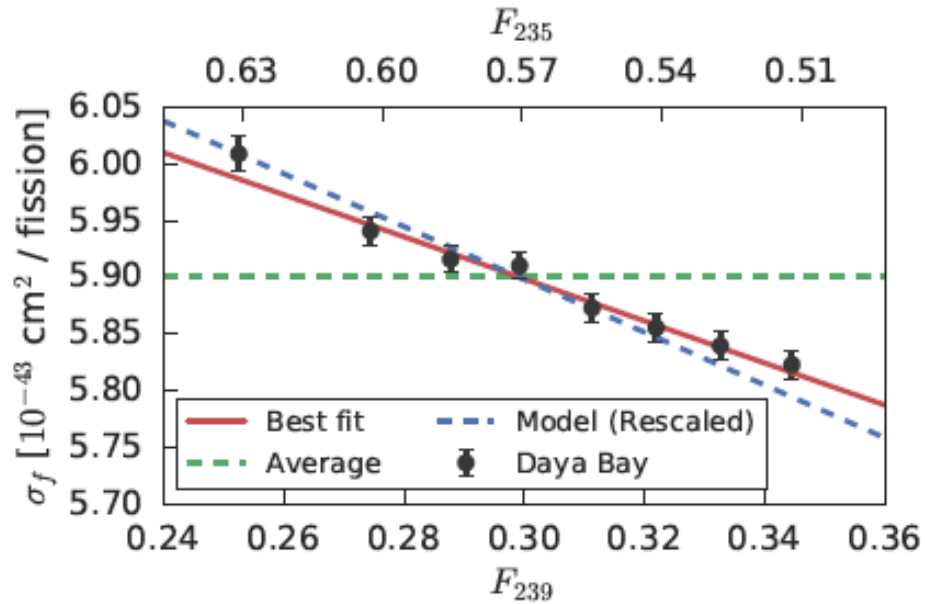
The forbidden transitions increase the uncertainty in the expected spectrum.

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Two equally good fits to the Schreckenbach β -spectra, lead to ν -spectra that differ by 4%.

The Total Number of Antineutrinos Decreases with Burnup, but the Huber-Mueller Model does not agree with the measured slope



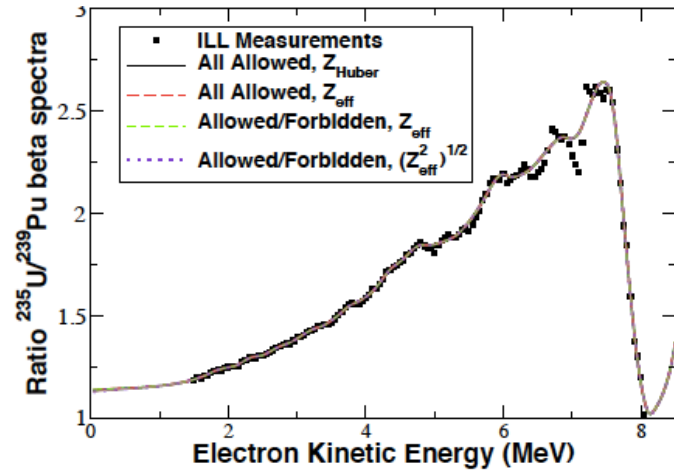
$$\sigma_f(F_{239}) = \bar{\sigma}_f + \frac{d\sigma_f}{dF_{239}}(F_{239} - \bar{F}_{239})$$

$$\begin{aligned} d\sigma_f/dF_{239} &= (-1.86 \pm 0.18) \times 10^{-43} \text{ cm}^2/\text{fission} \\ &(-2.46 \pm 0.06) \times 10^{-43} \text{ cm}^2/\text{fission} \end{aligned}$$

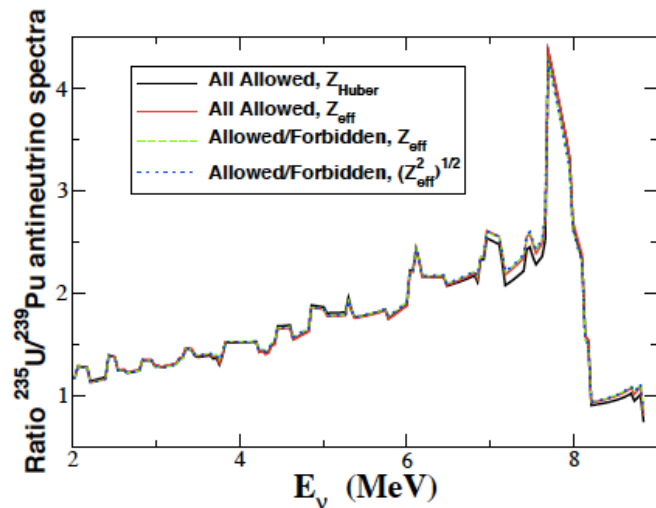
Experiment

Expected

The discrepancy between current Huber-Mueller model predictions and the Daya Bay results can be traced to the original Schreckenbach measured $^{235}\text{U}/^{239}\text{Pu}$ ratio



	all allowed $Z_{\text{eff}}^{\text{Huber}}$	all allowed Z_{eff}	allow.+forbid. Z_{eff}	allow.+forbid. $(Z_{\text{eff}}^2)^{1/2}$
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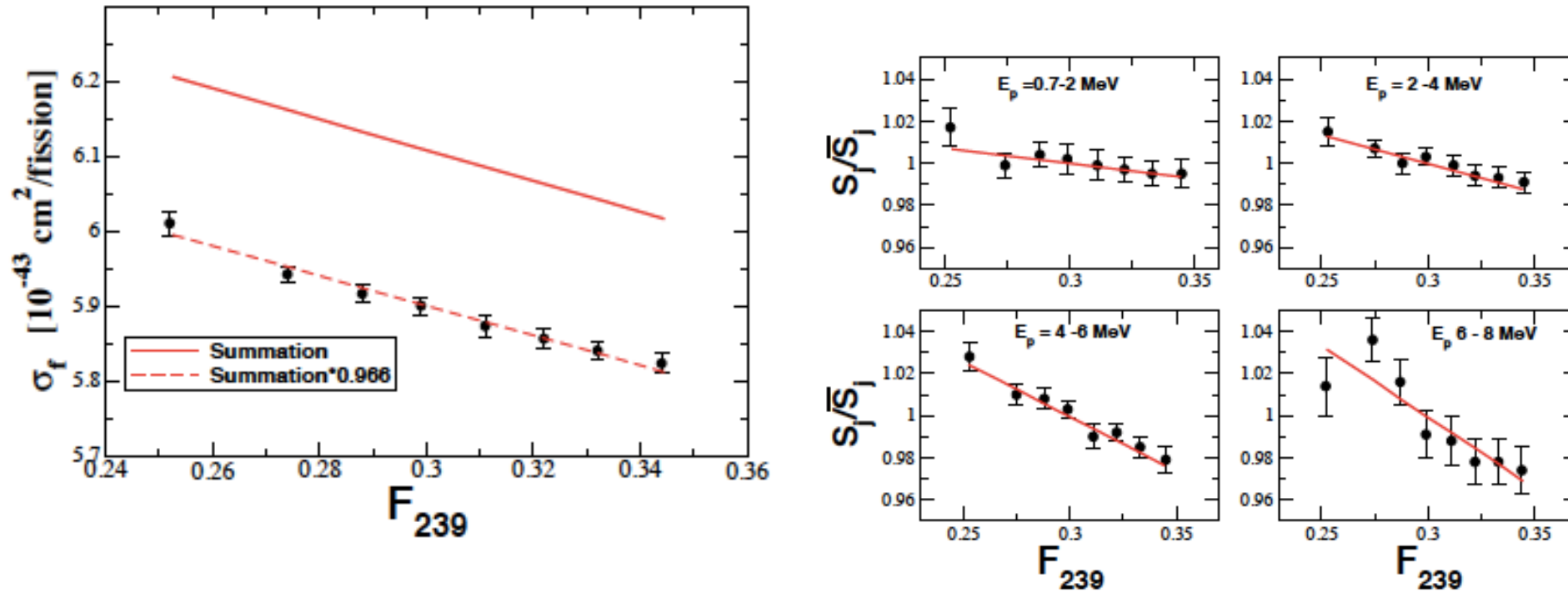


But the ratio of $^{235}\text{U}/^{239}\text{Pu}$ is fixed.

$$\sigma_5/\sigma_9 = 1.53 \pm 0.05 \text{ (Schreckenbach)}$$

$$\sigma_5/\sigma_9 = 1.445 \pm 0.097 \text{ (Daya Bay)}$$

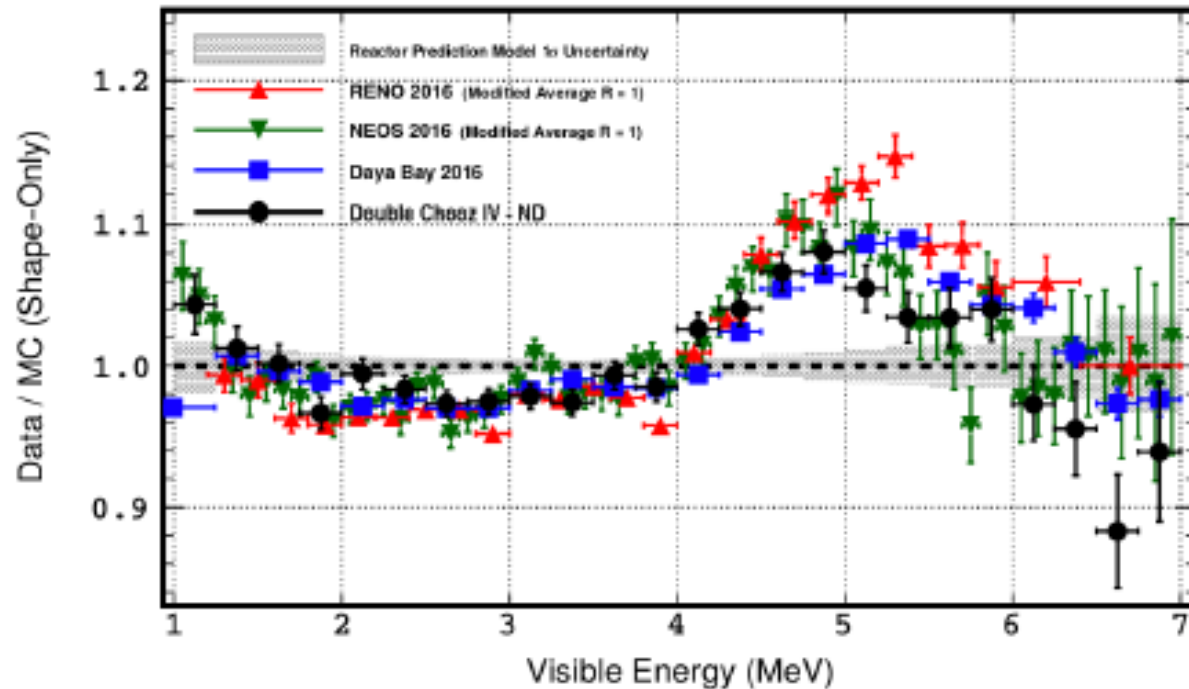
The Nuclear database explains all of the Daya Bay fuel evolution data, but still allows for a (smaller) anomaly



- The IBD yield is predicted to change with the correct slope.
- But the absolute predicted value is high by 3.5%.
- This anomaly is not statistically significant but it means that Daya Bay evolution data do not rule out sterile neutrinos.

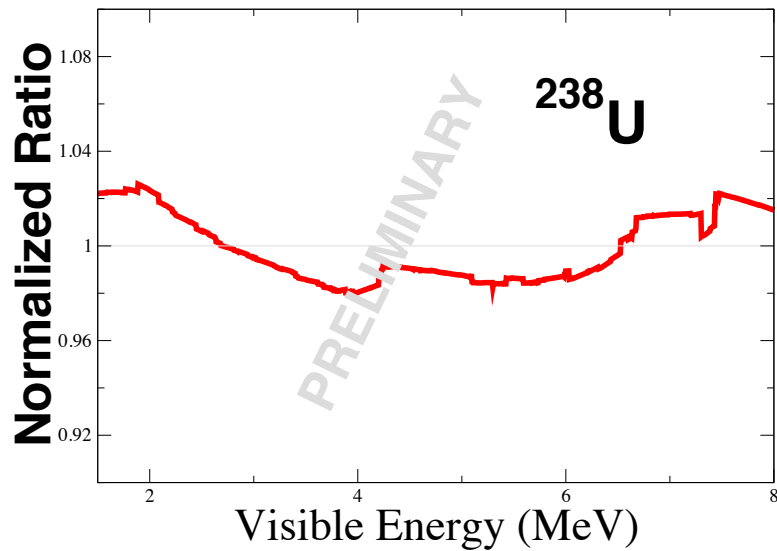
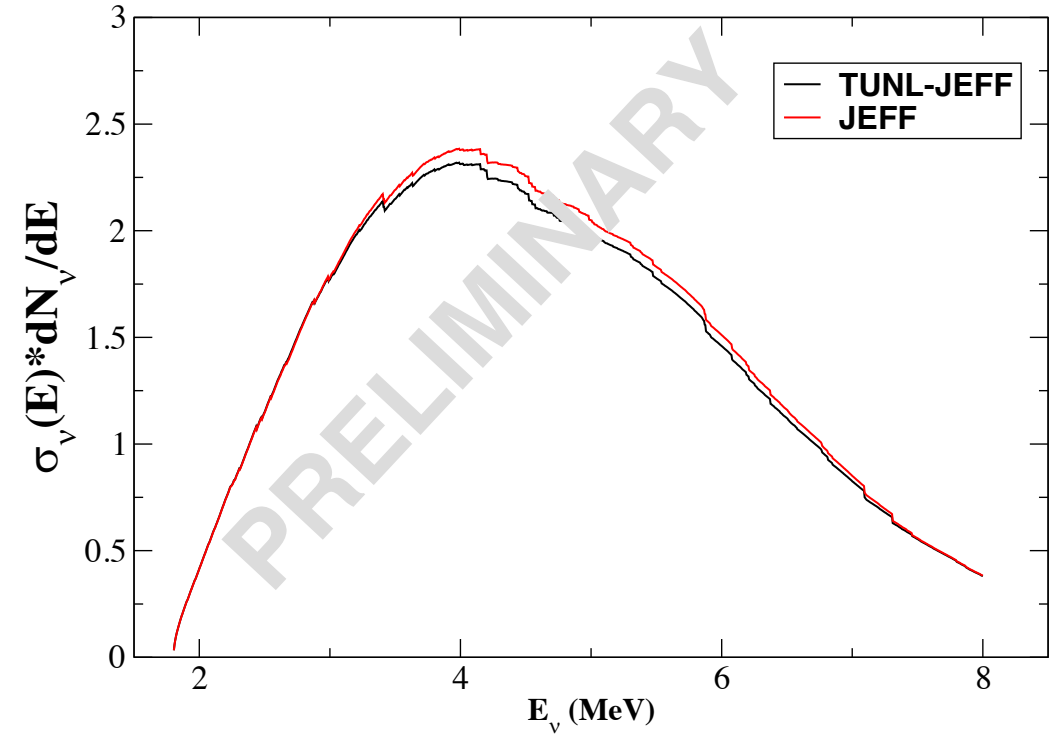
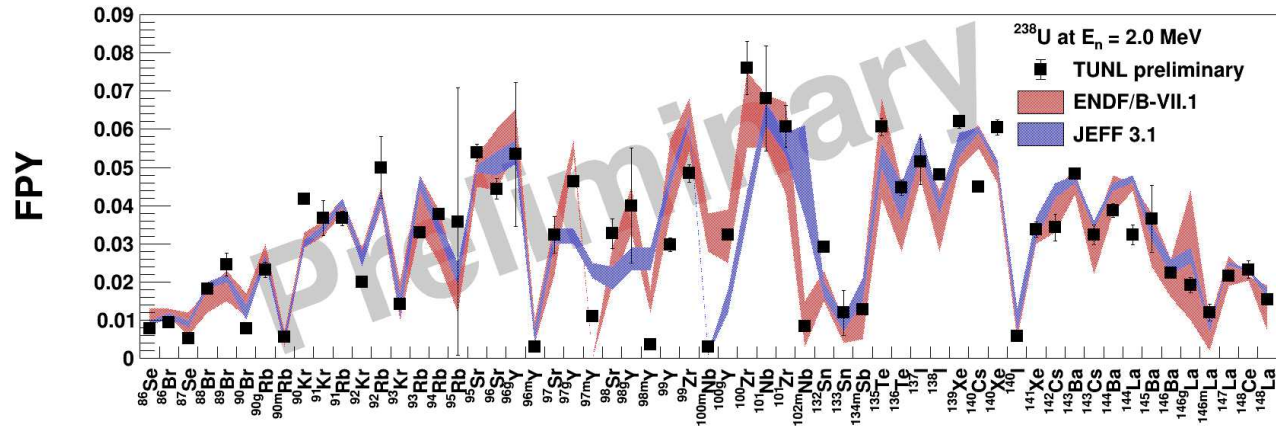
The Reactor Anti-Neutrino 'BUMP'

from Laura Bernard, STEREO (from Moriond 2019)



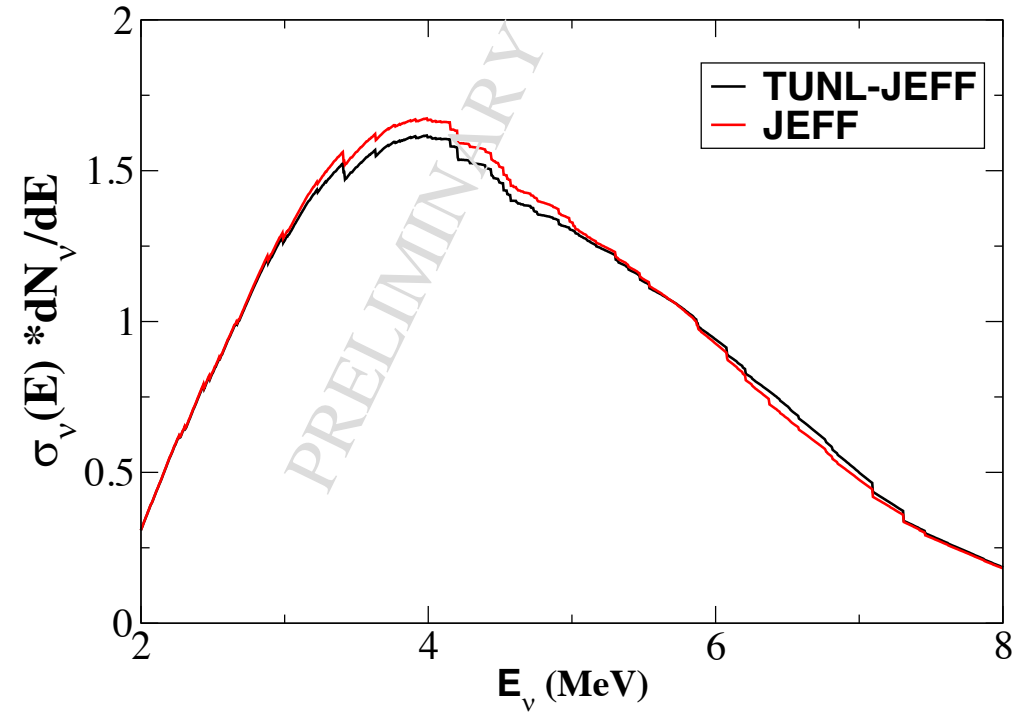
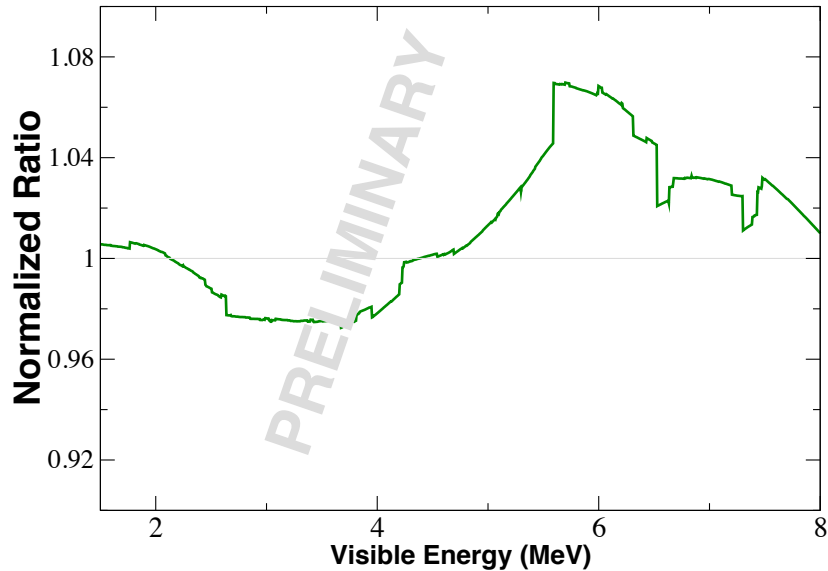
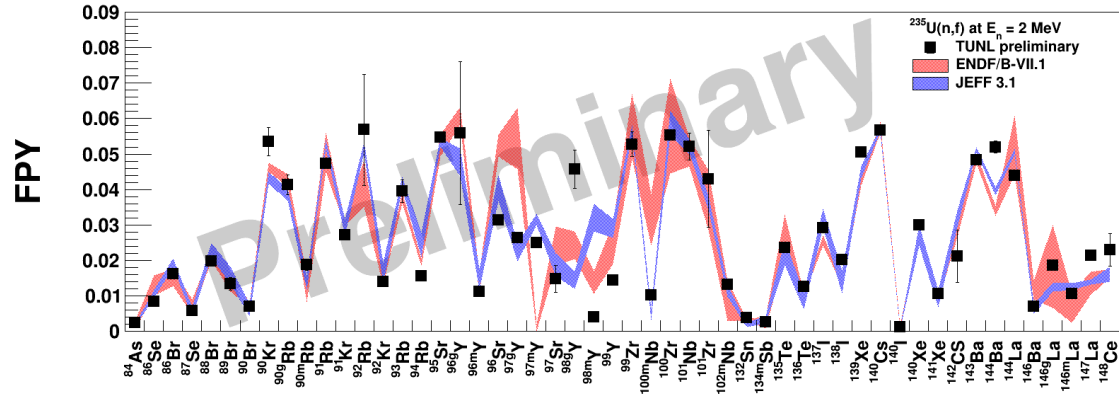
The shape of the measure anti-neutrino spectra differ from expectations

New data from TUNL suggest that this is largely a problem with the cumulative fission yields



^{238}U fast fission

Fast ^{235}U and ^{239}Pu do not contribute to reactors, but fast TUNL data are suggestive that thermal ^{235}U and ^{239}Pu fission yields may be an issue



^{235}U fast

Summary

- There are two methods used to determine anti-neutrino spectra from reactors; summation of individual spectra and conversion of aggregate β -spectra
- The summation method suffers from lack of modern nuclear data
- The conversion of β -spectra suffers from lack of accurate treatments of the Fermi functions and the forbidden transitions.
- The Schreckenbach $^{235}\text{U}/^{239}\text{Pu}$ ratio appears to be inconsistent with reactor burnup measurements.
- Modern fission yield measurements from TUNL appear to provide an explanation for the spectral shapes and the bump.