INTERNATIONAL ATOMIC ENERGY AGENCY

INDC(NDS)-0473 Distr. SD/EL

INDC INTERNATIONAL NUCLEAR DATA COMMITTEE

WORKSHOP

ON NUCLEAR STRUCTURE AND DECAY DATA:

THEORY AND EVALUATION

ADDENDUM - 2005

Editors: A.L.Nichols and P.K.McLaughlin IAEA Nuclear Data Section Vienna, Austria

July 2005

IAEA NUCLEAR DATA SECTION, WAGRAMER STRASSE 5, A-1400 VIENNA

INDC documents may be downloaded in electronic form from http://www-nds.iaea.org/indc_sel.html or sent as an e-mail attachment. Requests for hardcopy or e-mail transmittal should be directed to services@iaeand.iaea.org or to:

Nuclear Data Section International Atomic Energy Agency PO Box 100 Wagramer Strasse 5 A-1400 Vienna Austria

Produced by the IAEA in Austria June 2005

WORKSHOP

ON NUCLEAR STRUCTURE AND DECAY DATA: THEORY AND EVALUATION

ADDENDUM - 2005

ICTP Trieste, Italy

4 - 15 April 2005

Edited by A.L. Nichols and P.K. McLaughlin IAEA Nuclear Data Section Vienna, Austria

Abstract

A two-week Workshop on Nuclear Structure and Decay Data under the auspices of the IAEA Nuclear Data Section was organised and administrated at the Abdus Salam International Centre for Theoretical Physics (ICTP) in Trieste, Italy from 4 to 15 April 2005. This workshop constituted a further development of previous Nuclear Structure and Decay Data Workshops held in 2002 and 2003. The aims and contents of this workshop are summarized, along with the agenda, list of participants, comments and recommendations. Most of the workshop material can be found in the INDC report of the equivalent workshop of 17 to 28 November 2003 (INDC(NDS)-452). However, some new material was prepared for 4 to 15 April 2005, and these new and modified lectures are brought together in this addendum report. All of this material is freely available on CD-ROM (all relevant PowerPoint presentations and manuals along with appropriate computer codes):

e-mail: services@iaeand.iaea.org fax: (+43-1)26007 post to: International Atomic Energy Agency Nuclear Data Section P.O. Box 100 Wagramer Strasse 5 A-1400 Vienna Austria

TABLE OF CONTENTS

1. SUMMARY	1
1.1 OBJECTIVES	3
1.2 PROGRAMME	3
1.2.1 Agenda	3
1.2.2 List of participants	7
1.3 Presentations available in electronic form on cd-rom	13
1.4 Other workshop materials on cd-rom	13
1.5 ADDENDUM MANUAL	13
1.6 RECOMMENDATIONS AND CONCLUSIONS	13
ACKNOWLEDGEMENTS	14
REFERENCE	15
2. NUCLEAR THEORY	17
2.1 Notes on IBM-1 programmes	17
2.2 Interacting Boson Model	25
2.3 High spin states	51
2.4 Structure of odd-odd nuclei	79
3. EXPERIMENTAL NUCLEAR STRUCTURE PHYSICS	117
3.1 Lecture I – Experimental Nuclear Structure Physics	135
3.2 Lecture II – Experimental Nuclear Structure Physics at the extreme	177

WORKSHOP

ON NUCLEAR STRUCTURE AND DECAY DATA: THEORY AND EVALUATION – ADDENDUM, 2005

Summary

ICTP Trieste, Italy 4 – 15 April 2005

Prepared by A.L. Nichols IAEA Nuclear Data Section Vienna, Austria

Abstract

Basic aspects of a two-week Workshop on Nuclear Structure and Decay Data: Theory and Evaluation are outlined in this short summary note for the record. The aims and contents of this workshop are summarized, along with the agenda, list of participants, comments and recommendations. Much was achieved and one aim will be to hold this specific workshop at various time intervals for training purposes (with agreed changes and regular modifications) on the advice of the International Nuclear Data Committee (INDC) and the International Network of Nuclear Structure and Decay Data Evaluators.

July 2005

1.1 **OBJECTIVES**

The International Atomic Energy Agency sponsored a two-week Workshop on "Nuclear Structure and Decay Data: Theory and Evaluation" at the Abdus Salam International Centre for Theoretical Physics (ICTP) in Trieste from 4 to 15 April 2005. This workshop was organised and directed by A.L. Nichols (IAEA Nuclear Data Section), J. Tuli (NNDC, Brookhaven National Laboratory, USA) and A. Ventura (ENEA, Bologna, Italy).

As with earlier workshops [1,2], the primary objective was to familiarize nuclear physicists and engineers from both developed and developing countries with

- (i) modern nuclear models;
- (ii) relevant experimental techniques;
- (iii) statistical analyses procedures to derive recommended data sets;
- (iv) evaluation methodologies for nuclear structure and decay data;
- (v) international efforts to produce the Evaluated Nuclear Structure Data File (ENSDF).

Reliable nuclear structure and decay data are important in a wide range of nuclear applications and basic research. Participants were introduced to both the theory and measurement of nuclear structure data, and the use of computer codes to evaluate decay data.

Detailed presentations were given by invited lecturers, along with computer exercises and workshop tasks. Participants were also invited to contribute their own thoughts and papers of direct relevance to the workshop.

1.2 PROGRAMME

The workshop programme is listed in Section 1.2.1 of this brief summary.

1.2.1 Agenda

Monday, 4 April 2005

08:30 - 10:30	Registration and coffee
10:30 - 12:30	Opening Session Welcome (Alan Nichols (IAEA) and Jag Tuli (BNL)) Aims (Jag Tuli) NSDD – general features (Jag Tuli) IAEA-NDS – NSDD network and recent relevant CRPs (Alan Nichols)
12:30 - 14:00	Lunch break
14:00 - 15:30 15:30 - 16:00	Introduction ICTP computer facilities (Johannes Grassberger/Kevin McLaughlin) Coffee break
16:00 – 17:30	Introduction (cont.) Web capabilities (Tom Burrows and Alan Nichols) Bibliographic databases (Tom Burrows)

Tuesday, 5 April 2005

09:00 - 10:30	Nuclear theory (Piet Van Isacker)
10:30 - 11:00	Coffee break
11:00 - 12:30	ENSDF format + model exercises (Jag Tuli)
12:30 - 14:00	Lunch break
14:00 - 15:30	ENSDF programs (Tom Burrows)
15:30 - 16:00	Coffee break
16:00 – 17:30	Workshop activities (JagdishTuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)

Wednesday, 6 April 2005

09:00 - 10:30	Nuclear theory (Piet Van Isacker)
10:30 - 11:00	Coffee break
11:00 - 12:30	Experimental techniques (Peter von Brentano)
12:30 - 14:00	Lunch break
14:00 – 15:30	ENSDF programs+model exercise (Tom Burrows)
15:30 – 16:00	Coffee break
16:00 – 17:30	Students' presentations

Thursday, 7 April 2005

09:00 - 10:30 10:30 - 11:00 11:00 - 12:30	Experimental techniques (Peter von Brentano) Coffee break ENSDF – decay (Eddie Browne)
12:30 - 14:00	Lunch break
14:00 – 15:30 15:30 – 16:00 16:00 – 17:30	Model exercise – decay (lead by Eddie Browne) Coffee break Workshop activities (JagdishTuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)

Friday, 8 April 2005

09:00 - 10:30	ENSDF - evaluation (Jag Tuli)
10:30 - 11:00	Coffee break
11:00 - 12:30	ENSDF- reaction (Coral Baglin)
12:30 - 14:00	Lunch break
14:00 – 15:30	Model exercise- reaction (lead by Coral Baglin)
15:30 – 16:00	Coffee break
16:00 – 17:30	Students' presentations

Monday, 11 April 2005

09:00 - 10:30	ENSDF – Theory (Slobodan Brant)
10:30 - 11:00	Coffee break
11:00 - 12:30	ENSDF- adopted (Coral Baglin)
12:30 - 14:00	Lunch break
14:00 – 15:30	Workshop activities (JagdishTuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)

15:30 - 16:00	Coffee break
16:00 - 17:30	Students' presentations

Tuesday, 12 April 2005

09:00 - 10:30	ENSDF – Theory (Slobodan Brant)
10:30 - 11:00	Coffee break
11:00 – 12:30	Model exercises- adopted (Coral Baglin)
12:30 - 14:00	Lunch break
14:00 – 15:30	Workshop activities (JagdishTuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)
15:30 - 16:00	Coffee break
16:00 – 17:30	Workshop activities (JagdishTuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)

Wednesday, 13April 2005

09:00 - 10:30	ENSDF – Experimental techniques (Filip Kondev)
10:30 - 11:00	Coffee break
11:00 - 12:30	Data analyses (Desmond MacMahon)
12:30 - 14:00	Lunch break
14:00 – 15:30	Workshop activities (JagdishTuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)
15:30 - 16:00	Coffee break
16:00 – 17:30	Workshop activities (JagdishTuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)

Thursday, 14April 2005

12:30

09:00 - 10:30 10:30 - 11:00	ENSDF – Experimental techniques (Filip Kondev)
11:00 – 12:30	Data analyses (Desmond MacMahon)
12:30 - 14:00	Lunch break
14:00 – 15:30	Workshop activities (JagdishTuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)
15:30 - 16:00	Coffee break
16:00 – 17:30	Workshop activities (JagdishTuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)
Friday, 15 April 2005	
09:00 - 10:30	Workshop activities (JagdishTuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)
10:30 - 11:00	Coffee break
11:00 - 12:30	Review of workshop (JagdishTuli; Thomas Burrows; Eddie Browne;

0	Keview of workshop (Jaguish Tun, Thomas Durlows, Le	uu.
	Alan Nichols)	
	Close of workshop	

1.2.2 Participants

Twenty-seven participants (predominantly from developing countries) with full or partial support from the IAEA were selected to attend the workshop in April 2005. Selection was undertaken by Nuclear Data Section staff in association with the workshop directors and ICTP staff.



First row, standing from left to right:

Coral M. BAGLIN (USA), Elie SIMO (Cameroon), Elizabeth Brancaccio (Seated, ITALY), Ghania ISHAKI BOUSHAKI (Algeria), Caroline NESARAJA (USA), Mane MANE (Cameroon), Edgardo BROWNE-MORENO (USA), Mohamed KOUTHER (Sudan), Jagdish K. TULI (USA), Daniel ABRIOLA (Argentina),

Second row, standing from left to right:

Mirshod ERMAMATOV (Ubekistan), Stefan LALKOVSKI (Bulgaria), Adam GARNSWORTHY (UK), Nicholas THOMPSON (UK), Omidreza KAKUEE (Iran), Aurelian LUCA (Romania), Sami HADDAD (Syria), Marialena AVRIGEANU (Romania), Marie Martine BE (France), Thomas W. BURROWS (USA)

Third row, standing from left to right:

Kyoung LEE (Korea), Xiongiun CHEN (China), Nure ABDULLAH (Bangladesh), Alan NICHOLS (IAEA), Kevin MCLAUGHLIN (IAEA), Alexey CHEMEZOV (Russia), Alexander RODIONOV (Russia), Anwesa GHOSH (India)

Fourth row, standing from left to right:

Ruy CASTRO (Brazil), Ninel NICA (Romania), Peter VON BRETANO (Germany), Piet VAN ISACKER (France), Ameeya BHAGWAT (India), Sukhjeet DHINDSA (India)

LIST OF PARTICIPANTS

ABDULLAH Md. Nure Alam

Rajshahi University of Engineering & Technology (RUET) Department of Physics Rajshahi 6204 BANGLADESH E-mail mnaa031@yahoo.com

ABRIOLA Daniel Hugo Comision Nacional de Energia Atomica (CNEA) Centro Atomico Constituyentes Av. del Libertador 8250 1429 Buenos Aires ARGENTINA

AVRIGEANU (BOATA) Marilena

National R & D Institute For Physics and Nuclear Engineering 'Horia Hulubei' Str.. Atomistilor No.1 Magurele. Sector 5 79617 Bucharest ROMANIA E-mail <u>MVRIGI@ IFIN.NIPNE.RO</u>

BE Marie Martine Lab. National Henri Bequerel 911911 Gif sur Yvette Cedex FRANCE E-mail <u>mmbe@cea.fr</u>

BHAGWAT Ameeya Ashok

Indian Institute of Technology Dept. of Physics Powai Maharashtra 400 076 Mumbai INDIA E-mail <u>ameeya@phy.iitb.ac.in</u>

CASTRO Ruy Morgado

Centro Tecnico Aerospacial (Cta) Istituto de Estudos Avancados (leav) D. Physica Teorica Caixa Postal 6044 12231-970 Sao Jose Dos Campos BRAZIL E-mail rmcastro@ieav .cta.br

CHEMEZOV Alexey Victor

Petersburg Nuclear Physics Institute, PNPI Gatchina Orlova Rosha 1 188300 St Petersburg RUSSIAN FEDERATION E-mail chemezov2004@mail.ru

CHEN Xiongjun

China Institute of Atomic Energy (C.I.A.E.) Chinese Nuclear Data Centre P.O. Box 275- 41 102413 Beijing PEOPLE'S REPUBLIC OF CHINA E-mail <u>chenxj@iris.ciae.ac.cn</u>

DHINDSA Sukhjeet Singh

Guru Nanak Dev University Department of Physics 143005 Amritsar INDIA E-mail <u>dcse-gndu@yahoo.com</u>

EISA Mohamed Eltayeb

Sudan University of Science & Technology College of Science Department of Physics P .O. Box 9024 11111 Khartoum SUDAN E-mail MEMEISA@YAHOO.COM

ERMAMATOV Mirshod

Uzbek Academy of Sciences Institute of Nuclear Physics Department Theoretical Physics Ulugbek 702132 Tashkent UZBEKISTAN E-mail ermamat@inp.uz

GARNSWORTHY Adam Benjamin

University of Surrey Department of Physics GU2 7XH Guildford UNITED KINGDOM E-mail <u>a.garnsworthy@surrey.ac.uk</u>

GHOSH Anwesha

Indian Institute of Technology Roorkee {Former University of Roorkee} Department of Physics 247 667 Roorkee INDIA E-mail <u>anwesha-tua@yahoo.co.in</u>

HADDAD Sami

Atomic Energy Commission Department of Physics P .0. Box 6091 Gazaoui Street, 13 Damascus SYRIAN ARAB REPUBLIC E-mail <u>shaddad@aec.org.sy</u>

ISHAKI BOUSHAKI {MEDKOUR} Ghania

University of Sciences and Technology Houari Boumedienne Institute of Physics B.P. 32 El Alia Bab Ezzouar Algiers ALGERIA E-mail <u>gmedkour@yahoo.com</u>

KAKUEE Omidreza

Nuclear Research Center Karegar Ave P.O. Box 14155-1339 Tehran ISLAMIC REPUBIC OF IRAN E-mail <u>okakuee@aeoi.org.ir</u>

KOUTHER Mohamed Elhaj

Sudan University of Science & Technology College of Science Department of Physics P.O. Box 9024 11111 Khartoum SUDAN

LALKOVSKI Stefan Peshev University of Sofia 'St. Kliment Ohridski' Faculty of Physics Dept. of Atomic Physics 5 James Bourchier Blvd. 1164 Sofia BULGARIA E-mail <u>stl@phys.uni-sofia.bg</u>

LEE Kyoung

Korea Research Institute of Standards and Science Ionizing Radiation Group P.O. Box 102 Yuseong Daejeon 305-600 REPUBLIC OF KOREA E-mail lee@kriss.re.kr

LUCA Aurelian

National R & D Institute For Physics and Nuclear Engineering 'Horia Hulubei' Str.. Atomistilor No.1 Magurele. Sector 5 7 9617 Bucharest ROMANIA E-mail aluca@ifin.nipne.ro

MANE MANE

Ecole Nationale Superieure Polytechnique University of Yaounde'l Dept. of Maths and Physical Sc Bp 8390 Yaounde REPUBLIC OF CAMEROON

NESARAJA Caroline Delini

University of Tennessee Departments of Materials Science! Physics and Astronomy 218 South College Knoxville 37996-1508 TN UNITED STATES OF AMERICA E-mail <u>nesaraja@mail.phy.ornl.gov</u> NICA Ninel National R & D Institute For Physics and Nuclear Engineering 'Horia Hulubej' Str.. Atomistilor No.1 Magurele. Sector 5 79617 Bucharest ROMANIA E-mail <u>nica@comp.tamu.edu</u> Until when: 1 August 2006

RODIONOV Alexander

St. Petersburg Nuclear Physics Institute Neutron Research Department Pnpi Gatchina Leningrad Distrct. 188350 St Petersburg RUSSIAN FEDERATION E-mail Oz@rambler.ru

SIMO Elie

Universite' de Yaounde' I Faculte' des Sciences Departement de Physique P.O. Box 6052 Yaounde REPUBLIC OF CAMEROON E-mail <u>esimoch@yahoo.fr</u>

THIODJIO SENDJA Bridinette

Universite' de Yaounde' I Faculte' des Sciences Departement de Physique P.O. Box 6052 Yaounde REPUBLIC OF CAMEROON E-mail tsbridinette@yahoo.fr

THOMPSON Nicholas James

University of Surrey Department of Physics GU2 IXH Guildford UNITED KINGDOM E-mail M.Thompson@surrey.ac.uk

LIST OF LECTURERS

Ms. Coral M. BAGLIN

Nuclear Science Division Lawrence Berkeley National Laboratory University of California 1 Cyclotron Road MS 88R0192 Berkeley, CA 94720 USA Tel: +1-510-486-6152 Fax: +1-510-486-5757 E-mail: <u>cmbaglin@lbl.gov</u>

Mr. Slobodan BRANT

Permanent Institute: University of Zagreb Faculty of Science Department of Physics Bijenicka Cesta 32 P.O. Box 162 10000 Zagreb CROATIA E_mail Brant@phy.hr

Mr. Edgardo BROWNE-MORENO

Nuclear Science Division Lawrence Berkeley National Laboratory University of California 1 Cyclotron Road MS 88R0192 Berkeley, CA 94720-8101 USA Tel: +1-510-486-7647 Fax: +1-510-486-5757 E-mail: ebrowne@lbl.gov

Mr. Thomas W. BURROWS

National Nuclear Data Center Building 197D Brookhaven National Laboratory P.O. Box 5000 Upton, NY 11973-5000 USA Tel: +1 631 344 5084 Fax: +1 631 344 2806 E-mail: burrows@bnl.gov

Mr. Filip G KONDEV

Permanent Institute: Argonne National Laboratory Nuclear Data Program Nuclear Engineering Division 9700 South Cass Avenue IL 60439 Argonne Argonne IL 60439 UNITED STATES OF AMERICA E-mail: <u>kondev@anl.gov</u>

Mr. Desmond MACMAHON

Centre for Acoustics and Ionising Radiation National Physical Laboratory Queens Road Teddington, Middlesex TW11 0LW UNITED KINGDOM Tel: +44-20-8943-8573 Fax: +44-20-8943-6161 E-mail: desmond.macmahon@npl.co.uk

Mr. Jagdish K. TULI (Director)

National Nuclear Data Center Building 197D Brookhaven National Laboratory P.O. Box 5000 Upton, NY 11973-5000 USA Tel: +1-631-344-5080 Fax: +1-631-344-2806 E-mail: tuli@bnl.gov

Mr. Piet VAN ISACKER

Groupe Physique Grand Accelerateur National d'Ions Lourds (GANIL) BP 55027 F-14076 Caen Cedex 5 FRANCE Tel: +33-2-31 45 45 65 Fax: +33-2-31 45 44 21 E-mail: isacker@ganil.fr

Mr. Peter VON BRENTANO

Institut für Kernphysik der Universität Köln Zülpicher Strasse 77 D-50937 Köln GERMANY Tel: +49-221-470-6960 Fax: +49-221-470-5168 E-mail: brentano@ikp.uni-koeln.de

IAEA STAFF

Mr. Alan L. **NICHOLS** (Director) Nuclear Data Section International Atomic Energy Agency Wagramerstrasse 5 A-1400 Vienna Austria Tel: +43-1-2600-21709 Fax: +43-1-26007 E-mail: <u>a.nichols@iaea.org</u> Mr. Kevin **MCLAUGHLIN** (Tutor) Nuclear Data Section International Atomic Energy Agency Wagramerstrasse 5 A-1400 Vienna Austria Tel: +43-1-2600-21713 Fax: +43-1-26007 E-mail: p.mclaughlin@iaea.org

1.3 PRESENTATIONS AVAILABLE IN ELECTRONIC FORM ON CD-ROM

Presentations by Lecturers

Aims of the Workshop - General features of NSDD, J. Tuli

Nuclear Theory: Nuclear Shell Model, P. Van Isacker (November 2003) Interacting Boson Model, P. Van Isacker Structure of the odd-even nuclei in the interacting boson model, S. Brant (April 2005) High spin states in the interacting boson and boson-fermion model, S. Brant (April 2005) Structure of odd-odd nuclei in the interacting boson-fermion-fermion model, S. Brant (April 2005) β decay in the interacting boson-fermion model, S. Brant (April 2005) Geometrical Symmetries in Nuclei – An Introduction, A. Jain ((November 2003) Geometrical Symmetries in Nuclei, A. Jain (November 2003) Lectures on Geometrical Symmetries in Nuclei, A. Jain (November 2003) Hartree-Foch-Bogoliubov Method, D. Vretenar (November 2003) Self-consistent Mean-field Models – Structure of Heavy Nuclei, D. Vretenar (November 2003)

Experimental Nuclear Spectroscopy:

Introduction, P. Von Brentano

Lecture I – Nuclear Shapes, P. Von Brentano

Lecture II - Measurement of Lifetimes, P. Von Brentano

Lecture I – Experimental Nuclear Structure Physics, F. Kondev (April 2005)

Lecture II – Experimental Nuclear Structure Physics at the extreme, F. Kondev (April 2005)

Statistical Analyses:

Evaluation of Discrepant Data I, D. MacMahon

Evaluation of Discrepant Data II, D. MacMahon

Convergence of Techniques for the Evaluation of Discrepant Data: D. MacMahon, A. Pearce, P. Harris

Techniques for Evaluating Discrepant Data, M.U. Rajput, D. MacMahon

Possible Advantages of a Robust Evaluation of Comparisons, J.W. Muller (presented by D. MacMahon)

ENSDF:

Evaluated Nuclear Structure Data Base, J.K. Tuli Evaluations – A Very Informal History, J.K. Tuli Evaluated Nuclear Structure Data File – A Manual for Preparation of Data Sets, J.K. Tuli Guidelines for Evaluators, M.J. Martin, J.K. Tuli Bibliographic Databases, T.W. Burrows

ENSDF Analysis and Utility Codes, T.W. Burrows:

- Their Descriptions and Uses, T.W. Burrows
- FMTCHK (Format and Syntax Checking), T.W. Burrows
- PowerPoint presentations, T.W. Burrows
- LOGFT (Calculates log ft for beta decay), T.W. Burrows
- GTOL (Gamma to Level), T.W. Burrows
- HSICC (Hager-Seltzer Internal Conversion Coefficients), T.W. Burrows

ENSDF – Decay Data, E. Browne

Model Exercises - Decay, E. Browne

- ENSDF Reaction Data, C. Baglin
- ENSDF Adopted Levels and Gammas, C. Baglin

ENSDF – Examples 1, 2, 3, 4 and 5, C. Baglin

Additional Material:

IAEA: NSDD Network, Recent Relevant CRPs and Other Activities (PowerPoint presentation), A.L. Nichols

IAEA: NSDD Network, Recent Relevant CRPs and Other Activities (draft paper), A.L. Nichols

Nuclear Structure and Decay Data: Introduction to Relevant Web Pages (draft paper), T.W. Burrows, P.K. McLaughlin, A.L. Nichols

Presentations by Participants

Compton Add-Back Protocols for use with the EXOGAM Array, A. Garnswothy Experimental determination of photon emission probabilities, A. Luca Nuclear data activities for Astrophysics at Oak Ridge National Laboratory, C. Nesaraja Tandar Laboratory, CNEA. Argentina, D. Abriola Experimental approach to the dynamics of fission, G. Ishak Boushaki Laboratoire National Henri Becquerel, M.M. Be Nuclear structure by gamma-ray spectroscopy, a completeness perspective, N. Nica Radioactive beam spectroscopy of ²¹²Po and ²¹³At with the EXOGAM array, N. Thompson Developing ¹⁵²Eu into a standard for detector efficiency calibration, R.M. Castro

1.4 OTHER WORKSHOP MATERIALS ON CD-ROM

Atomic Masses Access to ENSDF Codes and Tools Isotope Explorer PCNuDat Access to NSDD Resources

NNDC Online Data Service Manual and Data Citation Guidelines

Introduction to International Nuclear Structure and Decay Data Network Contact names and addresses

Access to ENSDF Format Summary and Examples

Nuclear Structure Manuals

1.5 ADDENDUM MANUAL

Significant quantities of written material were prepared for the workshop. Their accumulation in various forms acted as an aid to the participants in their understanding of nuclear theory, measurement techniques, data analysis and ENSDF mass-chain evaluations, representing an important combination of technical information for future reference and other NSDD workshops. Therefore, a relatively large fraction of these presentations, background papers and manuals have been assembled for further use in the form of an earlier document [2] and this Addendum report.

Our intention is to use and develop this material in the years to come, particularly for other workshops of this type. Another aim is to ensure that such presentations are not lost, and can be readily at hand for new mass-chain and decay-data evaluators to assist them in their preparation of recommended data for the ENSDF files.

1.6 RECOMMENDATIONS AND CONCLUSIONS

A number of important points can be made concerning the workshop:

1. Twenty-seven participants were selected and attended a two-week workshop that covered nuclear theory and modeling, relevant experimental techniques, statistical analyses, and the philosophy and methodology for comprehensive mass chain evaluations. Support materials and information were also provided on the International Network of Nuclear Structure and Decay Data Evaluators and the most relevant CRPs organized by the IAEA Nuclear Data Section.

2. Workshop participants were introduced to mass chain evaluations through group and individual PC/computing activities (50% of the agenda of the second week) CD-ROM and hardcopy materials were provided by IAEA staff for all students/lecturers.

3. Administrative functions leading up to and during the course of the workshop worked smoothly, including visa arrangements, travel and subsistence payments to students and lecturers, additional banking transactions, and hotel/guest-house accommodation.

4. Specific participants were identified for future involvement in NSDD and mass chain evaluations.

5. Further lessons were learnt by the IAEA staff and lecturers involved in this ICTP workshop, and much experience was gained in ensuring future success in the organization of such events. This particular workshop ran extremely smoothly, and all participants were able to attend (i.e., 100% success with visas). Students were given the opportunity to review the workshop through a written questionnaire and direct discussions (on 15 April). Their major recommendations are as follows:

(a) $1\frac{1}{2}$ hour lectures should be broken down further to 2 x 40 min plus 10 min break;

- (b) provision of sample questions and answers prior to the workshops (answers also to be worked out during the course of individual lectures ICTP to note);
- (c) forewarn participants that they will be asked to give a short presentation on their own nuclear physics studies (ICTP to note);
- (d) every participant/student to operate their own individual PC, rather than share (ICTP to note);
- (e) outside activities during the middle weekend (ICTP to note);
- (f) further development of nuclear theory lectures beyond just IBM;
- (g) introduce ENSDF format to participants prior to the workshop (through IAEA-NDS web pages?);
- (h) additional presentations of the details of XUNDL and NSR databases;
- (i) allow an afternoon off from the intensive training (first Friday afternoon?).

As before, this combination of Wednesday/Thursday written questionnaire and Friday face-to-face review session produced constructive feedback. The overall opinion of all of the students was that they had thoroughly enjoyed the 2-week workshop, made useful new contacts with lecturers, IAEA-NDS staff and other students, and learnt much about nuclear structure and decay data; all of the primary objectives of the workshop were successfully achieved.

ACKNOWLEDGEMENTS

The authors wish to thank our fellow co-directors of the NSDD Workshop for their support leading up to April 2005, and particularly the lecturers (all experts in their fields) for their enthusiasm during the workshop and provision of the various technical input to this document. Administrative aspects of the workshop were considerable leading up to and during the course – as an ICTP-supported activity, all such features and problems were handled by Ms Elizabeth Brancaccio (ICTP), and her efforts were much appreciated. Finally, none of the lectures and associated

materials would have been delivered without the enthusiastic involvement of all participants at this workshop.

REFERENCE

1. PRONYAEV, V.G., NICHOLS, A.L., Summary Report on Workshop on Nuclear Structure and Decay Data Evaluation, 18-22 November 2002, INDC(NDS)-439, January 2003.

2. NICHOLS, A.L., MCLAUGHLIN, P.K., Workshop on Nuclear Structure and Decay Data: Theory and Evaluation, Manual, Parts 1 and 2, INDC(NDS)-452, November 2004.

Nuclear Theory: Notes on IBM-1 programmes

P. Van Isacker

GANIL, France

E.mail: isaker@ganil.fr

Notes on IBM-1 programmes

These notes contain a short and practical introduction on how to do numerical calculations with an IBM-1 code. Only the simplest version of the model is explained which corresponds to N identical s and d bosons. Numerical codes are available for several extensions of this approach, such as IBM-2 and IBM-3 or IBFM, and they function along similar principles.

1. Getting started

To run the IBM-1 programmes (energies and transitions), you should have the following files:

- cfp20.for: Fortran source of the programme that constructs a data file with the coefficients of fractional parentage (cfp's). This programme is selfcontained: no library is required with which to link it and no input file is required to run it. It produces two output files:
 - write(4): output file to be saved containing the cfp's (suggested name: cfp20.dat);
 - write(6): large output file that can be consulted should problems arise but can be discarded otherwise.

The programme is currently set for producing a data file so that a calculation up to $20 \ s$ and d bosons can be performed.

- ibm1.for: Fortran source of the programme that diagonalises an IBM-1 hamiltonian (*i.e.*, determines its eigenspectrum) with wave functions that are stored for subsequent use with ibm1t. The basis and the hamiltonian are described in section 2. It should be linked with ibm1 and eig500. Input and output files are:
 - read(3): input data file containing cfp's, usually named cfp20.dat;
 - read(5): input data file ibm1a.inp or ibm1b.inp;
 - write(1): output file containing the wave functions in unformatted form to be used by ibm1t;
 - write(6): output file summarising the results of the calculation.
- ibml.for: Fortran source of library routines to be linked with ibm1 and ibm1t.
- eig500.for: Fortran source of the diagonalisation routine to be linked with ibm1. The maximum dimension the matrix that can be diagonalised is set to 500.

- ibmlt.for: Fortran source of the programme that calculates E0 and E2 transition matrix elements and probabilities. The transition operators are described in section 3. It should be linked with ibml. Input and output files are:
 - read(3): input data file containing cfp's, usually named cfp20.dat;
 - read(5): input data file ibm1t.inp;
 - write(6): output file summarising the results of the calculation.
- ibm1a.inp: Input file for ibm1 with the two-body hamiltonian in multipole expansion.
- ibm1b.inp: Input file for ibm1 with the two-body hamiltonian in "standard" representation.
- ibm1t.inp: Input file for ibm1t.

2. Running ibm1

The first 20 characters on each input line are for description purposes: they contain the name(s) of the variable(s) that is (are) read on that line. Integer variables (NMIN, NMAX, LMIN, LMAX, MVMAX, I1, I2, I3, I4) are in I2, logical variables (MULT, CUB, DEF) are in L1 (*i.e.*, T for .TRUE. and F for .FALSE.) and real variables (all others) are in F9.6.

The integer and logical variables are defined as follows:

- NMIN, NMAX: minimum and maximum values of the number of bosons N;
- LMIN, LMAX: minimum and maximum values of the angular momentum L;
- LVMAX: number of levels calculated for each angular momentum;
- I1, I2, I3, I4: print control parameters with
 - all four zero: minimal output with relative energies;
 - I1 = 1: absolute energies;
 - I2 = 1: probability distribution in N;
 - I2 = 2: probability distribution in N and in n_d ;
 - I3 = 1: wave functions in a vibrational basis;
 - I4 = 1: hamiltonian matrix.
- MULT: whether the hamiltonian is in multipole or standard representation;
- CUB: whether three-body interactions are included or not;

• DEF: whether the deformation parameter γ is calculated or not.

The real variables are explained below as well as in the source code ibm1.for.

With the ibm1 programme one can perform "normal" calculations; these are described in section 2.1. Some extensions that are possible with ibm1 are summarised in section 2.2.

2.1. Standard IBM1 calculations

The following choice of parameters corresponds to a standard IBM-1 calculation:

- NMIN = NMAX: the total number of bosons N;
- CUB = .FALSE.: no three-body interactions;
- DEF = .FALSE.: no calculation of the deformation parameter γ .

It this case all lines after O6 (in ibm1a.inp) or U0 (in ibm1b.inp) are ignored.

The basis is the usual one in IBM-1, which consists of N s or d bosons. The hamiltonian can be specified in multipole form (if MULT = .TRUE.) or in terms of single-boson energies and two-body boson interaction matrix elements (in "standard" representation, if MULT = .FALSE.). The hamiltonian in multipole expansion is of the form

$$\hat{H} = ext{EPS}\,\hat{n}_d + ext{A}(0)\hat{P}^\dagger\hat{P} + ext{A}(1)\hat{L}\cdot\hat{L} + ext{A}(2)\hat{Q}_\chi\cdot\hat{Q}_\chi + ext{A}(3)\hat{T}_3\cdot\hat{T}_3 + ext{A}(4)\hat{T}_4\cdot\hat{T}_4,$$

where \cdot denotes a scalar product, $\hat{T}_{\lambda} \cdot \hat{T}_{\lambda} \equiv \sum_{\mu} \hat{T}_{\lambda\mu} \hat{T}_{\lambda\mu}$, and where the following multipole operators appear:

- *d*-boson number operator $\hat{n}_d = d^{\dagger} \cdot \tilde{d} = \sqrt{5} [d^{\dagger} \times \tilde{d}]^{(0)};$
- pairing operator $\hat{P}^{\dagger} = \frac{1}{2}(s^{\dagger}s^{\dagger} 06 d^{\dagger} \cdot d^{\dagger});$
- angular momentum operator $\hat{L} = \sqrt{10} [d^{\dagger} \times \tilde{d}]^{(1)};$
- quadrupole operator $\hat{Q}_{\chi} = [s^{\dagger} \times \tilde{d} + d^{\dagger} \times \tilde{s}]^{(2)} + \text{CHI}[d^{\dagger} \times \tilde{d}]^{(2)};$
- octupole operator $\hat{T}_3 = [d^{\dagger} \times \tilde{d}]^{(3)};$
- hexa decapole operator $\hat{T}_4 = [d^\dagger \times \tilde{d}]^{(4)}.$

The hamiltonian in standard representation is of the form

$$\begin{split} \hat{H} &= & \mathsf{C}(1)\hat{N} + \mathsf{C}(2)\hat{n}_d + \mathsf{C}(3)\frac{1}{2}[[d^{\dagger} \times d^{\dagger}]^{(0)} \times [\tilde{d} \times \tilde{d}]^{(0)}]^{(0)} \\ &+ \mathsf{C}(4)\sqrt{5}\frac{1}{2}[[d^{\dagger} \times d^{\dagger}]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}]^{(0)} \\ &+ \mathsf{C}(5)\frac{3}{2}[[d^{\dagger} \times d^{\dagger}]^{(4)} \times [\tilde{d} \times \tilde{d}]^{(4)}]^{(0)} \end{split}$$

$$\begin{split} + \mathbf{C}(6) &[[s^{\dagger} \times d^{\dagger}]^{(2)} \times [\vec{d} \times d]^{(2)} + [d^{\dagger} \times d^{\dagger}]^{(2)} \times [\tilde{s} \times d]^{(2)}]^{(0)} \\ + \mathbf{C}(7) &[[s^{\dagger} \times s^{\dagger}]^{(0)} \times [\vec{d} \times \vec{d}]^{(0)} + [[d^{\dagger} \times d^{\dagger}]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)}]^{(0)} \\ + \mathbf{C}(8) \sqrt{5} &[[s^{\dagger} \times d^{\dagger}]^{(2)} \times [\tilde{s} \times \vec{d}]^{(2)}]^{(0)} \\ + \mathbf{C}(9) &[[s^{\dagger} \times s^{\dagger}]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)}]^{(0)}. \end{split}$$

All parameters have units of energy, except CHI and O6 which are dimensionless.

2.2. Extensions

There are a number of options which allow the definition of a more general IBM-1 hamiltonian.

• Multi-particle-hole excitations. If NMIN \neq NMAX, the total boson number N is not a constant but can take the values N =NMIN, NMIN + 2, ..., NMAX. The different N values correspond to 0p-0h, 2p-2h, ..., 2np-2nh excitations, and each configuration is specified by its IBM-1 hamiltonian. Configurations differing by two bosons (e.g., N and N + 2) are mixed by the following hamiltonian

$$\texttt{ALPHA}(s^{\dagger}s^{\dagger} + \tilde{s}\tilde{s}) + \texttt{BETA}(d^{\dagger} \cdot d^{\dagger} + \tilde{d} \cdot \tilde{d}),$$

and, furthermore, configurations are separated by an offset DELTA.

• Three-boson interactions. The most general IBM-1 hamiltonian which contains all three-boson interactions can be considered if CUB = .TRUE. The three-boson interactions can be defined through matrix elements between normalised three-boson states in a vibrational or in an SU(3) basis. Definitions of these interactions are given in the source code ibm1.for. [There are two possible definitions of SU(3) basis, corresponding to SU3 = -1 (standard) or SU3 = +1.] Finally, with CUB = DEF = .TRUE., it is possible to add a Q^3 term

$$\mathbb{V}\mathbb{Q}\mathbb{Q}\mathbb{Q}[\bar{Q}_{\chi}\times\bar{Q}_{\chi}\times\bar{Q}_{\chi}]^{(0)},$$

with a quadrupole operator of the form

$$\hat{Q}_{\chi} = [s^{\dagger} \times \tilde{d} + d^{\dagger} \times \tilde{s}]^{(2)} + \text{CHIQ}[d^{\dagger} \times \tilde{d}]^{(2)}.$$

Note that all contributions to the hamiltonian are additive, which can thus be of one- + two- + three-body nature. However, if CUB = .FALSE., the three-body interactions are ignored; if either CUB = .FALSE. or DEF = .FALSE., the Q^3 term is ignored.

• Deformation parameter γ . For DEF = .TRUE. the programme calculates the deformation parameter γ in each eigenstate from the following expectation values in that state:

$$\langle [\hat{Q} \times \hat{Q}]_0^{(0)} \rangle = \sqrt{\frac{1}{5}} \beta^2, \qquad \langle [\hat{Q} \times \hat{Q} \times \hat{Q}]_0^{(0)} \rangle = -\sqrt{\frac{2}{35}} \beta^3 \cos 3\gamma.$$

The programme ibm1 also prints the classical limit (the potential in β and γ) of the one- and two-body part of the hamiltonian.

3. Running ibm1t

The first 20 characters on each input line are for description purposes: they contain the name(s) of the variable(s) that is (are) read on that line. The first line specifies NMIN and NMAX; their values should be consistent with those used in ibm1. The next two lines specify the boson E0 transition operator

$$T(\mathrm{E0}) = \mathrm{E0}(1)\hat{n}_s + \mathrm{E0}(2)\hat{n}_d,$$

where E0(1) and E0(2) are in units of efm. This operator can also be used for the calculation of nuclear radii. The next two lines specify the boson E2 transition operator

$$\hat{T}(\text{E2}) = \texttt{E2(1)}\left([s^{\dagger} \times \tilde{d} + d^{\dagger} \times \tilde{s}]^{(2)} + \texttt{E2(2)}[d^{\dagger} \times \tilde{d}]^{(2)}\right),$$

where E2(1) is a boson effective charge which has the units of $e \text{fm}^2$ and E2(2) is dimensionless. For E2(2) = CHI we obtain the so-called consistent-Q formalism. All subsequent input lines contain 7 integers (in I3) $L_1 n_{11} n_{12} L_2 n_{21} n_{22} \lambda$ with the following meaning:

- L_1 and L_2 : angular momenta of initial and final states;
- n_{i1} and n_{i2} : for angular momentum L_i , consider states from n_{i1} to n_{i2} ;
- λ : multipolarity of the transition (only $\lambda = 0, 2$ is implemented). If λ is negative, reduced matrix elements are computed, otherwise B(E2) values are given.

Nuclear Theory: Structure of the odd-even nuclei in the interacting boson model

S. Brant

University of Zagreb, Croatia

E.mail: Brant@phy.hr

Fermion degrees of freedom in the interacting boson model

Slobodan Brant

Department of Physics, Faculty of Science University of Zagreb Zagreb Croatia



1.

Structure of odd-even nuclei in the interacting boson fermion model

Shell model \longrightarrow matrix elements of the effective interaction between identical nucleons are strongly attractive when the two nucleons are in a J = 0 state and remain attractive when the two nucleons are in a J = 2 state. They become repulsive for $J \ge 4$.

Nucleons tend to form pairs with angular momentum J = 0 or J = 2



Generalized seniority scheme: generalization of the seniority scheme to several non-degenerate orbits. The number of active nucleons is counted in respect to the nearest closed shell (valence nucleons). Contributions from orbitals outside the valence shell can be neglected since they lie at a too high energy.

A collective J = 0 pair is generated by the operator

$$\begin{split} S^{\dagger} &= \sum_{j} \alpha_{j} S_{j}^{\dagger} \\ S_{j}^{\dagger} &= \frac{1}{2} \sqrt{2j+1} \ (c_{j}^{\dagger} \tilde{c}_{j})^{(0)} \end{split}$$

State with generalized seniority w = 0 and n = 2N particles

$$|n,\ J=0,\ w=0\rangle=(S^\dagger)^N\ |0\rangle$$

An excited 2^+ state is generated by the operator that creates a collective state with J=2 and w=2

$$D^{\dagger} = \sum_{jj'} \frac{1}{2} \beta_{jj'} \sqrt{1 + \delta_{jj'}} \ (c_j^{\dagger} \tilde{c}_{j'})^{(2)}$$

State with generalized seniority w = 2, J = 2 and n = 2N particles

$$|n, J=2, w=2\rangle = D^{\dagger}(S^{\dagger})^{N-1} |0\rangle$$

The structure coefficients α_j and $\beta_{jj'}$ can be obtained by diagonalizing the shell model interaction in the space of all w = 0, 2 states.

Instead of having to use the full shell model space, it is sufficient to consider the much smaller (S, D) subspace.

- Low-lying collective states can be described very well
- Non collective states can not be described
- The matrix elements of the fermion operators in the (S, D) subspace can be cumbersome
- The space built on S and D fermion pairs is mapped onto a corresponding space built on s and d boson degrees of freedom
- For states containing more than one *D* fermion pair, we have to map the components of the state which is orthogonal to all the states containing fewer *D* fermion pairs.



- By equating matrix elements in (*S*, *D*) and (*s*, *d*) spaces, the operators in the (*s*, *d*) space are obtained.
- Since the S and D fermion pairs are always pairs of like nucleons (two protons or two neutrons), one has proton (s_{π}, d_{π}) and neutron (s_{ν}, d_{ν}) bosons. The model is called IBM-2.
$$H_B = \epsilon_d \left(\hat{n}_{d_\nu} + \hat{n}_{d_\pi} \right) + \kappa \left(Q_\nu^B \cdot Q_\pi^B \right) + M_{\nu\pi} + V_{\nu\nu} + V_{\pi\pi}$$

$$Q_{\nu}^{B} = d_{\nu}^{\dagger} s_{\nu} + s_{\nu}^{\dagger} \tilde{d}_{\nu} + \chi_{\nu} [d_{\nu}^{\dagger} \tilde{d}_{\nu}]^{(2)}$$
$$Q_{\pi}^{B} = d_{\pi}^{\dagger} s_{\pi} + s_{\pi}^{\dagger} \tilde{d}_{\pi} + \chi_{\pi} [d_{\pi}^{\dagger} \tilde{d}_{\pi}]^{(2)}$$

$$M_{\nu\pi} = \frac{1}{2} \xi_2 \left((d_{\nu}^{\dagger} s_{\pi}^{\dagger} - d_{\pi}^{\dagger} s_{\nu}^{\dagger}) \cdot (\tilde{d}_{\nu} s_{\pi} - \tilde{d}_{\pi} s_{\nu}) \right) - \sum_{K=1,3} \xi_K \left([d_{\nu}^{\dagger} d_{\pi}^{\dagger}]^{(K)} \cdot [\tilde{d}_{\nu} \tilde{d}_{\pi}]^{(K)} \right) V_{\nu\nu} = \frac{1}{2} \sum_{L=0,2,4} c_L^{\nu} \left([d_{\nu}^{\dagger} d_{\nu}^{\dagger}]^{(L)} \cdot [\tilde{d}_{\nu} \tilde{d}_{\nu}]^{(L)} \right)$$

$$V_{\pi\pi} = \frac{1}{2} \sum_{L=0,2,4} c_L^{\pi} \left([d_{\pi}^{\dagger} d_{\pi}^{\dagger}]^{(L)} \cdot [\tilde{d}_{\pi} \tilde{d}_{\pi}]^{(L)} \right)$$

- The major part of the interaction between like particles is contained in the boson energies and a smaller in the $V_{\nu\nu}$ and $V_{\pi\pi}$ terms.
- The $Q^B_{\nu} \cdot Q^B_{\pi}$ interaction is the boson image of the neutron-proton quadrupolequadrupole interaction.
- $M_{\nu\pi}$ (Majorana term) shifts up all states that are not totally symmetric in the neutron-proton degree of freedom. It is a consequence of the truncation of the basis to s and d bosons only.

Introducing the concept of F spin, the IBM-1 Hamiltonian can be obtained by projecting out the part that acts only on the maximal F spin subspace (on states that are totally symmetric in the neutron-proton degree of freedom).

$$H_B = \varepsilon \hat{N} + \frac{1}{2} v_0 \left([d^{\dagger}d^{\dagger}]_{(0)} [\tilde{s}\tilde{s}]_{(0)} + h.c. \right)_{(0)} \\ + \frac{1}{\sqrt{2}} v_2 \left([d^{\dagger}d^{\dagger}]_{(2)} [\tilde{d}\tilde{s}]_{(2)} + h.c. \right)_{(0)} \\ + \sum_{L=0,2,4} \frac{1}{2} C_L \sqrt{2L+1} \left([d^{\dagger}d^{\dagger}]_{(L)} [\tilde{d}\tilde{d}]_{(L)} \right)_{(0)}$$

LIMITS

$$\begin{array}{lll} U(6) &\supset & U(5) \supset O(5) \supset O(3) \supset O(2) & vibrational \ limit \\ U(6) &\supset & SU(3) &\supset O(3) \supset O(2) & rotational \ limit \\ U(6) &\supset & O(6) \supset O(5) \supset O(3) \supset O(2) & \gamma - soft \ limit \end{array}$$

The Lie algebra U(6) admits:

- Schwinger boson realization in terms of 6 bosons s, d_μ
- Holstein-Primakoff boson realization in terms of 5 bosons b_{μ}

$$\begin{array}{rccc} d^{\dagger}_{2\mu} & \longleftrightarrow & b^{\dagger}_{2\mu} \\ s^{\dagger} & \longleftrightarrow & \sqrt{N - \sum_{\mu} b^{\dagger}_{2\mu} b_{2\mu}} \end{array}$$

- In the IBFM, an odd-nucleon operator a_j^i is introduced in addition to the *s* and *d* boson operators
- The states in the IBFM model space can be related to the shell model basis by using the generalized seniority scheme.
- The odd-nucleon operator a[†]_j should not be regarded as a nucleon creation operator (in the shell model sense) but as a generalized seniority raising operator.

$$\begin{array}{ccc} a_{j}^{\dagger} \mid \! s^{N} \rangle = & \mid \! j s^{N} \rangle & \longleftrightarrow & \mid \! n = 2N + 1, J = j, w = 1 \rangle \\ (a_{j}^{\dagger} d^{\dagger})^{(J)} \mid \! s^{N-1} \rangle = \mid \! (jd)^{(J)} s^{N-1} \rangle & \longleftrightarrow & \mid \! n = 2N + 1, J, w = 3 \rangle \end{array}$$

- The operator a_j^{\dagger} operating on an N boson state with $n_d d$ -bosons creates a state which corresponds to a shell model state with n = 2N + 1 and $w = 2n_d + 1$.
- For the shell model single-nucleon operator c_j^{\dagger} $c_j^{\dagger} | w = 2 \rangle = \alpha | w = 1 \rangle + \beta | w = 3 \rangle$
- For the odd-nucleon operator a_j^{\dagger} $a_j^{\dagger} | w = 2 \rangle = | w = 3 \rangle$

A microscopic theory for a system that includes both fermionic and bosonic degrees of freedom is complicated.

The dominant interaction in the coupling of the odd-particle to the bosons is the proton-neutron quadrupole interaction \rightarrow construction of the IBFM image of the shell model quadrupole operator.

There are several methods for obtaining the IBFM image of the shell model quadrupole operator. One of them is to introduce the pseudo particle operator \check{c}_{j}^{\dagger} (Scholten).

Condition:

The matrix elements of \check{c}_j^{\dagger} in the IBFM space are equal to the matrix elements of c_j^{\dagger} in the shel model space.

For $w \leq 1$ (α_j are the coefficients which enter in the definition of the S pair operator):

$$\hat{n} = \sum_{j} \alpha_{j}^{2} \sum_{m} c_{jm}^{\dagger} c_{jm} = \sum_{j} \alpha_{j}^{2} \hat{n}_{j}$$

$$\langle S^{N} | \ \hat{n} \ |S^{N} \rangle = 2N$$

Effective degeneracy

$$\Omega_e = \sum_j \alpha_j^2 \Omega_j$$

Here the spherical shell model OCCUPATION PROBABILITIES v_j^2 are introduced $(u_j^2 + v_j^2 = 1)$.

$$\begin{split} v_j^2 &= n_j/(2j+1) \\ n_j &= \langle S^N | \ \hat{n}_j \ | S^N \rangle \approx 2N \alpha_j^2 \frac{\Omega_j}{\Omega_e} \\ v_j^2 &= \alpha_j^2 N/\Omega_e \end{split}$$

$$\begin{split} \langle S^{N}j^{'} \parallel c_{j}^{\dagger} \parallel S^{N} \rangle &= -\hat{j}u_{j}\delta_{jj'} = u_{j}\langle s^{N}j^{'} \parallel a_{j}^{\dagger} \parallel s^{N} \rangle \\ \langle S^{N} \parallel c_{j}^{\dagger} \parallel S^{N-1}j^{'} \rangle &= \hat{j}v_{j}\delta_{jj'} = v_{j}\langle s^{N} \parallel (s^{\dagger}\tilde{a}_{j})^{(j)} \parallel s^{N-1}j^{'} \rangle / \sqrt{N} \end{split}$$

For $T \le 3$, similar expressions can be obtained. Finally, the IBFM image of the shell model single-nucleon creation operator is

$$\begin{split} \tilde{c}_{j}^{\dagger} &= u_{j}a_{j}^{\dagger} - \sum_{j'} \frac{v_{j}}{\sqrt{N}} \sqrt{\frac{10}{2j+1}} \beta_{j'j} (K_{\beta})^{-1} s^{\dagger} (\tilde{d} \ a_{j'}^{\dagger})^{(j)} \\ &+ \frac{v_{j}}{\sqrt{N}} (s^{\dagger} \tilde{a}_{j})^{(j)} + \sum_{j'} u_{j} \sqrt{\frac{10}{2j+1}} \beta_{j'j} (K_{\beta})^{-1} (d^{\dagger} \tilde{a}_{j'})^{(j)} \\ &K_{\beta}^{2} = \sum_{jj'} \beta_{j'j}^{2} \end{split}$$

The coefficients $\beta_{j'j}$ define the microscopic structure of the *d*-boson.

The matrix elements of the quadrupole operator $\sum_{jj'} Q_{jj'} (c_j^{\dagger} \tilde{c}_{j'})^{(2)}$ in the fermion space are replaced by the matrix elements of the pseudo particle operator \check{c}_j^{\dagger} acting in the boson space giving the quadrupole operator expressed in terms of boson and odd-particle operators.

$$\begin{aligned} Q^{(2)} &= Q^{(2)}_B + Q^{(2)}_F \\ Q^{(2)}_B &= [s^{\dagger} \tilde{d} + d^{\dagger} \tilde{s}]^{(2)} + \chi [d^{\dagger} \tilde{d}]^{(2)} \\ Q^{(2)}_F &= \sum_{jj'} Q_{jj'} (u_j u_{j'} - v_j v_{j'}) (a^{\dagger}_j \tilde{a}_{j'})^{(2)} \\ &- \sqrt{\frac{10}{N}} \sum_{jj'j''} Q_{jj'} (u_j v_{j'} + v_j u_{j'}) \beta_{j''j} [(d^{\dagger} \tilde{a}_{j''})^{(j)} (\tilde{s} a^{\dagger}_{j'})^{(j')}]^{(2)} (\hat{j} K_{\beta})^{-1} \end{aligned}$$

The boson-fermion interaction can be generated by the interaction between like particles or by the proton-neutron quadrupole interaction. The structure of the interactions is identical. The product of $Q_B^{(2)}$ and $Q_F^{(2)}$ contributes to the boson-fermion interaction. By mapping the basis from IBM-2 onto IBM-1 and taking terms up to second order in *d*-boson operators, the standard form of the boson-fermion interaction is obtained.

The IBFM-1 Hamiltonian for an odd-even nucleus

$$H = H_B + H_F + V_{BF}$$

 H_B is the boson Hamiltonian of IBM-1 describing a system of N interacting bosons (correlated S and D pairs) that approximate the valence nucleon pairs:

$$\begin{aligned} H_B &= \varepsilon \hat{N} + \frac{1}{2} v_0 \left([d^{\dagger} \times d^{\dagger}]_{(0)} \times [\tilde{s} \times \tilde{s}]_{(0)} + h.c. \right)_{(0)} \\ &+ \frac{1}{\sqrt{2}} v_2 \left([d^{\dagger} \times d^{\dagger}]_{(2)} \times [\tilde{d} \times \tilde{s}]_{(2)} + h.c. \right)_{(0)} \\ &+ \sum_{L=0,2,4} \frac{1}{2} C_L \sqrt{2L+1} \left([d^{\dagger} \times d^{\dagger}]_{(L)} \times [\tilde{d} \times \tilde{d}]_{(L)} \right)_{(0)} \end{aligned}$$

$$n_s = N - n_d$$

 H_F is the fermion Hamiltonian containing quasiparticle energies of odd protons or neutrons. The quasiparticle energies and occupation probabilities contained in the fermion Hamiltonian, and other terms, are obtained in a BCS calculation with some standard set of single fermion energies.

$$H_F = \sum_i \varepsilon_i \ a_i^{\dagger} \tilde{a}_i$$

 V_{BF} is the IBFM-1 boson-fermion interaction containing the dynamical, exchange and monopole term.

- The dynamical interaction V_{DYN} represents the direct component of the quadrupole interaction between the odd particle and the bosons.
- The exchange interaction V_{EXC} is due to the two-particle nature of the bosons, bringing the Pauli exclusion principle into play.
- The monopole interaction V_{MON} can result from a variety of causes, in particular from the blocking of certain degrees of freedom by the odd particle.

$V_{BF} = V_{DYN} + V_{EXC} + V_{MON}$

$$V_{DYN} = \prod_{j_1 j_2} \sqrt{5} \left(u_{j_1} u_{j_2} - v_{j_1} v_{j_2} \right) \left\langle j_1 \parallel Y_2 \parallel j_2 \right\rangle \left(\left[a_{j_1}^{\dagger} \times \tilde{a}_{j_2} \right]^{(2)} \times Q_B^{(2)} \right)^{(0)}$$

 $Q_B^{\left(2\right)}$ is the standard boson quadrupole operator

$$Q_B^{(2)} = [s^{\dagger} \times \tilde{d} + d^{\dagger} \times \tilde{s}]^{(2)} + \chi [d^{\dagger} \times \tilde{d}]^{(2)}$$

$$\begin{split} V_{EXC} &= \Lambda_0 \sum_{j_1 j_2 j_3} (-2) \sqrt{\frac{5}{2j_3 + 1}} \left(u_{j_1} v_{j_3} + v_{j_1} u_{j_3} \right) \left(u_{j_2} v_{j_3} + v_{j_2} u_{j_3} \right) \\ &\langle j_3 \parallel Y_2 \parallel j_1 \rangle \; \langle j_3 \parallel Y_2 \parallel j_2 \rangle \; : \left([a_{j_1}^{\dagger} \times \tilde{d} \;]_{j_3} \times [\tilde{a}_{j_2} \times d^{\dagger} \;]_{j_3} \right)^{(0)} : \end{split}$$

$$V_{MON} = A_0 \sum_j \sqrt{5} \ (2j+1) \ \left([a_j^{\dagger} \times \tilde{a}_j]^{(0)} \times [d^{\dagger} \times \tilde{d}]^{(0)} \right)^{(0)}$$



(a), (b), (c) exchange terms (d), (e), (f) direct terms

The structure coefficients:

- The coefficients v_j are related to the structure coefficients of the fermion Spair state, which is the microscopic equivalent of the s boson. In practice, they are the occupation probabilities of the single-particle orbits, as follows from a spherical BCS calculation.
- The coefficients $\beta_{j_a j_b} = (u_{j_a} v_{j_b} + v_{j_a} u_{j_b}) \langle j_a \parallel Y_2 \parallel j_b \rangle$ are the structure coefficients of the *d* boson.

The electromagnetic operators have the form:

$$M(E2) = M_B(E2) + M_F(E2)$$

 $M_B(E2) = \frac{3}{4\pi} R_0^2 e^{VIB} \left([s^{\dagger} \times \tilde{d} + d^{\dagger} \times \tilde{s}]^{(2)} + \chi [d^{\dagger} \times \tilde{d}]^{(2)} \right)$

$$R_0^2 = 0.0144 \ A^{\frac{2}{3}} \qquad barn$$

$$M_F(E2) = \frac{3}{5} R_0^2 e_F Y_2$$

Common notation:

$$\frac{3}{4\pi} R_0^2 \ e^{VIB} \ = \ e_B$$

$$\begin{split} \vec{M}(M1) &= \vec{M}_B(M1) + \vec{M}_F(M1) \\ \vec{M}_B(M1) &= \sqrt{\frac{3}{4\pi}} \sqrt{10} \ g_R \ [d^{\dagger} \times \tilde{d}]^{(1)} \\ \vec{M}_F(M1) &= \sqrt{\frac{3}{4\pi}} \ [g_l \ \vec{l} \ + \ g_s \ \vec{s} \ + \ g_T \ (Y_2 \ \times \ \vec{s} \)_1 \] \end{split}$$

Common notations:

$$\sqrt{\frac{3}{4\pi}} g_R = g_B$$

IBFM (and its extensions) provide a consistent description of nuclear structure phenomena in:



🜻 deformed nuclei





Spherical nuclei



Scholten



Scholten

Deformed nuclei



The IBFM generates bands that are analogous to the bands which can be constructed in the Nilsson model. In addition it generates bands that could be called β and γ bands. While they arise automatically here, in the Nilsson model they must be either placed ad hoc or calculated by use of other methods.









IBFA-calculated excitation energies for negative-parity states in the odd-mass Os isotopes compared with experimental data. States are labeled with 2J

The choice of the model space has a strong influence on the model parameters. Even if there is a large separation between shells, the mixing due to the strong core-particle quadrupole interaction does not allow for restricting the model space to a single j shell. For example: Levels based on the $g_{9/2}$ particle. Here the $d_{5/2}$ particle from the next major shell has to be included due to the large non-spinflip matrix element $\langle d_{5/2} \parallel Y_2 \parallel g_{9/2} \rangle$. The same situation appears in the case of $h_{11/2}$ ($f_{7/2}$ has to be included in the model space). Restricting the model space requires a renormalization of the interactions. For unique-parity states:

- Strengths of boson-fermion interactions obtained in a single j calculation are effective strengths
- Strengths of boson-fermion interactions obtained in a multi j calculation are real strengths

Intruder deformed bands in odd Ag isotopes



Fig.2 Partial level scheme of ¹¹³Ag and ¹¹⁵Ag. Energies and relative intensities of γ rays are given with their uncertainties in parentheses. The energies of the intruder band members are compared to the values calculated with the rotational formula K = ½.



Fig. 3. Intruder positive-parity states of ^{109,111,113,115}Ag in comparison to IBFM calculations. Indicated spin values are twice the actual value.

Only the monopole fermion-boson interaction strength is slightly changed from isotope to isotope. All other interaction strengths and occupation probabilities are the same for all isotopes.





Scholten



Scholten

Transitional nuclei









¹⁰⁵ Pd	

J of Level		1,	
From	То	Expt.	Theor,
3/21	5/21+	100	100
7/2	3/21		0.0005
	5/21+	100	100
5/22	7/21		0.0002
	3/21	0.12	0.7
	5/21	100	100
1/21	5/22		0
	3/21	27	93
	5/21	100	100
7/22	5/22		2
	7/21		0.0006
	3/21		0.03
	5/2+	100	100
3/22	7/22		0.0005
	1/2+	2.4	1
	5/22		2
	7/2+		0.005
	3/21		0.4
	5/21	100	100
3/23	3/22	0.8	0.4
	7/22		0.08

Halflives

J	Expt. a	Theor.
3/21	0.067 ns	0.03 ns
5/22	0.04 ns	0.1 ns
7/2*	3.8 ps	7.7 ps
1/21	0.88 ns	0.22 ns
3/22	1.9 ps	6.3 ps
1/22	>2 ps	1.6 ps

BOSON-FERMION SYMMETRIES SUPERSYMMETRIES

If the Hamiltonian can be expressed in terms of Casimir invariants of the chain of subgroups, the energy spectrum can be obtained ANALYTICALLY. Other observables (B(E2), B(M1), static moments, spectroscopic factors, ...) can be expressed in analytical form, too.

The symmetry group related to IBM-1 is U(6). The six dimensions are formed by the s boson and five components of d_{μ} boson. Since the number of bosons is invariant, the group is unitary. There are three chains of subgroups:

U(6)	\supset	$U(5) \supset O(5) \supset O(3) \cap O(3) \supset O(3) \cap $	O(2)	vibrational limit
U(6)	\supset	$SU(3) riangle O(3) \supset O(3)$	$\mathcal{O}(2)$	$rotational\ limit$
U(6)	\supset	$O(6) \supset O(5) \supset O(3) \supset O(3)$	$\mathcal{O}(2)$	$\gamma - soft \ limit$

For boson-fermion systems, many group chains have been investigated. Example: A j = 3/2 particle coupled to an O(6) core (j = 3/2 has four different *m*-states, and therefore forms a representation of the U(4) group).

$$U^{B}(6) \otimes U^{F}(4) \supset O^{B}(6) \otimes U^{F}(4) \supset Spin(6) \supset Spin(5) \supset Spin(3) \supset Spin(2)$$

$$E = -\frac{A}{4}[\sigma_1(\sigma_1 + 4) + \sigma_2(\sigma_2 + 2) + \sigma_3^2] + \frac{B}{6}[\tau_1(\tau_1 + 3) + \tau_2(\tau_2 + 1)] + CJ(J+1) + D\Sigma(\Sigma + 4)$$

$U^B(6)$	$quantum \ numbers$	[N]
$U^F(4)$	$quantum\ numbers$	$\{M\}$
$O^{B}(6)$	$quantum\ numbers$	Σ
Spin(6)	$quantum\ numbers$	$(\sigma_1, \sigma_2, \sigma_3)$
Spin(5)	$quantum\ numbers$	(au_1, au_2)
Spin(3)	$quantum\ numbers$	J
Spin(2)	$quantum\ numbers$	M_J

 $O^B(6) \otimes U^F(4) \supset Spin(6) \longrightarrow$ Parameters describing the boson system are in a unique relation to the parameters describing the boson-fermion system.



Problems:

- The symmetry approach to boson-fermion systems is more phenomenological in nature
- It can be applyed only in special cases when one or few fermion configurations are coupled to boson cores in one of the symmetry limits of IBM

Advantages:

- This approach was extended to boson-fermion-fermion systems (odd-odd nuclei)
- The spectra of neighboring even-even, odd-even and odd-odd nuclei can be described with the same set of parameters
- Analytical expressions are available
- Evidence that collective and single-particle degrees of freedom are closely related

Nuclear Theory: High spin states in the interacting boson and interacting boson-fermion model

S. Brant

University of Zagreb, Croatia

E.mail: Brant@phy.hr

High spin states in the interacting boson and interacting boson-fermion model

2.

Interacting Boson Model (IBM-1) based models constructed to describe the physics of high-spin states in nuclei (10 $\hbar \le J \le 30$ \hbar):

- Interacting boson plus broken pairs model (IBBPM) for eveneven nuclei
- Interacting boson fermion plus broken pairs model (IBFBPM) for odd-even nuclei

In the formulation of these models, one has to go beyond the boson approximation and include selected non-collective fermion degrees of freedom. By including part of the original shell-model fermion space through successive breaking of correlated S and D pairs, IBM can describe the structure of high-spin states.

The models are based on the IBM-1; the boson space consists of s and d bosons, with no distinction between protons and neutrons. To generate high-spin states, the models allow one or two bosons to be destroyed and to form non-collective fermion pairs, represented by two- and four-quasiparticle states which recouple to the boson core. High-spin states are described in terms of broken pairs.

Advantages of using models based on the IBM over more traditional approaches based on the cranking approximations:

- No assumption has to be made about the geometrical picture of high-spin bands
- The bands result from a consistent calculation of the complete excitation spectrum, which includes also the ground state band
- Polarization effects directly result from the model fermion-boson interactions
- All calculations are performed in the laboratory frame, and therefore the results can be directly compared with experimental data
- This extension of the model is especially relevant for transitional regions, where single-particle excitations and vibrational collectivity are dominant modes, and the traditional cranking approach to high-spin physics is not adequate



The model space for an even-even nucleus with 2N valence nucleons is

 $|N \text{ bosons } > \oplus | (N-1) \text{ bosons } \otimes 1 \text{ broken } pair > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ broken } pairs > \oplus | (N-2) \text{ bosons } \otimes 2 \text{ broken } pairs > \oplus | (N-2) \text{ broken } pairs$

This means that the fermion basis can contain two-proton, two-neutron, four-proton, four-neutron and two-proton-two-neutron configurations.

For odd-A nuclei, two-broken pair configurations are not included in the model space. They would generate five-quasiparticle configurations resulting in exhaustive numerical calculations. The IBFBPM can describe one- and threefermion structures. The two fermions in a broken pair can be of the same type as the unpaired fermion, resulting in a space with three identical fermions. If the fermions in the broken pair are different from the unpaired fermion, the fermion basis contains two protons and one neutron or vice versa.



The Interacting boson plus broken pairs model (IBBPM) Hamiltonian for an even-even nucleus:

$$H = H_B + H_{\nu F} + H_{\pi F} + V_{\nu BF} + V_{\pi BF} + V_{\nu}^{mix} + V_{\pi}^{mix} + V_{\nu \pi}$$

The label π stands for protons and ν for neutrons. If broken pairs contain both protons and neutrons, the full model Hamiltonian is used. Otherwise, when broken pairs contain only protons ($\alpha = \pi$) or neutrons ($\alpha = \nu$), the model Hamiltonian is reduced to:

$$H = H_B + H_{\alpha F} + V_{\alpha BF} + V_{\alpha}^{mix}$$

In description of high-spin states in odd-even nuclei we employ the Interacting boson fermion plus broken pairs model (IBFBPM).

- When the two fermions in a broken pair are of the same type as the unpaired fermion, the reduced Hamiltonian is used, where α labels the type of fermion (proton or neutron).
- If the fermions in the broken pair are different from the unpaired fermion, the full Hamiltonian is used, without the pair breaking interaction of the unpaired fermion and with the fermion Hamiltonian of the unpaired fermion containing only single-fermion energies.

 H_B is the boson Hamiltonian of IBM-1 describing a system of N interacting bosons (correlated S and D pairs) that approximate the valence nucleon pairs:

$$H_{B} = \sum_{k=0}^{\infty} \hat{N} + \frac{1}{2} \frac{v_{0}}{v_{0}} \left([d^{\dagger} \times d^{\dagger}]_{(0)} \times [\tilde{s} \times \tilde{s}]_{(0)} + h.c. \right)_{(0)} + \frac{1}{\sqrt{2}} \frac{v_{2}}{v_{2}} \left([d^{\dagger} \times d^{\dagger}]_{(2)} \times [\tilde{d} \times \tilde{s}]_{(2)} + h.c. \right)_{(0)} + \sum_{L=0,2,4} \frac{1}{2} \frac{C_{L}}{C_{L}} \sqrt{2L+1} \left([d^{\dagger} \times d^{\dagger}]_{(L)} \times [\tilde{d} \times \tilde{d}]_{(L)} \right)_{(0)}$$

$$n_s = N - n_d$$

 $H_{\alpha F}$ is the fermion Hamiltonian which contains single-fermion (quasiparticle) energies and fermion-fermion interactions. The quasiparticle energies and occupation probabilities contained in the fermion Hamiltonian and other terms, are obtained in a BCS calculation with some standard set of single fermion energies.

$$H_{\alpha F} = \sum_{i} \varepsilon_{\alpha_{i}} a_{\alpha_{i}}^{\dagger} \tilde{a}_{\alpha_{i}} + \frac{1}{4} \sum_{abcd JM} \sum_{JM} V_{\alpha a b c d}^{J} A_{JM}^{\dagger}(\alpha_{a} \alpha_{b}) A_{JM}(\alpha_{c} \alpha_{d})$$
$$A_{JM}^{\dagger}(\alpha_{a} \alpha_{b}) = \frac{1}{\sqrt{1 + \delta_{ab}}} [a_{\alpha_{a}}^{\dagger} a_{\alpha_{b}}^{\dagger}]_{J}^{M}$$

 $V^{J}_{\alpha a b c d} = (u_{\alpha_{a}} u_{\alpha_{b}} u_{\alpha_{c}} u_{\alpha_{d}} + v_{\alpha_{a}} v_{\alpha_{b}} v_{\alpha_{c}} v_{\alpha_{d}}) G(\alpha_{a} \alpha_{b} \alpha_{c} \alpha_{d}J) + 4 v_{\alpha_{a}} u_{\alpha_{b}} v_{\alpha_{c}} u_{\alpha_{d}} F(\alpha_{a} \alpha_{b} \alpha_{c} \alpha_{d}J)$

 $V_{\alpha BF}$ is the interaction between the unpaired fermions and the boson core containing the dynamical, exchange and monopole interactions of the IBFM-1:

$$V_{\alpha BF} = V_{\alpha DYN} + V_{\alpha EXC} + V_{\alpha MON}$$

$$V_{\alpha DYN} = \prod_{\alpha j_1 \alpha j_2} \sqrt{5} \left(u_{\alpha j_1} u_{\alpha j_2} - v_{\alpha j_1} v_{\alpha j_2} \right) \left\langle \alpha j_1 \parallel Y_2 \parallel \alpha j_2 \right\rangle \left([a_{\alpha j_1}^{\dagger} \times \tilde{a}_{\alpha j_2}]^{(2)} \times Q_B^{(2)} \right)^{(0)}$$

 $Q_B^{\left(2\right)}$ is the standard boson quadrupole operator

$$Q_B^{(2)} = [s^{\dagger} \times \tilde{d} + d^{\dagger} \times \tilde{s}]^{(2)} + \chi [d^{\dagger} \times \tilde{d}]^{(2)}$$

$$V_{\alpha EXC} = \Lambda_0 \sum_{\alpha j_1 \alpha j_2 \alpha j_3} (-2) \sqrt{\frac{5}{2 \alpha j_3 + 1}} \left(u_{\alpha j_1} v_{\alpha j_3} + v_{\alpha j_1} u_{\alpha j_3} \right) \left(u_{\alpha j_2} v_{\alpha j_3} + v_{\alpha j_2} u_{\alpha j_3} \right)$$
$$\langle \alpha j_3 \parallel Y_2 \parallel \alpha j_1 \rangle \langle \alpha j_3 \parallel Y_2 \parallel \alpha j_2 \rangle : \left([a^{\dagger}_{\alpha j_1} \times \tilde{d}]_{\alpha j_3} \times [\tilde{a}_{\alpha j_2} \times d^{\dagger}]_{\alpha j_3} \right)^{(0)} :$$

$$V_{\alpha MON} = A_0 \sum_{\alpha j} \sqrt{5} (2\alpha j + 1) \left([a^{\dagger}_{\alpha j} \times \bar{a}_{\alpha j}]^{(0)} \times [d^{\dagger} \times \tilde{d}]^{(0)} \right)^{(0)}$$

The pair breaking interaction V_{α}^{mix} which mixes states with different numbers of fermions, conserving the total nucleon number only:

$$V_{\alpha}^{mix} = -U_{0} \left\{ \sum_{\alpha j_{1} \alpha j_{2}} u_{\alpha j_{1}} u_{\alpha j_{2}} (u_{\alpha j_{1}} v_{\alpha j_{2}} + u_{\alpha j_{2}} v_{\alpha j_{1}}) \langle \alpha j_{1} \parallel Y_{2} \parallel \alpha j_{2} \rangle^{2} \frac{1}{\sqrt{2 \alpha j_{2} + 1}} \left([a_{\alpha j_{2}}^{\dagger} \times a_{\alpha j_{2}}^{\dagger}]^{(0)} \cdot \bar{s} \right) + hc \right\}$$
$$-U_{2} \left\{ \sum_{\alpha j_{1} \alpha j_{2}} (u_{\alpha j_{1}} v_{\alpha j_{2}} + u_{\alpha j_{2}} v_{\alpha j_{1}}) \langle \alpha j_{1} \parallel Y_{2} \parallel \alpha j_{2} \rangle \left([a_{\alpha j_{1}}^{\dagger} \times a_{\alpha j_{2}}^{\dagger}]^{(2)} \cdot \bar{d} \right) + hc \right\}$$

The proton-neutron interaction is:

$$V_{\nu\pi} = \sum_{\nu\nu'\pi\pi'} \sum_{J} h_{J} (\nu\nu'\pi\pi') (u_{\nu}u_{\nu'} - v_{\nu}v_{\nu'}) (u_{\pi}u_{\pi'} - v_{\pi}v_{\pi'}) \left(\left[a_{\nu}^{\dagger} \times \bar{a}_{\nu'} \right]^{(J)} \cdot \left[a_{\pi}^{\dagger} \times \bar{a}_{\pi'} \right]^{(J)} \right)$$

The coefficients $h_J(\nu\nu'\pi\pi')$ are connected to the two-body matrix elements of

the residual proton-neutron interaction by:

$$h_J(\nu\nu'\pi'\pi) = (-)^{j_\nu+j_\pi} \sum_{J'} (-)^{J'} \sqrt{2J'+1} \langle (j_\nu j_\pi) J' \parallel V(1,2) \parallel (j_{\nu'} j_{\pi'}) J' \rangle W(j_\nu j_\pi j_{\nu'} j_{\pi'}; J'J)$$

The residual proton-neutron interaction is usually taken in the form:

$$H_{\delta} = 4\pi V_{\delta} \,\delta(\vec{r}_{\pi} - \vec{r}_{\nu}) \,\delta(r_{\pi} - R_0) \,\delta(r_{\nu} - R_0)$$

The strength parameters of the boson-fermion interactions should be those obtained in the analysis of the neighboring nuclei. For example, the boson-fermion strength parameters for the couplings of two and four-proton configurations to the boson core in an even-even nucleus, have to be the same as for coupling of one-proton configurations to the boson core in the neighboring odd-even nucleus. This is the case for spherical, transitional and γ -soft nuclei. However, approaching the rotational SU(3) limit of IBM, the boson-fermion interaction strengths are not identical for an even-even nucleus and its odd-even neighbor. The effective core for configurations based on broken pairs in a deformed nucleus can be somewhat different from the one obtained by a simple decrease of the boson number by one.

$$T(E2) = \frac{3}{4\pi} e^{\text{vib}} R_0^2 [(d^{\dagger} \times \tilde{s} + s^{\dagger} \times \tilde{d})^{(2)} + \chi (d^{\dagger} \times \tilde{d})^{(2)}] - \frac{u_{j_1} v_{j_2}}{\sqrt{N}} [(a_{j_1}^+ \times a_{j_2}^+)^{(2)} \times \tilde{s}]^{(2)} + \frac{u_{j_2} v_{j_1}}{\sqrt{N}} [(\bar{a}_{j_1} \times \bar{a}_{j_2})^{(2)} \times s^{\dagger}]^{(2)}]$$

where

 $q_{j_1j_2} = \langle j_1 || r^2 Y_2 || j_2 \rangle$.

We take $\langle r^2 \rangle = \frac{3}{5} R_0^2$, and $R_0 = 0.12 A^{1/3} \times 10^{-12}$ cm. N is the number of bosons.

$$T(M1) = \sqrt{30/4\pi} g_R(d^{\dagger} \times \tilde{d})^{(1)} \\ - \frac{1}{\sqrt{4\pi}} \sum_{j_1 j_2} [g_1 \langle j_1 \| \vec{j} \| j_2 \rangle + (g_s - g_1) \langle j_1 \| \vec{s} \| j_2 \rangle] \\ \times \{ (u_{j_1} u_{j_2} + v_{j_1} v_{j_2}) (a_{j_1}^{\dagger} \times \tilde{a}_{j_2})^{(1)} - \frac{u_{j_1} v_{j_2}}{\sqrt{N}} [(a_{j_1}^{\dagger} \times a_{j_2}^{\dagger})^{(1)} \times \vec{s}]^{(1)} + \frac{u_{j_2} v_{j_1}}{\sqrt{N}} [(\vec{a}_{j_1} \times \vec{a}_{j_2})^{(1)} \times \vec{s}^{\dagger}]^{(1)} \}$$





¹⁰⁴Cd



¹⁰⁴Cd







¹⁰¹Ag

State				Lifetime (ps)			
E_x (keV)	1"	DDCM	RDDS	DSA/NGTB	Adopted	Theory	
Positive pa	rity						
98	7/2+					425	
687	11/2+	2.7(3)			2.7(3)	2.0	
861	13/2+	11.7(10)			11.7(10)	3.4	
1573	15/2+	2.1(5)			2.1(5)	0.4	
1769	17/2+	1.9(2)			1.9(2)	1.9	
2017	19/2+	9(1)			9(1)	5.0	
2621	21/2+	0.6(1)			0.6(1)	0.4	
2922	21/22					1700 ^a	
						2.8 ^b	
2956	23/2+	1.8(3)			1.8(3)	0.9	
3578	25/2+		<2.0		<2.0	0.4	
4159	27/2+		<2.5		<2.5	0.3	
4572	$(29/2^+)$		14(1)		14(1)		
5300	(31/2+)			<1.7	<1.7		
Negative pa	arity						
750	3/2(-)					10.2	
797	5/2(-)					30.5	
3870	23/2(-)	11.4(11)			11.4(11)	8.4	
4217	25/2(-)	1.1(2)			1.1(2)	1.2	
4749	27/2(-)		1.1(1)		1.1(1)	1.1	
5134	29/2(-)			0.83(8)	0.83(8)	0.81	
5678	31/2(-)			0.41(5)	0.41(5)	0.45	
6197	33/2(-)			0.30(4)	0.30(4)	0.34	
6917	35/2(-)			0.18(5)	0.18(5)	0.13	
7393	37/2(-)			<1.3	≤1.3	0.39	
No parity a	assigned					$\pi = +$	$\pi = -$
2115	17/2		199(7)		199(7)	120	
3210	21/2	1.2(1)			1.2(1)	1.0	2.9
3801	23/2					0.5	0.9
4315	25/2					0.5	0.4

 *If wave function predominantly $\pi^3(g_{9/2})$ hIf wave function predominantly $\pi(g_{9/2})$



Structure of isomers in spherical nuclei

Fig. 1. Calculated states in 97 Y of (a) positive parity and (b) negative parity in comparison to the available data. Above 2 MeV of excitation energy only the calculated yrast states are shown and, in the energy interval between 2 and 3 MeV the calculated $\frac{1}{2}$ and $\frac{1}{2}$ states (dashed lines).



Assuming a possible error of 200 - 300 keV for the predicted energies, a $27/2^-$ isomer with a halflife in the μ s - ms range could be found in ⁹⁹Nb _

Deformed nuclei



This nucleus displays a transitional structure between deformed nuclei (lighter Ce isotopes) described by the SU(3) limit of the IBM, and γ -soft nuclei (heavier Ce isotopes) which correspond to the O(6) limit of the of the IBM. The SU(3)-O(6) transition can be described by the boson Hamiltonian

$$H_{IBM} = -\frac{\alpha}{10} Q \cdot Q + \frac{\beta}{10} L \cdot L$$

and is determined by the value of the parameter χ in the quadrupole boson operator. The limiting cases are: $\chi = 0$ corresponds to the O(6) limit of the IBM-1, and $\chi = -\frac{\sqrt{7}}{2}$ describes a prolate shape in the SU(3) dynamical symmetry limit.

Here: $\alpha = 0.19$ MeV, $\beta = 0.13$ MeV, $\chi = -1.0$ and the boson number N = 12.

$$\begin{aligned} v^2(\pi h_{11/2}) &= 0.06\\ \varepsilon(\pi h_{11/2}) &= 1.70 \text{ MeV}\\ v^2(\nu h_{11/2}) &= 0.40\\ \varepsilon(\nu h_{11/2}) &= 1.32 \text{ MeV} \end{aligned}$$



This band is based on the $\nu h_{11/2}$ orbital for the states with $I \leq 27/2^-$, and on the threefermion configuration $\nu h_{11/2} (\pi h_{11/2})^2$ for $I \geq$ $29/2^-$. The structure of this band is very simple. The neutron $\nu h_{11/2}$ orbital couples to the yrast sequence of states in the core nucleus 124 Ce.



The band 2 is based on the $\nu d_{5/2}$ and $\nu g_{7/2}$ neutron orbitals. The band 3, in addition, contains sizeable components based on the $\nu d_{3/2}$ and $\nu s_{1/2}$ states. While the alignment of a proton pair is not observed in band 2, the states with $I \geq 25/2^+$ of band 3 are based on the one-neutron plus $(\pi h_{11/2})^2$ configuration.



Transitional nuclei



Wave functions

$$\begin{split} |I_{k}^{\pi}\rangle &= \sum_{jn_{d}vR} \xi_{j,n_{d}vR;I} |\pi \tilde{j}, n_{d}vR;I\rangle \\ &+ \sum_{jj'j''I_{\alpha\alpha}I_{\pi\alpha\alpha'}n_{d}vR} \eta_{jj'j''I_{\alpha\alpha'}I_{\pi\alpha\alpha'},n_{d}vR;I} \\ &\times |[\pi \tilde{j}, (\alpha \tilde{j}', \alpha \tilde{j}'')I_{\alpha\alpha'}]I_{\pi\alpha\alpha'}, n_{d}vR;I \end{split}$$

Here $\pi \tilde{j}$ stands for a proton quasiparticle, and $\alpha \tilde{j}', \alpha \tilde{j}''$ for neutron quasiparticles ($\alpha = v$), or proton quasiparticles ($\alpha = \pi$), which are coupled to the angular momentum $I_{\alpha\alpha}$. Angular momenta j and $I_{\alpha\alpha}$ are coupled to the threequasiparticle angular momentum denoted by $I_{\pi\alpha\alpha}$. In the boson part of the wave function, the $n_d d$ bosons are coupled to the total boson angular momentum R. The additional quantum number v is used to distinguish between the n_d -boson states having the same angular momentum R. We note that the number of s bosons associated with the boson state $|n_d v R\rangle$ is $n_s = N - n_d$, where N is the total number of bosons.

To make it easier to follow the origin of states, for the indexing of the theoretical states we use I_{qp_i} for the quasiparticle+phonon states, I_{bp_i} for proton broken pair states and I_{bn_i} for neutron broken pair states. Here the index *i* denotes the *i*th state of the denoted type. In the standard notation I_k , the index *k* is used as total label obtained from the IBFBPM calculation. The indexing I_k is pointed out only for states where $i \neq k$. Otherwise, the indexes *i* and *k* are equal.




$\rightarrow I_{j}^{\pi}$ (h)	$E_i \rightarrow E_f$ Expt. Expt.	B(E2)(e ² b ²) IBFBPM	$B(M1)(\mu_N^2)$ IBFBPM	Expt.	I, IBFBPM
2	167-0	0.013	0.058	100	100
"@1	359-167	0.027	0.000	100	1.8
"@1 " @1		0.003	0.006	100	100
······································	878-350	1×10 ⁻⁵	0.013	4	6
1/2 mg	167	2×10-5	0.072	10	27
1.2		0.027	0.022	100	100
2 1/2	911-828	0.0027	0.028	100	0.0
·@2 ··· @2	250	0×10 ⁻⁵	0.0005	2.2	0.5
1/2- 1/2-		0.000	0.0000	2.2	2.0
1/2.001		0.003	0.010	100	100
	1082 011	0.0004	0.010	100	0.0007
@2	- 878	0.0003	0.533	12	37
5/2·@2	- 150	0.003	0.555		17
1/2/01	- 167	0.005	0.030	100	100
1/201		0.021	0.000	100	100
5/2 m	1202 011	0.021	0.003	33	122
"@1 → 5/2 @2	1202-911	0.002	0.131		0 0001
5/2 m2		0.0004	0.012		0.0004
5/2 m		0.002	0.012	31	14
5/2 m1		0.033	0.002	100	100
"ຫ2//2 ຫງ	1412→1202	0.002	0.002	20	0.06
5/2 _{@2}		0.002	0.011	20	5
5/2 _{(#2}	$\rightarrow 828$	0.001			0.2
5/2 _{(p1}	\rightarrow 359	0.024	0.008	100	100
3/2 _{@1}	→0	3×10		20	0.3
$2_{\varpi_1} \rightarrow 7/2_{\varpi_2}$	$1519 \rightarrow 1412$	0.003	0.018		0.03
7/2 _{sp1}	\rightarrow 1202	0.0007	0.005		0.2
5/2 _{@2}	$\rightarrow 911$	0.0003			0.02
5/2 _{@1}	\rightarrow 359	0.049		100	100
$2_{qp_1} \rightarrow 9/2_{qp_1}$	$2653 \rightarrow 1519$	0.001	0.007		5
7/2 42	\rightarrow 1412	0.0004			0.4
7/2	\rightarrow 1202	0.051		100	100
$2_{\mathfrak{P}_1} \rightarrow 11/2_{\mathfrak{P}_1}$	$3160 \rightarrow 2653$	0.0003	0.004		0.1
9/2 _{@1}	\rightarrow 1519	0.065		100	100
$2_{\phi_1} \rightarrow 13/2_{\phi_1}$	$4280 \rightarrow 3160$	0.0009	0.004		1.5
11/2 _{ep1}	$\rightarrow 2653$	0.056		100	100
$2_{\varphi_1} \rightarrow 15/2_{\varphi_1}$	$4750 \rightarrow 4280$	8×10 ⁻⁵	0.002	12	0.04
13/2 _{m1}	\rightarrow 3160	0.067		100	100
$2_{bn_2} \rightarrow 21/2_{bn_1}$	$5744 \rightarrow 5225$	2×10^{-6}	0.003	100	100
19/2 _{bo}	$\rightarrow 5085$	0.0003	0.002	24	161
$2_{bn_1} \rightarrow 21/2_{bn_2}$	$6185 \rightarrow 5744$	0.0004	0.718	30	89
21/2 _{b0}	→5225	0.0007	0.007		10
19/2mg	\rightarrow 5085	0.025		40	40
$2_{bm} \rightarrow 23/2_{bm}$	$6589 \rightarrow 6185$	0.0006	0.006		0.4
21/2m,	\rightarrow 5744	8×10 ⁻⁶			0.002
21/2mg	→5225	0.036		100	100
$2_{bn} \rightarrow 25/2_{bn}$	$6870 {\rightarrow} 6589$	0.007	0.598		20
23/2m	$\rightarrow 6185$	0.0002	0.0004		0.2
21/2m	→5744	4×10^{-5}	0.003	36	6
21/2m	→5225	0.008	0.0003	100	100
1000	6086	3 1/10-7			0.005

$I_l^{\pi} \rightarrow I_l^{\pi}$	$E_f \rightarrow E_f$	$B(E2)(e^{2}b^{2})$	$B(M1)(\mu_N^2)$		I,
(ħ) (ħ)	Expt. Expt.	IBFBPM	IBFBPM	Expt.	IBFBPN
27/2 [−] _{bs} →23/2 [−] _{bo}	$7958 {\rightarrow} 6870$	0.041		50	259
→ 25/2m	$\rightarrow 6589$	0.005	0.0001	100	100
$\rightarrow 23/2_{bb}$	$\rightarrow 6185$	9×10^{-6}			0.6
$13/2^+_{cm} \rightarrow 9/2^+_{cm}$	$3031 \rightarrow 2073$	0.037		100	100
$11/2^{4} \rightarrow 13/2^{4}$	$3190 \rightarrow 3031$	0.001	0.119		0.4
→ 9/2 m	$\rightarrow 2073$	0.041	0.058	100	100
$15/2^+_{bb} \rightarrow 11/2^+_{ab}$	$3577 \rightarrow 3190$	0.002		3.4	33
$\rightarrow 13/2^+_{cm}$	$\rightarrow 3031$	0.0009	4×10^{-5}	100	100
$17/2^+_{rec} \rightarrow 15/2^+_{rec}$	3855→3577	0.006	0.039	1.2	22
$\rightarrow 13/2^+$	$\rightarrow 3031$	0.015		100	100
17/2 +	$4198 \rightarrow 3855$	6×10 ⁻⁵	0.095	90	6
$\rightarrow 15/2_{bc}^{+}$	→ 3577	0.001	0.028		11
$\rightarrow 13/2^{+}$	$\rightarrow 3031$	0.043		100	100
19/2 ⁺ → 17/2 ⁺	$4290 \rightarrow 4198$	0.002	0.001		0.02
→17/2 ⁺	→ 3855	0.004	9×10 ⁻⁵	1.6	1.2
→15/2 ²	→ 3577	0.029		100	100
17/2 + - 19/2 +	$4349 \rightarrow 4290$	6×10^{-5}	0.004		0.02
$\rightarrow 17/2^+$	$\rightarrow 4198$	3×10^{-5}	0.008		0.8
→ 17/2 ⁺	→ 3855	0.0004	0.001	52	5
$\rightarrow 15/2^{21}$	→ 3577	0.0001	0.012	<10	146
$\rightarrow 13/2$	\rightarrow 3031	0.0014		100	100
21/2 ⁺ → 17/2 ⁺	$4995 \rightarrow 4349$	0.035			24
$\rightarrow 19/2^+$	→4290	2×10^{-5}	0.033	100	100
$\rightarrow 17/2^{+1}$	$\rightarrow 4198$	0.009			17
→ 17/2 m	→ 3855	0.0002			2.7
$21/2^+_{21} \rightarrow 21/2^+_{21}$	$5186 \rightarrow 4995$	0.0002	0.005		0.03
$\rightarrow 17/2^+$	$\rightarrow 4349$	0.0002			0.04
$\rightarrow 19/2_{10}^{+2}$	$\rightarrow 4290$	0.003	0.106	31	67
$\rightarrow 17/2^{+1}$	$\rightarrow 4198$	0.0003			0.2
→ 17/2 [*]	→ 3855	0.040		100	100
$21/2^+ \rightarrow 21/2^+$	$5491 \rightarrow 5186$	0.0002	0.029	2.1	0.7
→ 21/2 [±]	$\rightarrow 4995$	1×10 ⁻⁵	0.001		0.1
$\rightarrow 17/2$	-+4349	0.008		27	9
-+ 19/2 ⁺²		0.0004	0.004	27	7
$\rightarrow 17/2^{+1}$	$\rightarrow 4198$	0.048		100	100
- (P)	51.54				



⁶⁷Ga



Interactions ?

The strength of the exchange interaction is adjusted to reproduce the energy spacings of negative-parity states in 82Sr. It differs considerably from that used for odd-even iso-In order to understand the origin of this anomaly, topes one may consider the coupling of unpaired protons to proton bosons in the 82Sr. To create multiproton states in the eveneven nucleus we destroy proton bosons and the effective coupling of the exchange interaction is reduced. In the IBM-2 framework this reduction would be implicit and no adjustment of strength parameters should be needed. However, in our model based on IBM-1, we couple to all the core bosons, irrespective of their nature and the suppression of coupling is greatly diminished. Thus, the need to empirically reduce the strength of coupling parameter. This effect should be especially pronounced near closed shells, and in our case the reduction of the exchange interaction might be due to the subshell closure at Z=40.





A detailed comparison of states calculated for 84 Zr (a) in the positive-parity sequence (b) in the negative-parity sequence

Some of the positive-parity states calculated for ⁸⁴Zr. The first ten states of each spin are shown for configurations involving core vibrations coupled to quasiproton states.





B(E2) strengths measured for the yrast sequence in 86 Zr compared with geometrical models and the present calculation.



The pair breaking interaction V^{mix} , which mixes states with different number of fermions, and conservs only the total number of valence nucleons, in general does not induce sufficient mixing as can be deduced, for example, from observed transition strengths. It is the lowest order contribution to a pair-breaking interaction. Since the interaction contains only fermion operators of rank 0 and 2, it cannot connect in first order the ground state band with two-fermion states of higher fermion angular momenta. In order to enhance the mixing, interactions that contain fermion operators of higher rank could be included in the model Hamiltonian. However, such an interaction would also require higher order boson operators, with parameters that cannot be determined from available experimental data, or from the intrinsic structure of the model.







h11/2 d3/2 neutron pair





137Nd₇₇

- Band 1 (v h11/2)
- Band 6 (v h11/2)3
- Band 7 (v h11/2) (π h11/2)⁴



Bands 3, 5, 7 (π h11/2)² Band 8 (v h11/2)²

Bands 10, 11 (π h11/2)² (ν h11/2)² !!!

136 60 (Parity	1 ₇₆ (**-1)		60 Nd 76
TRENT	RXPERTMENT		Parity- 11
		TREORY	EXPERIMENT.
hend 2	Pand 2		
		19 Band 12	Dend 13
19	21	,	
Rand J		22	
	Rand 1	2120	
15		2 - 19 2 - 19	
		₩ s17	17
		5	<u>1</u> 5
		<u> </u>	
;			







FIG. 4. Positive-parity levels calculated with the IBFBPM model for the following configuration: •: $\pi g9/2$, \Box : $\pi g9/2\nu(h_1^2_{12})$ small *I*, •: $\pi g9/2\nu(h_1^2_{12})$ I=10,

*: $\pi g9/2\nu(d5/2 \text{ and/or } g7/2)$, +: $(\pi p1/2 \text{ and/or } \pi p3/2 \text{ and/or } \pi \int 5/2) \nu h11/2\nu(d5/2 \text{ or } g7/2)$, Δ : $\pi g \frac{3}{9/2}$, ∇ : $\pi g9/2$ ($\pi p1/2$ and/or $\pi p3/2$ and/or $\pi \int 5/2$).



Fig. 5. Comparison of positive parity levels in the 0 - 1.6 MeV range observed in experiments and calculated using the IBFBPM model. All the calculated levels are shown for a given spin.

Fig.6. Comparison of positive parity levels in bands 1 and 2 observed in the present experiment (Exp.) and calculated using the IBFBPM model (Th.) up to spin I = 45/2.

Band	1	π 99/2
Band	2	$\pi g9/2 (v h11/2)^2$





Fig. 7. Negative-parity levels calculated with the IBFBPM model for the following configurations: •: $\pi p_{1/2}$ from $1/2^-$ to $21/2^-$ and $\pi f_{5/2}$ for $23/2^-$ and $25/2^-$, \Box : $\pi g_{5/2}v(d_{5/2}h_{11/2})$, •: $\pi g_{5/2}v(g_{7/2}h_{11/2})$, #: $\pi p_{1/2}v(h_{11/2}^2)_{10}$ from $I = 19/2^-$ to $37/2^-$ and $\pi f_{5/2}v(h_{11/2}^2)_{10}$ from $I = 39/2^-$ to $41/2^-$, +: $\pi (p_{1/2}$ and/or $p_{3/2}$ and/or $f_{5/2})v(h_{11/2}^2)_{\text{small }I}$, ∇ : $\pi (p_{1/2}$ and/or $p_{3/2}$ and/or $f_{5/2})\pi g_{3/2}^2$, \circ : $\pi (p_{1/2}$ and/or $p_{3/2}$ and/or $f_{5/2})v(d_{5/2})\pi g_{3/2}^2$, σ : $\pi (p_{1/2}$ and/or $p_{3/2}$ and/or $f_{5/2})v(d_{5/2})\pi g_{3/2}^2$.



Fig 8. Comparison of negative–parity levels in the 0 - 1.5 Mev range observed in experiments and calculated using IBFBPM model. For a given spin, all the calculated levels are shown.



Fig 9. Comparison of negative-parity levels in Bands 3, 4, 5 observed in the present experiment (EXP.) and calculated using IBFBPM model (Th) up to spin I = 41/2.













Comparison between calculated (0 and 2qp) (circles) and experimental (squares) negative-parity states in 194 Hg. Only the first five calculated levels of each angular momentum J are shown. Sets of states with similar structure (bands) are joined together a





Nuclear Theory: Structure of the odd-odd nuclei in the interacting boson-fermion-fermion model

S. Brant

University of Zagreb, Croatia

E.mail: Brant@phy.hr



Structure of odd-odd nuclei in the interacting boson fermion-fermion model

The IBFFM is able to give an accurate description of the structure of oddodd nuclei. Odd-odd nuclei constitute a very stringent test of the model:

A detailed knowledge of even-even cores and odd-mass neighbours is required

• Odd-odd nuclei do not provide the same sort of smoothly varying systematics as do other types of nuclei



Fig. 8 Classification of the IBFFM levels into multiplets on the basis of largest components in the wave functions. The experimental levels of ¹⁴⁰La are compared with the theoretical spectra on the basis of level energies, electromagnetic de-excitation, and transfer properties. IBFFM and experimental levels are presented by solid circles and triangles respectively.

The IBFFM Hamiltonian for an odd-odd nucleus is:

 $H = H_B + H_{\nu F} + H_{\pi F} + V_{\nu BF} + V_{\pi BF} + V_{\pi \nu}$

 H_B is the boson Hamiltonian of IBM-1 describing a system of N interacting bosons (correlated S and D pairs) that approximate the valence nucleon pairs:

$$\begin{split} H_B &= \varepsilon \hat{N} + \frac{1}{2} v_0 \left([d^{\dagger} \times d^{\dagger}]_{(0)} \times [\tilde{s} \times \tilde{s}]_{(0)} + h.c. \right)_{(0)} \\ &+ \frac{1}{\sqrt{2}} v_2 \left([d^{\dagger} \times d^{\dagger}]_{(2)} \times [\tilde{d} \times \tilde{s}]_{(2)} + h.c. \right)_{(0)} \\ &+ \sum_{L=0,2,4} \frac{1}{2} C_L \sqrt{2L+1} \left([d^{\dagger} \times d^{\dagger}]_{(L)} \times [\tilde{d} \times \tilde{d}]_{(L)} \right)_{(0)} \end{split}$$

 $n_s = N - n_d$

 $H_{\pi F}$ and $H_{\nu F}$ are the fermion Hamiltonians containing quasiparticle energies of protons and neutrons, respectively. The quasiparticle energies and occupation probabilities contained in the fermion Hamiltonian, and other terms, are obtained in a BCS calculation with some standard set of single fermion energies. For protons ($\alpha = \pi$) and for neutrons ($\alpha = \nu$).

$$H_{\alpha F} = \sum_{i} \varepsilon_{\alpha_{i}} a^{\dagger}_{\alpha_{i}} \tilde{a}_{\alpha_{i}}$$

 $V_{\nu BF}$ and $V_{\pi BF}$ are the IBFM-1 boson-fermion interactions containing the dynamical, exchange and monopole interactions. For protons ($\alpha = \pi$) and for neutrons ($\alpha = \nu$).

$$V_{\alpha BF} = V_{\alpha DYN} + V_{\alpha EXC} + V_{\alpha MON}$$

$$V_{\alpha DYN} = \Gamma_{0} \sum_{\alpha j_{1} \alpha j_{2}} \sqrt{5} \left(u_{\alpha j_{1}} u_{\alpha j_{2}} - v_{\alpha j_{1}} v_{\alpha j_{2}} \right) \left\langle \alpha j_{1} \parallel Y_{2} \parallel \alpha j_{2} \right\rangle \left(\left[a_{\alpha j_{1}}^{\dagger} \times \tilde{a}_{\alpha j_{2}} \right]^{(2)} \times Q_{B}^{(2)} \right)^{(0)}$$

 $Q_B^{(2)}$ is the standard boson quadrupole operator

$$Q_B^{(2)} = [s^{\dagger} \times \tilde{d} + d^{\dagger} \times \tilde{s}]^{(2)} + \chi [d^{\dagger} \times \tilde{d}]^{(2)}$$

$$\begin{split} V_{\alpha EXC} &= \Lambda_{0} \sum_{\alpha j_{1} \alpha j_{2} \alpha j_{3}} (-2) \; \sqrt{\frac{5}{2 \; \alpha j_{3} + 1}} \; \left(u_{\alpha j_{1}} v_{\alpha j_{3}} + v_{\alpha j_{1}} u_{\alpha j_{3}} \right) \; \left(u_{\alpha j_{2}} v_{\alpha j_{3}} + v_{\alpha j_{2}} u_{\alpha j_{3}} \right) \\ &\left\langle \alpha j_{3} \parallel Y_{2} \parallel \alpha j_{1} \right\rangle \; \left\langle \alpha j_{3} \parallel Y_{2} \parallel \alpha j_{2} \right\rangle \; : \left(\left[a_{\alpha j_{1}}^{\dagger} \times \tilde{d} \; \right]_{\alpha j_{3}} \times \left[\tilde{a}_{\alpha j_{2}} \times d^{\dagger} \; \right]_{\alpha j_{3}} \right)^{(0)} : \end{split}$$

$$V_{\alpha MON} = A_6 \sum_{\alpha j} \sqrt{5} (2\alpha j + 1) \left([a^{\dagger}_{\alpha j} \times \tilde{a}_{\alpha j}]^{(0)} \times [d^{\dagger} \times \tilde{d}]^{(0)} \right)^{(0)}$$

 $V_{\pi\nu}$ is the residual proton-neutron interaction taken in the form of spin-spin, surface-delta, spin-spin-delta, tensor or multipole-multipole interaction.

$$H_{\sigma\sigma} = -\sqrt{3} \frac{V_{\sigma\sigma}}{V_{\sigma\sigma}} [\vec{\sigma}_{\pi} \cdot \vec{\sigma}_{\nu}]$$

$$H_{\delta} = 4\pi \frac{V_{\delta}}{V_{\delta}} \delta(\vec{r}_{\pi} - \vec{r}_{\nu}) \delta(r_{\pi} - R_{0}) \delta(r_{\nu} - R_{0})$$

$$H_{\sigma\sigma\delta} = 4\pi \frac{V_{\sigma\sigma\delta}}{V_{\sigma\sigma\delta}} [\vec{\sigma}_{\pi} \cdot \vec{\sigma}_{\nu}] \delta(\vec{r}_{\pi} - \vec{r}_{\nu}) \delta(r_{\pi} - R_{0}) \delta(r_{\nu} - R_{0})$$

$$H_{T} = \frac{V_{T}}{V_{T}} \left(3 \frac{[\vec{\sigma}_{\pi} \cdot \vec{r}_{\pi\nu}] [\vec{\sigma}_{\nu} \cdot \vec{r}_{\pi\nu}]}{r_{\pi\nu}^{2}} - [\vec{\sigma}_{\pi} \cdot \vec{\sigma}_{\nu}] \right)$$

$$H_{MM} = 4\pi \frac{\delta(r_{\pi} - r_{\nu})}{r_{\pi}r_{\nu}} \sum_{\kappa\mu} \frac{V_{\kappa}}{r_{\kappa\mu}} Y_{\kappa\mu}^{*}(\pi) Y_{\kappa\mu}(\nu)$$

$$\vec{r}_{\pi\nu} = \vec{r}_{\pi} - \vec{r}_{\nu} \qquad R_{0} = 1.2 A^{\frac{1}{3}} fm$$

The electromagnetic operators have the form (for protons $(\alpha = \pi)$ and for neutrons $(\alpha = \nu)$):

$$M(E2) = M_B(E2) + M_{\pi}(E2) + M_{\nu}(E2)$$

$$M_B(E2) = \frac{3}{4\pi} R_0^2 e^{V/B} \left([s^{\dagger} \times \tilde{d} + d^{\dagger} \times \tilde{s}]^{(2)} + \chi [d^{\dagger} \times \tilde{d}]^{(2)} \right)$$
$$R_0^2 = 0.0144 A^{\frac{2}{3}} \quad barn$$

$$M_{\alpha}(E2) = \frac{3}{5} R_0^2 \epsilon_{\alpha} Y_2(\alpha)$$

$$\vec{M}(M1) = \vec{M}_B(M1) + \vec{M}_{\pi}(M1) + \vec{M}_{\nu}(M1)$$

$$ec{M_B}(M1) ~=~ \sqrt{rac{3}{4\pi}} ~\sqrt{10} ~g_R ~[d^\dagger imes { ilde d}]^{(1)}$$

$$\vec{M}_{\alpha}(M1) = \sqrt{\frac{3}{4\pi}} \left[g_{l}(\alpha) \ \vec{l}(\alpha) + g_{s}(\alpha) \ \vec{s}(\alpha) + g_{T}(\alpha) \ (Y_{2}(\alpha) \times \vec{s}(\alpha))_{1} \right]$$

Spherical nuclei

Parabolic rule for proton-neutron multiplets in the particle-vibration model

Exchange of the quadrupole phonon

The particle-quadrupole vibration interaction is

$$H_{2} = \sqrt{20\pi a_{2}} |Y_{2}(b_{2}^{\dagger} + b_{2})|_{0}$$
$$a_{2} = \frac{1}{3} (4\pi)^{\frac{1}{2}} \frac{1}{ZR_{0}^{2}} \langle k \rangle [B(E2; 2_{1}^{+} \to 0_{1}^{+})_{vib}]^{\frac{1}{2}}$$

For the quasiparticle, we also include the usual blocking factors U and V in the interaction strength a_2 . The symbol b_2^{\dagger} denotes the creation operator of the quadrupole phonon.





The contribution to the splitting of the multiplet $|(j_p, j_n)I = |j_p - j_n|, ..., j_p + j_n\rangle$ coming from the exchange of quadrupole phonons (fig. a) is

$$\begin{split} \delta E_2 &= -\alpha_2 \psi \cdot \frac{\left[I(I+1) - j_{\mathsf{n}}(j_{\mathsf{n}}+1) - j_{\mathsf{p}}(j_{\mathsf{p}}+1)\right]^2 + \left[I(I+1) - j_{\mathsf{n}}(j_{\mathsf{n}}+1) - j_{\mathsf{p}}(j_{\mathsf{p}}+1)\right]}{2j_{\mathsf{n}}(2j_{\mathsf{n}}+2)2j_{\mathsf{p}}(2j_{\mathsf{p}}+2)} \\ &+ \frac{\alpha_2 \psi}{12} \,, \end{split}$$

 $\alpha_2 = 15a_2^2/\hbar\omega_2$

Here $\hbar\omega_2$ is the energy of the quadrupole phonon. We assume the coupling strength a_2 to be equal both for protons and for neutrons.

We rewrite the I-dependent terms

$$\delta E_2(I) = A [I(I+1)]^2 + BI(I+1)$$

where A and B stand for the factors which multiply $[I(I+1)]^2$ and I(I+1) respectively.

The quantity \mathscr{V} is the occupation number defined as $\mathscr{V} = 1$ if $|j_p\rangle$ and $|j_n\rangle$ are both particle-like or both hole-like: $\mathscr{V} = -1$ if $|j_n\rangle$ is particle-like and $|j_n\rangle$ is note-like, or vice versa.

Inclusion of the spin-vibrational 1+ phonon

The particle-spin-vibration interaction reads

$$H_1 = \sqrt{3}a_1[\sigma_1 \times (b_1^{\dagger} + b_1)]_0$$

Here a_1 is the coupling strength defined as $a_1 = \kappa_1 (\hbar \omega_1 / 2c_1)^{\frac{1}{2}}$, σ_1 is the spin operator and $b_1^{\frac{1}{2}}$ the creation operator of the $\lambda = 1^+$ spin vibration

We derive the expression for the contribution to the energy shift of the $|(j_p j_n)I\rangle$ states due to the exchange of the 1⁺ phonon

$$\delta E_1(I) = B_1 I(I+1) + \alpha_1 \frac{j_n(j_n+1) + j_p(j_p+1)}{(2j_n+2)(2j_p+2)}$$

$$B_1 = -\alpha_1 \frac{\xi}{(2j_p+2)(2j_n+2)} \qquad \alpha_1 = 4 \frac{a_1^2}{\hbar\omega_1}$$

$$\xi = \begin{cases} 1 & \text{if } \mathcal{N} = -1, \frac{(2j_p+2)(2j_n+2)}{2j_p 2j_n} & \text{if } \mathcal{N} = 1, \\ -\frac{2j_p+2}{2j_p} & \text{if } \mathcal{N} = 0^-, -\frac{2j_n+2}{2j_n} & \text{if } \mathcal{N} = 0^+. \end{cases}$$

 \mathcal{N} is the Nordheim number defined as

$$\mathcal{N} = j_{p} - l_{p} + j_{n} - l_{p}$$

For $\mathcal{N} = 0$, we use the labels – and +. The symbols $\mathcal{N} = 0^-$ and $\mathcal{N} = 0^+$ denote the situations $j_n - l_n = -\frac{1}{2}$, $j_p - l_p = \frac{1}{2}$ and $j_n - l_n = \frac{1}{2}$, $j_p - l_p = -\frac{1}{2}$, respectively.









For spherical nuclei, with only the dynamical interaction and $\chi=0$

96Y



Fig. 4. Negative and positive parity levels of 96 Y calculated in IBFFM. Negative parity levels are denoted by full circles and positive parity by crosses. The states classified into the same multiplet on the basis of largest components in the wave function, are connected by full line.



 $\blacktriangleright \quad \Gamma_0 = a \ \sqrt{\frac{4 \ \pi}{5 \ N}}$

Fig. 5. IBFFM positive- and negative-parity energy spectrum of 96 Y in comparison to the available experimental data. The excitation energy of the 8* level is not known, but it should amount to ≥ 1 MeV, see text.



Sb isotopes



Fig. 6. IBFFM energy spectrum of ¹¹⁶Sb in comparison with experimental data. The solid lines connect the members of the given multiplets. The leading proton-neutron configurations for several multiplets were identified on the basis of the $({}^{3}\text{He}, d)$ proton transfer results [4].

TABLE III. Comparison of measured and calculated in IBFFM spectroscopic factors for the ¹²¹Sb(p, d)¹²⁰Sb one neutron transfer reaction. The first two columns contain the energy, spin and parity of the final states involved in the reaction taken from ref.[8].

TABLE IV. Comparison of measured and calculated in IBFFM spectroscopic factors for the one neutron transfer reaction 122 Sb. The first two columns contain the energy, spin and parity of the final states involved in the reaction taken from ref [13].

					Energy (keV)	J.4	ž4	Sout *	ė.	Sea.	Stappy 4	\$	\$ (6.9)	Sugar
	~~~~~~				5	2	\$	0.25		0.65	0.45			
Ebergy (keV)	30	éa.	Seg. 4	Same	8t	3*	9	0.23	(0)	0.46	ē.13	$\langle 2 \rangle$	0.66	80.6
8	1,	****	0.15	0.15	75	\$		0.46			0.52			
28	30	6	0.19	0.12	121	£7	819 <u>8</u> 0		S. 220			(3)	0,30	9.67
140	5*	8	9.66	0.08	137	37	- 3-	6.38	(#)	6.37	0.20			
\$66	3	3	0.63	0.44	154	8	. <u>P</u> .	Q.50			0.53			
193	?°	ö	0.11	0.08	197	2	2	0.19			0.14	(4)	9.62	6.09
		- 2	0.07	0.05	153	8 20	2	0.95	101	0.92	U.afi			
233	27	8	¢.06	6.83	220	4.	6. <b></b>	0.39	(4)	15 2.8	0.13			e 14
		2	0.12	8.85	854 2012	2.					0.66	×	N 200	0.00
334	4643	8	6.69	6.19	1000	÷.		0.89			0.5%			
9.63	40.00	- fi	0.64	<u>0.89</u>	410	an f		6.36			0.68			
387	(35)	11. <b>5</b> . 11	0.64	0.68	282	age 1					() () () () () () () () () () () () () (	(85)	2.40	6.48
390	(2, 3)*	8	\$-98	Q 07	91.1	4.						25	6.40	0.69
436	(2)	e	0.63	0.02	323	24						0	8.30	0.60
		2	0.82	0.13	334	34	2	2.67		8.09	9.93	200	6.16	0.63
					284	47	0.020					1	8.45	9.46
a Deference	a [0]				397	3+	2	0.63		8.10		3	6.45	¢.56
a Kererene					414	671						5		0.88
b S(2jtarget -	+1)				sa 420	$\tau$	6	0.05			0.36	3(3)	3.70	0.45
	- Í 🛛				<b>\$</b> 25	37.1						3		0.49
					461	- 4*	2	0.39	(2)	8.50	9.22			
					48.4	3*	84.0 G	0.010.010	848-84			2	0.51	0.80
					a Reference b Reference c Reference	[9] [7] [13]								

Nucleus	E"	J*	jlengs *	FIRFYM	Quero *	QIBFED
120,555	0 keV	1+	+2.34(22)	+2.35		-0.10
	78 keV	3 ⁴	+2.584(6)	+2.67	±0.41(4)	-0.47
		87	±2.34(4)	+2.45		-0.51
322Sb	0 keV	2	-1.905(20)	-2.33	+0.85(13)	-0.08
	61 keV	3*	+2.983(12)	+3.07	±0.41(4)	-0.48
	137 keV	5*	+3.05(10)	+3.07		~0.62
12655	0 keV	3"	±1.20(2)	-1.23	+1.87(38)	-0.35
an a	41 keV	3*	+2.070(33)	+3.01	er en en en en en en	-0.46
	125 keV	67	+0.384(12)	+0.36		40.16





40K

State (keV)	$T_{1/2}$					
	Exp	The				
31 (30)	4.24(8) ns	5.2 ns				
21 (800)	0.28(4) ps	0.23 ps				
51 (891)	0.87(14) ps	0.6 ps				
22 (2047)	0.34(4) ps	0.3 ps				
32 (2070)	0.47(10) ps	0.1 ps				
1 [ (2104)	0.52(10) ps	0.14 ps				
42 (2397)	0.035(14) ps	0.03 ps				
01 (2626)	0.21(4) ps	0.2 ps				
11 (2290)	0.083(14) ps	0.25 ps				
31 (2787)	<0.04 ps	0.07 ps				
61 (2879)	0.27(10) ps	0.6 ps				

Transition*	E _{tr} /keV	Gamma	branching	
		Expb	The	
$2_{1}^{-}(800) \rightarrow 4_{1}^{-}(0)$	800	0.15	0.17	
-+3 ⁻ ₁ (30)	770	100	100	
$5_{1}(891) \rightarrow 4_{1}(0)$	891	99	99	
→ 3 ⁻ ₁ (30)	862	1	0.1	
$2_{2}^{-}(2047) \rightarrow 4_{1}^{-}(0)$	2047	29	128	
$\rightarrow 3_{1}^{-}(30)$	2018	29	100	
$\rightarrow 2_{1}^{-}(800)$	1247	41	41	
$3_{2}^{-}(2070) \rightarrow 4_{1}^{-}(0)$	2070	36	56	
$\rightarrow 3_{1}^{-}(30)$	2040	49	49	
$\rightarrow 2^{-}(800)$	1270	9	6	
→ 5 ⁻ (891)	1178	7	3	
$1_{1}^{-}(2104) \rightarrow 3_{1}^{-}(30)$	2074	70	70	
-+2, (800)	1304	29	96	
→ 2 ⁻ ₂ (2047)	57	-	0.1	
→ 3 ⁻ (2070)	34		0.0	
$4_{1}(2397) \rightarrow 4_{1}(0)$	2397	26	7	
→ 3 ⁻ (30)	2367	67	67	
-+ 2 ⁻ (800)	1597	-	0.4	
→ 5 (891)	1506		0.1	
→ 25 (2047)	350	-	0.0	
→ 3 ⁻ ₇ (2070)	327	7	1	
→ 35 (2291)	106	-	0.01	
$0_{1}^{-}(2626) \rightarrow 2_{1}^{-}(800)$	1826	30	41	
$\rightarrow 2^{-}_{2}(2047)$	579		0.001	
→ 1, (2104)	522	70	70	
$\rightarrow 2^{-}_{2}(2419)$	207	-	0.0	
$1^+(2290) \rightarrow 0^+(1644)$	646	56	56	
→2 ⁺ (1959)	331	9	10	
-+ 3 ⁺ (2260)	30	_	0.0	
$3^+_{(2787)} \rightarrow 2^+_{(1959)}$	828	17.4	17.4	
$\rightarrow 3^+(2260)$	527	_	0.5	
→ 1 ⁺ (2290)	497	-	0.0	
→2 ⁺ (2576)	211		0.0	
-+2+(2757)	30	_	0.0	



The structure of ¹⁰⁶Ag is very complex. The ground states of odd-mass Ag nuclei are 7/2⁺ states based on the proton g9/2 configuration. The IBFFM is even successful in the description of such nuclei.

### ⁴⁰K

# **Deformed nuclei**



IBFFA-calculated excitation energies for states in odd-odd ¹⁸²Re compared with experimental data. Because the experimental separation between the 7⁺ triplet and the 2⁻ singlet couplings is not known, the figure is divided into two parts, using each coupling as the reference point. Otherwise the triplet coupling is shown on the left of a pair of bands, the singlet coupling on the right. For example; 9⁻ triplet, 0⁻ singlet.



IBFFA-calculated excitation energies for states in odd-odd ¹⁸⁴Re compared with experimental data. The bands are plotted in pairs, with the triplet coupling on the left, the singlet coupling on the right of each pair.

Scholten PRC 37

	and the second se		
Isotope	J	Q _{1BA} (e b)	$Q_{exp}$ (e b)
180Re	6-	6.50	
	1-	6.45	
182Re	7+	5.66	< 6.4
	2+	5.40	> 6.6
184Re	3-	5.06	7.9±0.7
	8+	4.83	

Ouadrupole moments for odd-odd Re isotopes.

Magnetic moments for odd-odd Re isotopes.

Isotope	J≭	$\mu_{\rm IBA}(\mu_{\rm N})$	$\mu_{exp}(\mu_N)$
¹⁸⁰ Re	6-	2.41	
	1-	2.35	
¹⁸² Re	7+	2.33	2.76±0.07
	2+	3.28	3.07±0.24
¹⁸⁴ Re	3-	3.19	2.50±0.19
	8+	2.09	2.89±0.13

The investigations of odd-A nuclei have revealed the following decoupled-strongly coupled rule:

(i) The strongly coupled band pattern arises if the odd fermion is a particle coupled to an oblate core, or a hole coupled to a prolate core.

(ii) The decoupled band pattern arises if the odd particle is coupled to a prolate core or a hole to the oblate core.

This rule was extended to odd-odd nuclei in the case of counling two quasiparticles to the asymmetric rotor.

Rules (i) and (ii) can be expressed in terms of quadrupole moments of the odd particle and the core Taking into account the signs of quadrupole moments Q(j) < 0,  $Q(j^{-1}) > 0$ ,  $Q(2_1^{\text{rolate}}) < 0$ , and  $Q(2_1^{\text{oblate}}) > 0$ , the rule reads

if 
$$Q(\tilde{j}) \cdot Q(2_1^{\text{core}}) > 0$$
, the band is strongly coupled  $> 0$ , the band is decoupled ,

where  $\vec{j}$  denotes the odd quasiparticle coupled to the core.

This rule is of more general character, independent of the particular nature of the core. It applies as well to the IBFM and IBFFM.

We note that the case of two particles or holes coupled to the core, referred to as the "peaceful" case in the particle-plus-asymmetric rotor model, corresponds to the inverted parabola of the parabolic rule for odd-odd nuclei;² this yields the bandheads with angular momenta  $J = j_p + j_n$  and  $J = |j_p - j_n|$  as the lowest states on two branches of the parabola. By coupling the proton particle  $j_1$  and the neutron particle  $j_2$  to the SU(3) boson core, there arise  $2(j_1 + \frac{1}{2})(j_2 + \frac{1}{2})$  bands, based on the states of angular momenta  $J = J_1 \pm J_2$ , with  $J_1 = j_1, j_1 - 1, \ldots, \frac{1}{2}$  and  $J_2 = j_2, j_2 - 1, \ldots, \frac{1}{2}$ . For the particular interaction strengths  $\Gamma_{1}^{SUSY}$  the band based on the lowest  $J = j_1 + j_2$  state exhibits an exact J(J + 1) energy rule, with the same moment of inertia as for the ground-state band of the boson core. Furthermore, the states of this band are characterized by the exact quantum numbers  $(K_1 = j_1, K_2 = j_2)K = j_1 + j_2$ , defined according to the IBFFM relation The other IBFFM bands in the odd-odd system deviate from the J(J + 1) energy rule; in general, more so with increasing energy (decrease of  $K_1, K_2$ ). Simultaneously, the IBFFM wave functions expressed in the KR basis are a mixture of different K values. However, in each state a particular KR basis state dominates. In this way we can attribute approximate quantum numbers  $(K_1, K_2)K$  to each state.



Twelve ground state bands calculated in the IBFFM for the odd-odd system with  $j_1 = \frac{1}{2}$  proton hole and  $j_2 = \frac{5}{2}$  neutron hole coupled to the SU(3) prolate boson core. The J(J + 1) scale is employed for the angular momentum axis.



The IBFFM ground state bands for the odd-odd system with  $j_1 = \frac{1}{2}$  proton particle and  $j_2 = \frac{5}{2}$  neutron hole coupled to the SU(3) prolate boson core.



Lowest bands in the odd-odd system with  $j_1 = \frac{11}{2}$  proton hole and  $j_2 = \frac{11}{2}$  neutron hole coupled to the SU(3) prolate boson core.

### Realistic case: Dynamical and exchange interactions different from zero and not limited by supersymmetry constraints



Calculated IBFFM yrast states  $(0_1, 1_1, ..., 20_1)$ for  $\overline{j}_{\pi} = \prod_{11/2}^{-1}$ ,  $\overline{j}_{\nu} = \overline{i}_{13/2}$  coupled to an O(6) core, as a function of  $V_{13/2}^2$ .

*E/J* plots for the lowest-lying high-spin bands for  $v_{1}^{2}_{3/2} = 0$  a, and  $v_{1}^{2}_{3/2} = 1$  b.

*E/J* plots for the levels 1, 2, ..., 12, for  $v_1^2 {}_{3/2}=0$  a,  $v_1^2 {}_{3/2}=0.5$  b and  $v_1^2 {}_{3/2}=1$  c. The scale on the abscissa is given by J(J+1). The points for levels are connected by curves to guide the eye.



The head of the lowest high-spin band is

 $J = j_{\pi} + j_{\nu} - 3 = 9 \text{ for } \nu_{1}^{2} _{3/2} \cdot 0.2,$   $J = j_{\pi} + j_{\nu} - 2 = 10 \text{ for } .0.2\nu_{1}^{2} _{3/2} \cdot 0.2,$   $J = j_{\pi} + j_{\nu} - 1 = 11 \text{ for } .0.5\nu_{1}^{2} _{3/2} \cdot 0.8$ and  $J = j_{\pi} + j_{\nu} = 12 \text{ for } .0.8\nu_{1}^{2} _{3/2}.$ 

A pronounced feature is rather broad region with  $J = J_{\pi} + J_{\nu} - 1 = 11$  level as the lowest highspin state. This resembles the J = j - 1anomaly for rather broad region around  $\nu^2$ = 0.5 in odd-even nuclei.

Characteristic lowest high-spin band patterns associated with O(6) limit for several values of  $v_{1}^2$  _{3/2} in the illustrative IBFFM calculation. Examples with all possible high-spin band heads are presented. For comparison the experimental bands ^{190, 192}Au are presented to the right. We note that the Au-region is approximately associated with an O(6) symmetry, but the present calculation is not fitted to Au isotopes

Calculated IBFFM yrast states  $(0_1, 1_1, \dots, 20_1)$  for  $\bar{j}_{\pi} = h_{11/2}^{-1}, \bar{j}_{\gamma} = \bar{i}_{13/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  and  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  and  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  and  $v_2^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  coupled to an SU(3) core, as a function of  $v_1^2_{1/2}$  cor





*E/J* plots for the levels  $1_1$ ,  $2_1$ , ...,  $12_1$ ,  $v_1^2 x_2 = 0$  a,  $v_1^2 x_2 = 0.5$  b and  $v_2^2 x_2 = 1$  c.



E/J plots for the lowest-lying high-spin bands for  $v_{\perp 3/2}^2 = 0$  a, and  $v_{\perp 3/2}^2 = 0.5$  b

# **Transitional nuclei**





Experimental and theoretical IBFFM energy spectra of the low lying states in ⁶⁸As. Up to 0.6 MeV, the positive parity levels  $3^+$ ,  $4^+$ ,  $5^+$  are shown. In addition the high spin isomer  $9_1^*$  is shown. For negative parity the calculated states with  $J^{\pi} = 4_1^-$ ,  $5_1^-$ ,  $5_2^-$ ,  $6_1^-$ ,  $6_2^-$ ,  $6_3^-$ ,  $7_1^-$ ,  $7_2^-$ ,  $7_3^-$ ,  $8_1^-$ ,  $8_2^-$ ,  $9_1^-$  are shown.

$E^*_{exp}$	J [#] _{cxp}	$J_i^{\pi} \rightarrow J_j^{\pi}$	l _y		
(MeV)	(九)	(theor.)	Exp.	Theor	
0.158	3+	$3^+_2 \rightarrow 3^+_1$	100(6)	100	
0.214	4+	$4_1^+ \rightarrow 3_2^+$	6(2)	5	
		→ 3 ⁺	100(14)	100	
0.313	3+	$3^+_3 \rightarrow 4^+_1$	<2	4	
		$\rightarrow 3^+_2$	16(2)	86	
		$\rightarrow 3^+_1$	100(2)	100	
0.500	4+	$4^+_3 \rightarrow 3^+_3$	5(1)	12	
		$\rightarrow 4_1^+$	5(1)	5	
		$\rightarrow 3^+_1$	100(2)	100	
0.550	4+	$4_2^+ \rightarrow 3_3^+$	9(1)	100	
		$\rightarrow 4_1^+$	36(2)	16	
		$\rightarrow 3^+_2$	100(4)	9	
		$\rightarrow 3_1^+$	48(2)	10	
0.733	51	$5^+_1 \rightarrow 4^+_2$	17(2)	3	
		$\rightarrow 4_3^+$	<6	26	
		$\rightarrow 4_1^+$	100(5)	100	
3.183	11(+)	$11_1^+ \rightarrow 9_1^+$	100(3)	100	
0.965	5(-)	$5^1 \rightarrow 4^1$	100(3)	100	
1.304	7(-)	$7_1^- \rightarrow 5_1^-$	100(3)	100	
1.323	6(-)	$6^1 \rightarrow 5^1$	100(4)	100	
		$\rightarrow 4_1^-$	<3	4	
1.427	6(-)	$6_2^- \to 5_2^-$	8(2)	2	
		$\rightarrow 5^1$	100(5)	100	
1.956	(8-)	$8^1 \rightarrow 6^1$	100(8)	100	
		$\rightarrow 7^1$	23(6)	16	
2.094	8(-)	$8^2 \rightarrow 7^1$	100(3)	100	
2.251	(7-)	$7_1^- \rightarrow 6_2^-$	<55	75	
		- 6,	100(15)	100	
			<40	45	

Calculated in IBFFM branching ratio for ⁶⁸As compared with the experimental data





Fig. 4. Low-lying yields in ¹⁰²Rh calculated with the IBFFM model and compared to experimental ones. For the clarity of the figures, only the three lowest calculated levels for a given spin are shown. The left- hand and right-hand sides of the figure are for the negative- and positive-parity states, respectively. Levels with unidentified parity are shown in the central column. Experimental states of uncertain parity are denoted by *, states interpreted as intruders are denoted by **.

			102	Rh				
Transition		B(E2)(e ²	$B(E2)(e^2b^2)$		$B(MI)(\mu_N^2)$		ly	
$I_i^{\pi} \rightarrow I_j^{\pi}$	$E_i \rightarrow E_f$	Exp.	IBFFM	Exp.	IBFFM	Exp.	IBFFN	
$2^1 \rightarrow 2^1$	42 → 0		0.0089	0.0054(18)	0.0033	100	100	
3 2-	76 - 42		0.0206		0.0731		68	
→ 2 ⁻	$\rightarrow 0$		0.0065		0.0097		100	
1 3.	124 -> 76		0.0092				0.0	
- 27	- 42		0.0068	0.0072(54)	0.0169	12.9	3.4	
- 2-	→ 0		0.0307	0.034(18)	0.1436	100	100	
$5^+_{,} \rightarrow 6^+_{,}$	$155 \rightarrow 141$		0.0003		0.0515	100	100	
21 - 11	157 -+ 105		0.0246		1.1948	100	100	
3+ - 2	179 - 157		0.0109		1.2919	824	43*	
- 17	105		0.0232				0.1ª	
7: - 5;	242 155		0.0033				0.0	
- 6	-+ 141		0.0342		0.1286	100	100	
$7^*_1 \rightarrow 7^*_2$	297 - 242		0.0057		0.0019		8	
- 51			0.0178				20	
$\rightarrow 6^+_1$	141	≥0.039 ^h	0.0164	≥ 0.093 ^b	0.0007	100	100	
$6^+_5 \rightarrow 7^+_1$	$360 \rightarrow 297$		0.0322		0.2138		100	
$\rightarrow 7^+_2$	-+ 242		0.0181		0.0287		90	
$6_1^+ \rightarrow 6_2^+$	378 -+ 360		0.0099		0.0002		0.0	
$\rightarrow 7^{+}_{1}$	- 297		0.0009	0.045 ^b	0.5392	92	69	
$\rightarrow 7^+_2$	- 242		0.0023	0.016 ^b	0.0046	58	3	
- 5°	-+ 155	0.0063 ^b	0.0006		0.0375	83	100	
$\rightarrow 6^+_1$	- 141	0.0044 ^b	0.0038	0.0018 ^b	0.0309	100	100	
$7_1^+ \rightarrow 6_1^+$	570 -> 378		0.0343		0.1572		19	
$\rightarrow 6^+_2$	-> 360		0.0000		0.0088		1.4	
$\rightarrow 7^+_1$	- 297		0.0018		0.0000	5	0.1	
- 72	- 242		0.0000		0.0035		2.1	
$\rightarrow 5^{\circ}_{1}$	- 155		0.0076		0.0000		1.1	
$\rightarrow 6^1$	→ 141		0.0016		0.0728	100	100	
$8_1^+ \rightarrow 7_3^+$	616 - 570		0.0080		0.2705		0.3	
$\rightarrow 6^+_1$	378		0.0009				0.0	
$\rightarrow 6^+_2$	~ 360		0.0228				0.2	
$\rightarrow 7^{*}_{1}$	- 297		0.0374		0.2572	100	100	
- 7	-+ 242		0.0022		0.0006		0.5	
$\rightarrow 6^+_1$	- 141		0.0020				0.4	

### ¹⁰²Rh



### $\pi$ g9/2 v h11/2

**Full line** 



The  $\Delta I=2$  positive-parity band 4

In the present IBFFM calculation we obtain two positive-parity high-spin bands, based on  $\pi p_{1/2}\nu h_{11/2}$  and  $\pi g_{9/2}\nu g_{7/2}$  two-quasiparticle configurations, respectively These bands are clearly formed above the 10⁺ state, while for lower spins there is a stronger configuration mixing. The two lowest 10⁺ states are based on the  $(\pi p_{1/2}\nu h_{11/2})$ 6, 24; 10 (50%) and  $(\pi g_{9/2}\nu g_{7/2})$ 8, 12; 10 (59%) configurations These two calculated bands appear close lying and they cross at angular momentum  $t \approx 15h$ . Contrary to the experimental band 4 which is of  $\Delta t = 2$  type with signature

tum  $l \approx 15h$ . Contrary to the experimental band 4 which is of  $\Delta I = 2$  type with signature  $\alpha = 0$ , the calculated bands show doublet-type structures. The  $\pi p_{1/2} \nu h_{11/2}$  configuration is associated with a much larger signature splitting than the  $\pi g_{3/2} \nu g_{3/2}$  configuration and exhibit a pronounced tendency towards decoupled band i Dn this basis we attribute the  $\pi p_{1/2} \nu h_{11/2}$  configuration to band 4.



B(E2) and B(M1) reduced transition probabilities calculated between states of the  $\pi p_{1/2} \nu h_{11/2}$  configuration with N = 7 in ¹⁰²Rh and comparison of the intensities of  $\gamma$ -rays observed in band 4 with those calculated for the above configuration

Transition	$B(E2)(e^2b^2)$ IBFFM	$B(M1)(\mu_N^2)$	l _y	
			Exp.	IBFFM
$12^+ \rightarrow 11^+$	0.0016	0.0073		0.0
$12^+ \rightarrow 10^+$	0.4295		100	100
$14^+ \rightarrow 13^+$	0.0021	0.0001		0.0
$14^+ \rightarrow 12^+$	0.1003		100	100
$16^+ \rightarrow 15^+$	0.0005	0.0049		0.1
$16^+ \rightarrow 14^+$	0.0418		100	100
$18^+ \rightarrow 17^+$	0.0010	0.0019		0.0
$18^+ \rightarrow 16^+$	0.1076		100	100
$20^+ \rightarrow 19^+$	0.0011	0.0061		0.1
$20^+ \rightarrow 18^+$	0.0634		100	100

The two remaining experimental bands, bands 2 and 3, are expected to be based on four-quasiparticle states involving broken neutron pairs, in particular the  $w_{l_1}^2 _{1/2}$ broken pair, so that four-quasiparticle states should be coupled to the boson core. The model including boson pairs of fermions has not been applied yet to odd-odd nuclei and therefore the corresponding theoretical states are missing in the present IBFFM calculations.






On the basis of the IBFFM analysis, we propose that the negative-parity bands presented have a  $\pi(d_{5/2g7/2})vh_{11/2}$  configuration in their low-spin part and, starting from  $I \approx 12$ , are almost pure  $\pi h_{11/2vg7/2}$ , with band 2 being the yrast structure and bands 3 and 5 the yrare structures. Thus the collective band structures start at spin  $I \approx 12$ .



Level scheme of ¹³²Pr deduced from the present work. The transition intensities are proportional to the width of the arrows. The inset shows the IBFFM identification of levels populated in the decays of bands 1 and 2.

## Mixing of configurations with different parity both for protons and for neutrons (high with low spin states)



parity neutron configurations

The formation of regular  $\Delta I = 1$  high-spin band pattern in odd-even and odd-odd nuclei with O(6) boson core presents a challenging problem for the IBFM and IBFFM models, respectively. Namely, both models predict normal or decoupled band patterns with close-lying unfavored and favored bands, resulting in characteristic "doublet structures" and  $\Delta I = 2$  transitions. This is because the standard IBFM interactions can not induce a sufficiently strong effective interaction which would establish a regular  $\Delta I = 1$  band pattern, without sizeably altering the low-energy part of the spectrum.



Schematic presentation of the conflicting pattern in odd-odd nuclei associated with unique-parity proton and neutron quasiparticles of opposite occupancy coupled to the O(6) boson core (n is an integer).

In order to account for this problem, we introduce a new term in the quadrupole operator of the dynamical boson-fermion interaction.

$$H_{dyn} = \Gamma_0 \sqrt{5} \sum_{j_1 j_2} (u_{j_1} u_{j_2} - v_{j_1} v_{j_2}) \times \langle j_1 \parallel Y_2 \parallel j_2 \rangle \{Q_2^B(c_{j_1}^{\dagger} \tilde{c}_{j_2})_2\}_0$$
$$Q_{2\mu}^B = d_{2\mu}^{\dagger} s + s^{\dagger} \tilde{d}_{2\mu} + \chi (d^{\dagger} \tilde{d})_{2\mu} + \eta \sum_{L_1 L_2} [(d^{\dagger} \tilde{d})_{L_1} (d^{\dagger} \tilde{d})_{L_2}]_{2\mu}$$

The effect of the last term is to induce an effective deformation of a soft O(6) core due to polarization caused by odd fermions. In this way a regular  $\Delta I = 1$  yrast band is established in odd-odd nuclei with O(6) boson core. Without the  $\eta$ -term a familiar "doublet structure" is obtained in IBFFM, while taking  $\eta \neq 0$  leads to a regular  $\Delta I = 1$  pattern, both in energies and M1 transitions on the yrast.



IBFFM calculation for yrast states in odd-odd system with inclusion of  $h_{11/2}$  proton ( $v^2 = 0.1$ ) and  $h_{11/2}$ neutron ( $v^2 = 0.8$ ) quasiparticles coupled to the O(6) boson core. The non vanishing parameters are: core parameters from R. F. Casten and P. von Bretano, *Phys. Lett.* **B152** (1985) 22, N=7,  $\Gamma_0^{\pi} = \Gamma_0^{\nu} = 0.4$  MeV. These values are used in other illustrative figures as well as: g = 1, g = 0, g = 0.7 g, g = 0.7 g,  $g_R = 0.43$ . The energies of  $8_1^+$  states are taken as zero. In this figure  $\chi =$ 0 (full and dashed lines corresponds to  $\eta = -0.4$  and 0, respectively).



Calculated energy difference between the neighbouring yrast states for  $\eta = 0$  (dashed line) and  $\eta = -0.4$  (full line).



Calculated B(M1) values from transitions between the yrast states for  $\eta = 0$  (dashed line) and n = -0.4 (full line).



Signature inversion has been investigated in various models. Taking a very strong (!!??) boson-fermion exchange interaction, a change of signature pattern from inverted to the normal with increasing spin can be achieved within the unique-parity twoquasiparticle band. However, the strength of the exchange interaction is not a free parameter. A sizable increase of the strength of exchange interaction would destroy the agreement of the calculation with the low-spin data.

$J_i^{\pi} \rightarrow J_j^{\pi}$	$J_i^\pi \to J_j^\pi$	$B(E2) (e^2b^2)$	$B(M1) \ (\mu_N^2)$		1,	B(MI),	/B(E2)	$J_i^{\pi} \rightarrow J_f^{\pi}$	B(E2) (e ² b ² )	$B(M1) (\mu_N^2)$		1,	B(M1)/	B(E2)
	IBFFM	IBFFM	EXP.	IBFFM	EXP.	IBFFM		IBFFM	IBFFM	EXP.	IBFFM	EXP.	IBFFM	
7 ₁ ⁻ →6 ₁ ⁻	0.698	0.949	100	100	-	-	13++→12+	0.430	1.236	100	100	2.6(3)	1.47	
$8^{-}_{1} \rightarrow 7^{-}_{1}$	0.668	0.727	100	100	0.97(2)	1.62	→11 ⁺	0.837	-	45	79			
61	0.448	-	5	3			14+	0.347	1.194	47	40	17(2)	1 44	
$9^1 \rightarrow 8^1$	0.585	0.538	100	100	0.75(10)	0.95	-12+	0.828		100	100		1.45	
-→7 ₁ ⁻	0.567	-	36	28			15+	0.263	1.152	100	100	24(1)	1.48	
$10^{-}_{1} \rightarrow 9^{-}_{1}$	0.490	0.459	95	110	0.61(5)	0.71	131 - 141	0.283	1.152	00	100	2.4(1)	1.40	
8 ⁻	0.648	-	100	100			-13 ₁	0.778	-	6/	141			
$11_{1}^{-} \rightarrow 10_{1}^{-}$	0.406	0.375	61	48	0.66(5)	0.52	$16^+_1 \rightarrow 15^+_1$	0.203	0.856	32	23	1.6(5)	1.15	
9 ⁻ 1	0.716	-	100	100			$\rightarrow 14^+_1$	0.748	-	100	100			
$12^1 \rightarrow 11^1$	0.330	0.337	29	27	0.47(6)	0.45	$17_{1}^{+} \rightarrow 16_{1}^{+}$	0.132	1.128	61	57	1.95(10)	1.83	
$\rightarrow 10^{-}_{1}$	0.751	-	100	100			$\rightarrow 15^+_1$	0.615	-	100	100			
$13^1 \rightarrow 12^1$	0.273	0.294	30	18	0.62(5)	0.38	$18_{1}^{+} \rightarrow 17_{1}^{+}$	0.092	0.351	24	11	1.3(3)	0.61	
$\rightarrow 11^{-}_{1}$	0.763	-	100	100				0.580	-	100	100			
$14_1^- \rightarrow 13_1^-$	0.219	0.261	20	13	0.52(5)	0.34	$19^{+}_{1} \rightarrow 18^{+}_{1}$	0.051	1.103	35	51	1.6(5)	2.32	
	0.760	-	100	100			→17 ⁺	0.476	-	100	100			
$15_1^- \rightarrow 14_1^-$	0.179	0.246	-	11	-	0.34	$20^{+}_{1} \rightarrow 19^{+}_{1}$	0.031	0.233	26	14	1.3(3)	0.71	
	0.727	1.77		100			-→18 ⁺	0.328	-	100	100			
$16_1^- \rightarrow 15_1^-$	0.142	0,201	14	8	0.5(1)	0.29	21 ⁺ →20 ⁺	0.018	0.980	-	63	-	3.52	
-+141	0.697	-	100	100			-19+	0 279	-	-	100			
$17_1^- \rightarrow 16_1^-$	0.113	0.215	9	9	0.35(5)	0.34	-101	0.217		_	100			
-15	0.635	-	100	100										
$18^{-}_{1} \rightarrow 17^{-}_{1}$	0.087	0.146	-	6	-	0.25								
→16 ₁	0.581	-	-	100										
$19^{-}_{1} \rightarrow 18^{-}_{1}$	0.066	0.192	-	9	-	0.38								
17_1	0.508	-	-	100										
8 ⁺ ₂ →7 ⁺ ₅	0.721	2.103	100	100	-	-			132	Dr				
$9_1^+ - 8_2^+$	0.714	1.365	-	100	-	2.69								
$\rightarrow 7^+_5$	0.507	-	-	0.5										
$10^+_1 \rightarrow 9^+_1$	0.727	1.684	100	100	3.3(5)	2.87								
	0.587	-	3	3										
111 101	0.629	1.408	100	100	2.3(3)	1.86								
$\rightarrow 9^+_1$	0.758	-	16	19										
$12^+_1 \rightarrow 11^+_1$	0.530	1.457	100	100	2.5(3)	1.83								
→10 ⁺	0.798		66	91										



## Transitional SU(3) - O(6) ¹²⁶Pr nucleus



Fig. 12. The negative parity (a) and positive parity (b) yrast states in the Z-nucleus  $^{125}\mathrm{Pr}$  are compared with the experimental states of  $^{127}\mathrm{Pr}.$ 





Fig. 13. The yrast sequence of high angular momentum positive parity states based on the  $\pi h_{11/2} \otimes v h_{11/2}$  configuration is compared with experimental counterpart in ¹²⁶Pr. The assignment for the band head (i.e., the lowest observed state) is 8⁺.

Fig. 14. IBFFM (dashed line) and experimental (solid line) [E(I) - (E(I-1))]/2I vs *I* plots for the  $\pi h_{11/2} \otimes v h_{11/2}$  yrast band in ¹²⁶Pr. The state 8+ is assigned to the band head.



The IBM prediction for low angular momentum states in ¹²⁶Pr. These states are based on positive parity proton and neutron configurations.

The IBFFM analysis predicts the existence of another isomer in ¹²⁶Pr 5⁺ at  $\approx$  150 keV excitation energy - isomeric character depends strongly on the choice of the proton-neutron interaction. In the present calculation  $5_1^+$  is below  $4_1^+$  and therefore it is an isomer with a possible  $\gamma$  decay to  $2_1^+$ . This transition is slow enough to allow for a  $\beta$  decay that has been reported

## ¹⁹⁸Au





### Chiral bands in IBFFM

The rotation of triaxial nuclei may give rise to pairs of identical  $\Delta I = 1$ bands with the same parity in odd-odd nuclei - chiral doublet bands. These structures arise from configurations in which the angular momenta of the valence proton, the valence neutron and the core are mutually perpendicular, and can be arranged to form two systems that differ by intrinsic chirality, a left- and a right-handed system. When chiral symmetry is thus broken in the bodyfixed frame, the restoration of the symmetry in the laboratory frame results in the occurrence of degenerate doublet  $\Delta I = 1$  bands. It has been suggested that such nearly degenerate rotational bands might be observed in the region of transitional nuclei with  $A \approx 130$ . A number of nuclei in this region are susceptible to triaxial deformation and the yrast bands of odd-odd nuclei are built on the  $\pi h_{11/2}$  particle-like –  $\nu h_{11/2}$  hole-like configuration. The existence of self-consistent rotating mean field solutions with chiral character has been demonstrated for ¹³⁴Pr and ¹⁸⁸Ir. The theoretical prediction of chiral doublet structures has prompted quite a number of experimental studies of odd-odd N = 75 and N = 73 isotones in the  $A \approx 130$  region, and nearly degenerate  $\Delta I = 1$  bands built on the  $\pi h_{11/2} \otimes \nu h_{11/2}$  configuration have been identified in many of these nuclei. Sideband partners of the  $\pi h_{11/2} \otimes \nu h_{11/2}$  yrast band have been identified in  ${}_{55}$ Cs,  ${}_{57}$ La and  ${}_{61}$ Pm N = 75 isotones of  134 Pr. For  134 Pr the energy spacing between the doublet rotational bands gradually decreases from  $\approx 0.19$  MeV at low spin to the point where the two bands cross between I = 14and I = 15. For the other N = 75 isotones the two lowest  $\pi h_{11/2} \otimes \nu h_{11/2}$ bands are almost parallel in the E vs I plot, and the energy spacing between the corresponding states with the same spin is  $\approx 0.3$  MeV.

# +

In ¹³⁴Pr the two lowest, yrast and yrare  $\pi h_{11/2} \otimes \nu h_{11/2}$  bands have been interpreted as chiral restored doublet bands. In order to explain similar doublet bands in the other N = 75 odd-odd nuclei, it has been suggested that in these cases the triaxial core deformation is not stable, but perhaps more  $\gamma$ -soft, resulting in collective chiral vibration of the core angular momentum between the left- and right-handed chiral systems. It has to be emphasized that in all studies of chiral doublet bands it has been argued that the empirical separation of  $\leq 300$  keV is too small for the sideband to be interpreted as a band built either on the unfavored signature of the proton orbital, or on the  $\gamma$ -vibrational excitation. A  $\gamma$ -vibration coupled to the yrast band has been ruled out because in this region the  $\gamma$ -vibration energies are  $\geq 600$  keV.



IBM/IBFM calculations are performed in the laboratory frame, and the results can be directly compared with experimental data. All states within the model space and their electromagnetic properties are compared with experiment, rather than just band-head energies. In the particular case of ¹³⁴Pr, it will be assumed that the nucleus is triaxial but, unlike in a geometric description based on the tilted axis cranking model, the occurrence of nearly degenerate doublet  $\Delta I = I$  bands is not related to a definite alignment of the angular momenta of the odd proton and odd neutron along the body-fixed axes of the core nucleus.

The spectrum of the core nucleus  ${}^{134}_{58}\text{Ce}_{76}$  is described by the Hamiltonian

 $H_{IBM} = \epsilon_d \ \hat{n}_d + p \ P \cdot P + k' \ L \cdot L + k \ Q \cdot Q + \Theta_3 \left[ (d^{\dagger} \ d^{\dagger})_2 \ d^{\dagger} \right]_3 \cdot \left[ (\tilde{d} \ \tilde{d})_2 \ \tilde{d} \right]_a$ 

The first four terms represent the standard Hamiltonian of the Interacting Boson Model (IBM-1). The cubic interaction in the last term, with the strength parameter  $\Theta_3$ , introduces a degree of triaxiality. The best agreement with the experimental spectrum is obtained for the following choice of parameters:

 $\epsilon_d = 0.75, \ p = 0.25, \ k' = 0.014, \ k = -0.003, \ \Theta_3 = 0.025$  (all in MeV), and  $\chi = -0.3$  in the quadrupole operator  $Q_2$ . This value of  $\chi$  is also used in the boson quadrupole operator appearing in the boson-fermion dynamical interaction, as well as in the E2 operator.

With the inclusion of the three-body term in the boson Hamiltonian, the boson quadrupole operator appearing in the dynamical boson-fermion interaction and in the E2 operator should also be extended to higher order. The standard boson quadrupole operator is modified by including the additional term

$$\eta \left[ (d^{\dagger} \ \tilde{d})_3 \ (d^{\dagger} \ \tilde{d})_3 \right]_2$$

This term is included in the dynamical boson-fermion interaction and in the E2 operator, with the strength parameter  $\eta = -0.46$  MeV.



The spectrum of positive-parity states in ¹³⁴Pr, based on negative-parity orbitals of the odd proton and odd neutron, is calculated by using the quasiparticle energies, occupation probabilities and boson-fermion interaction strengths obtained in the IBFM calculations of negative-parity spectra in ¹³⁵₅₉Pr₇₆ and ¹³³₅₈Ce₇₅. Most of the model parameters, therefore, are determined by the structure of collective and single-nucleon states in the even-even and odd-even neighbors of ¹³⁴Pr and, in principle, only the residual interaction between the odd proton and odd neutron has to be adjusted to the experimental data in the odd-odd nucleus.





The yrast and yrare  $\pi h_{11/2} \otimes \nu h_{11/2}$  bands in ¹³⁴Pr calculated in the IBFFM for the ¹³⁴Ce core without triaxiality (left panel,  $\Theta_3 = 0$ ), and with stable triaxial deformation (right panel,  $\Theta_3 = 0.03$  MeV).

For  $\Theta_3 = 0$  the core is  $\gamma$ -soft and the two bands do not cross or become degenerate. Rather, an almost constant energy spacing  $\approx 400$  keV between the two bands is predicted. For a stable triaxial deformation ( $\Theta_3 = 0.03$  MeV) the energy difference between the yrast and yrare bands gradually decreases, and between angular momenta 16⁺ and 17⁺ the two bands cross. Except for the exact position of the band crossing (which can be also affected by four-quasiparticle configurations not included in the model space), the "energy vs spin" diagram is in excellent agreement with the experimentally observed evolution of the two lowest positive-parity bands in ¹³⁴Pr. The results are also in agreement with the conclusions of calculations in the body-fixed frame, where it has been suggested that in the odd-odd N = 75 nuclei other than ¹³⁴Pr the triaxial core deformation is not stable, rather it is  $\gamma$ -soft, resulting in the two lowest  $\pi h_{11/2} \otimes \nu h_{11/2}$ bands being almost parallel in the E vs I plot, with the energy spacing of  $\approx 0.3$ MeV. In ¹³⁴Pr, on the other hand, the stable triaxial deformation causes the two lowest positive-parity bands to become almost degenerate.



The yrast band is basically built on the ground-state band of the even-even core. With increasing angular momentum the admixture of the  $\gamma$ -band of the core becomes more pronounced. The structure of the second – yrare band, however, is that of the odd proton and odd neutron coupled to the  $\gamma$ -band of the core, especially in the lower part of the band. With increasing angular momentum both ground-state band and  $\gamma$ -band components contribute to the wave functions of the yrare band in ¹³⁴Pr. In the region of band crossing, in particular, the wave functions of the yrare band contain sizeable components of the higher-lying core structures. The IBFFM prediction, therefore, is that the two lowest  $\pi h_{11/2} \otimes \nu h_{11/2}$  bands in ¹³⁴Pr are built, in leading order, on the ground-state band and the  $\gamma$ -band of the core nucleus, respectively. Their wave functions closely follow the triaxial structure of the core nucleus. This result is at variance with previous analyses of the doublet bands in ¹³⁴Pr, based on the tilted axis cranking approach, which have basically excluded the possibility that the second band might be built on the  $\gamma$ -band of the core. This was done only on the basis of the relatively high excitation energy of the  $\gamma$ -vibration.

As the IBFFM calculations are performed in the laboratory frame, they cannot determine the alignment of the odd particles along the body fixed axes. The IBFFM analysis indicates that the  $\gamma$  degree of freedom plays an important role in the formation of these bands.

# Nuclear Theory: β decay in the interacting boson-fermion model

## S. Brant

# University of Zagreb, Croatia

# E.mail: Brant@phy.hr

# 4.

# β decay in the interacting boson-fermion model

## OBJECTIVES



To test the nuclear model by analyzing experimental data

Wave functions (two odd-even and one even-even nucleus are involved)

Transition operators

*

To provide reliable information for astrophysical applications

The process is very sensitive to configuration mixing both in the initial and final states. A detailed knowledge of the wavefunctions is required. Beta decay properties can be calculated by using:

Shell model (in light nuclei and in medium-mass and heavy nuclei in the neighborhood of doubly magic nuclei)

Other models for medium-mass and heavy nuclei.

Example: Simple pairing theory



Overestimates the Gamow-Teller strengths by a large factor (up to a factor 70 !!!!!)

How to account for the large hindrance:

Nuclear deformation Mixing with 2p-2h states Mesonic degrees of freedom

In the IBFM there is NO quenching factor (once the wave functions have been calculated, the calculation of beta decay properties is parameter free), or the quenching factor is SMALL.





Comparison of experimental Gamow-Teller matrix elements with pairing theory and results of the calculation using the interacting boson-fermion model (IBFM).

Comparison between experimental Gamow – Teller matrix elements (triangles) and those obtained using IBFM renormalized by a factor 3.5 (continuous line).



$$+\frac{1}{2}\xi_{2}\left(\left(d_{\nu}^{\dagger}s_{\pi}^{\dagger}-d_{\pi}^{\dagger}s_{\nu}^{\dagger}\right)\cdot\left(\tilde{d}_{\nu}s_{\pi}-\tilde{d}_{\pi}s_{\nu}\right)\right)+\sum_{K=1,3}\xi_{K}\left(\left[d_{\nu}^{\dagger}d_{\pi}^{\dagger}\right]^{(K)}\cdot\left[\tilde{d}_{\pi}\tilde{d}_{\nu}\right]^{(K)}\right)\\+\frac{1}{2}\sum_{L=0,2,4}c_{L}^{\nu}\left(\left[d_{\nu}^{\dagger}d_{\nu}^{\dagger}\right]^{(k)}\cdot\left[\tilde{d}_{\nu}\tilde{d}_{\nu}\right]^{(k)}\right)+\frac{1}{2}\sum_{L=0,2,4}c_{L}^{\pi}\left(\left[d_{\pi}^{\dagger}d_{\pi}^{\dagger}\right]^{(k)}\cdot\left[\tilde{d}_{\pi}\tilde{d}_{\pi}\right]^{(k)}\right)$$

$$Q_{\nu}^{\mathsf{B}} = d_{\nu}^{\dagger} s_{\nu} + s_{\nu}^{\dagger} \tilde{d}_{\nu} + \chi_{\nu} [d_{\nu}^{\dagger} \tilde{d}_{\nu}]^{(2)}$$
$$Q_{\pi}^{\mathsf{B}} = d_{\pi}^{\dagger} s_{\pi} + s_{\pi}^{\dagger} \tilde{d}_{\pi} + \chi_{\pi} [d_{\pi}^{\dagger} \tilde{d}_{\pi}]^{(2)}$$

IBM2 parameters. The unit is MeV except for the dimensionless  $\chi_{\nu}$ . The parameters  $\chi_{\pi} = -0.80$  and  $\xi_1 = \xi_2 = 0.24$  MeV,  $\xi_3 = -0.18$  MeV are fixed.

odd nuclei	core nucleus	$\epsilon_d$	$\kappa$	$\chi_{ u}$	$c_0^{\nu}$	$c_2^{\nu}$
¹²⁵ Cs	¹²⁴ Xe	0.70	-0.145	0.00	0.05	-0.10
¹²⁵ Xe, ¹²⁷ Cs	¹²⁶ Xe	0.70	-0.155	0.20	0.10	-0.10
¹²⁷ Xe, ¹²⁹ Cs	¹²⁸ Xe	0.70	-0.170	0.33	0.30	0.00
¹²⁹ Xe	¹³⁰ Xe	0.76	-0.190	0.50	0.30	0.10

$$H^{\mathsf{F}} = \sum_{i} \epsilon_{i} n_{i}$$
 Hamiltonian of the odd fermion  
 $\epsilon_{i}$  is the quasi-particle energy of the *i*th orbital

BCS

 $\epsilon_i$  is the quasi-particle energy of the  $i{\rm th}$  orbital  $n_i$  is its number operator

Interaction between bosons and the odd fermion :

$$V^{\mathsf{BF}} = \sum_{i,j} \Gamma_{ij} \left( [a_i^{\dagger} \tilde{a}_j]^{(2)} \cdot Q_{\rho}^{\mathsf{B}} \right) + \sum_{i,j} \Gamma_{ij}' \left( [a_i^{\dagger} \tilde{a}_j]^{(2)} \cdot Q_{\rho'}^{\mathsf{B}} \right) + \sum_{i,j} A_i n_i n_{d\rho} + \sum_{i,j} A_i' n_i n_{d\rho'} + \sum_{i,j} \Lambda_{ki}^j \left\{ : \left[ \left[ d_{\rho}^{\dagger} \tilde{a}_j \right]^{(k)} a_i^{\dagger} s_{\rho} \right]^{(2)} : \cdot \left[ s_{\rho'}^{\dagger} \tilde{d}_{\rho'} \right]^{(2)} + H.c. \right\} + B J \cdot L_{\rho} + B' J \cdot L_{\rho'}.$$

 $\rho$  and  $\rho'$  denote  $\pi$  ( $\nu$ ) and  $\nu$  ( $\pi$ ) if the odd fermion is a proton (a neutron).

### Orbital dependence of the interaction strengths



**Electromagnetic transition operators** 





	$d_{5/2}$	97/2	$s_{1/2}$	d3/2	$h_{11/2}$
¹²⁵ Xe	0.00	0.30	1.55	2.00	1.30
¹²⁷ Xe	0.00	0.35	1.55	2.00	1.30
¹²⁹ Xe	0.00	0.40	1.60	2.00	1.30
	isotop	ре Г		A	Λ
100 Contraction 1	125X	e 0.3	39 -0	).42	0.40
	¹²⁷ X	e 0.4	4 -0	).42	0.40
	129X	e 0.5	50 -0	).42	0.40
	125×e 127×e 129×e	$\begin{array}{c c} & d_{5/2} \\ \hline 125 \times e & 0.00 \\ 127 \times e & 0.00 \\ \hline 129 \times e & 0.00 \\ \hline 129 \times e & 0.00 \\ \hline 125 \times \\ 127 \times \\ 129 \times \\ 129 \times \end{array}$	$\begin{array}{c ccccc} & d_{5/2} & g_{7/2} \\ \hline 125 \times e & 0.00 & 0.30 \\ 127 \times e & 0.00 & 0.35 \\ 129 \times e & 0.00 & 0.40 \\ \hline & & & & & \\ 125 \times e & 0.3 \\ 127 \times e & 0.4 \\ 129 \times e & 0.5 \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$





B(E2) values and magnetic moments. The symbol • with the error bar denotes the experimental data, while  $\times$  shows the calculated values

B(E2) and B(M1) values, static moments, branching ratios are calculated for the sequence of Cs and Xe nuclei



Wave functions are realistic The Fermi  $\sum_k t^{\pm}(k)$  and the Gamow-Teller  $\sum_k t^{\pm}(k)\sigma(k)$  transition operators can be expressed in the framework of IBFM2. They can be constructed from the transfer operators.

$$A_m^{\dagger(j)} = \zeta_j a_{jm}^{\dagger} + \sum_{j'} \zeta_{jj'} s^{\dagger} [\tilde{d}a_{j'}^{\dagger}]_m^{(j)}$$
$$(\Delta n_j = 1, \ \Delta N = 0)$$
$$B_m^{\dagger(j)} = \theta_j s^{\dagger} \tilde{a}_{jm} + \sum_{j'} \theta_{jj'} [d^{\dagger} \tilde{a}_{j'}]_m^{(j)}$$
$$(\Delta n_j = -1, \ \Delta N = 1)$$

The former creates a fermion, while the latter annihilates a fermion simultaneously creating a boson. Either operator increases the quantity  $n_j + 2N$  by one unit. The conjugate operators are:

The asterisks mean complex conjugate. These operators decrease the quantity  $n_j + 2N$  by one unit.

The IBFM image of the Fermi  $\sum_k t^{\pm}(k)$  and the Gamow-Teller transition operator  $\sum_k t^{\pm}(k)\sigma(k)$ 

$$O^{\mathsf{F}} = \sum_{j} -\sqrt{2j+1} \left[ P_{\nu}^{(j)} P_{\pi}^{(j)} \right]^{(0)}$$
$$O^{\mathsf{GT}} = \sum_{j'j} \eta_{j'j} \left[ P_{\nu}^{(j')} P_{\pi}^{(j)} \right]^{(1)}$$

$$\eta_{j'j} = -\frac{1}{\sqrt{3}} < l'\frac{1}{2}; j'||\sigma||l\frac{1}{2}; j >$$



$$\langle M_{\rm F} \rangle^2 = \frac{1}{2I_i + 1} |\langle I_f||O^{\rm F}||I_i \rangle|^2$$

$$\langle M_{\rm GT} \rangle^2 = \frac{1}{2I_i + 1} |\langle I_f||O^{\rm GT}||I_i \rangle|^2$$

$$ft = \frac{6163}{\langle M_{\rm F} \rangle^2 + (G_{\rm A}/G_{\rm V})^2 \langle M_{\rm GT} \rangle^2 }$$

in units of second where  $(G_{\rm A}/G_{\rm V})^2=1.59$ 

The coefficients  $\eta_j,~\eta_{jj'},~\theta_j,~\theta_{jj'}$  appearing in transfer operators

$$\begin{split} \zeta_{j} &= u_{j} \frac{1}{K_{j}'} & N \text{ is } N_{\pi} \text{ or } N_{\nu}, \text{ depending on the transfer operator, and } K, K_{j}', K_{j}'' \text{ are determined by} \\ \zeta_{jj'} &= -v_{j}\beta_{j'j} \left(\frac{10}{N(2j+1)}\right)^{1/2} \frac{1}{KK_{j}'} & K = \left(\sum_{jj'}\beta_{jj'}^{2}\right)^{1/2} \\ \theta_{j} &= \frac{v_{j}}{\sqrt{N}K_{j}''} & \sum_{\alpha J} < \operatorname{odd}(\alpha J) |A^{\dagger j}||\operatorname{even}(0_{1}^{+})^{2} = (2j+1)u_{j}^{2} \\ \theta_{jj'} &= u_{j}\beta_{j'j} \left(\frac{10}{2j+1}\right)^{1/2} \frac{1}{KK_{j}''} & \sum_{\alpha J} < \operatorname{even}(0_{1}^{+}) |B^{\dagger j}||\operatorname{odd}(\alpha J)^{2} = (2j+1)v_{j}^{2} \end{split}$$

When the odd fermion is a hole in respect to the boson core,  $\boldsymbol{u}_j$  and  $\boldsymbol{v}_j$  have to be interchanged

Beta-decay rates from  A Cs to  A Xe shown in terms of log₁₀ ft values. The symbol • with the error bar denotes experimental data, while  $\times$  presents the calculated value.



Rh 📥 Pd	
---------	--

A	=	105,	107,	109

U(5) ← → O(6) nuclei









 $\log_{10} ft$  values in the decays  ${}_{45}\text{Rh}_{N+1} \rightarrow {}_{46}\text{Pd}_N$ . The experimental data are presented by • while the calculated values are shown by  $\times$ .





## The exclusion of intruder components does not influence strongly the theoretical values of static moments

## and branching ratios



### Branching ratios in ⁷¹As

level (MeV)	transition	$I_{\gamma}(IBFM2)$	$I_{\gamma}(EXP)$
0.143	$1/2^1 \rightarrow 5/2^1$	100	100
0.147	$3/2^{-}_{1} \rightarrow 1/2^{-}_{1}$	0.0	
	$3/2^{-}_{1} \rightarrow 5/2^{-}_{1}$	100	100
0.506	$3/2^{-}_{2} \rightarrow 3/2^{-}_{1}$	100	100 (5)
	$3/2^{-}_{2} \rightarrow 1/2^{-}_{1}$	7.1	27 (14)
	$3/2^{-}_{2} \rightarrow 5/2^{-}_{1}$	8.2	
0.829	$3/2^{-}_{3} \rightarrow 3/2^{-}_{2}$	9.3	
	$3/2^{-}_{3} \rightarrow 3/2^{-}_{1}$	100	100 (14)
	$3/2^3 \rightarrow 1/2^1$	30.3	9.3 (7)
	$3/2^{-}_{3} \rightarrow 5/2^{-}_{1}$	29.4	
0.870	$5/2^{-}_{2} \rightarrow 3/2^{-}_{3}$	0.0	
	$5/2^{-}_{2} \rightarrow 3/2^{-}_{2}$	28.8	
	$5/2^{-}_{2} \rightarrow 3/2^{-}_{1}$	36.4	40 (1)
	$5/2^{-}_{2} \rightarrow 1/2^{-}_{1}$	27.0	1.8 (7)
	$5/2^{-}_{2} \rightarrow 5/2^{-}_{1}$	100	100.0(7)
0.925	$7/2^{-}_{1} \rightarrow 5/2^{-}_{2}$	0.0	
	$7/2^{-}_{1} \rightarrow 3/2^{-}_{3}$	0.0	
	$7/2^{-}_{1} \rightarrow 3/2^{-}_{2}$	0.0	
	$7/2^{-}_{1} \rightarrow 3/2^{-}_{1}$	1.1	5.8 (16)
	$7/2^{-}_{1} \rightarrow 5/2^{-}_{1}$	100	100 (3)

### Branching ratios in 69Ge

level (MeV)	transition	$I_{\gamma}(IBFM2)$	$I_{\gamma}(EXP)$
0.087	$1/2^1 \rightarrow 5/2^1$	100	100
0.233	$3/2^{-}_{1} \rightarrow 1/2^{-}_{1}$	43.2	48.3 (13)
	$3/2^{-}_{1} \rightarrow 5/2^{-}_{1}$	100	100 (3)
0.374	$3/2^{-}_{2} \rightarrow 3/2^{-}_{1}$	0.7	4.6 (8)
	$3/2^{-}_{2} \rightarrow 1/2^{-}_{1}$	100	100.0 (15)
	$3/2^{-}_{2} \rightarrow 5/2^{-}_{1}$	0.1	31.5 (8)
0.862	$7/2_1^- \rightarrow 3/2_2^-$	0.4	0.76 (13)
	$7/2^{-}_{1} \rightarrow 3/2^{-}_{1}$	0.1	8.4 (21)
	$7/2^{-}_{1} \rightarrow 5/2^{-}_{1}$	100	100 (3)
0.933	$5/2^{-}_{2} \rightarrow 7/2^{-}_{1}$	0.0	
	$5/2^{-}_{2} \rightarrow 3/2^{-}_{2}$	0.5	32 (7)
	$5/2^{-}_{2} \rightarrow 3/2^{-}_{1}$	16.7	8
	$5/2^{-}_{2} \rightarrow 1/2^{-}_{1}$	35.5	24 (7)
	$5/2^{-}_{2} \rightarrow 5/2^{-}_{1}$	100	100 (5)
0.995	$1/2^{-}_{2} \rightarrow 5/2^{-}_{2}$	0.0	
	$1/2^{-}_{2} \rightarrow 3/2^{-}_{2}$	7.9	9 (6)
	$1/2^{-}_{2} \rightarrow 3/2^{-}_{1}$	26.8	41 (9)
	$1/2^{-}_{2} \rightarrow 1/2^{-}_{1}$	0.7	
	$1/2^{-}_{2} \rightarrow 5/2^{-}_{1}$	100	100 (21)

 $\log_{10} ft$  values for levels in ⁶⁹Ge.

 $\log_{10} ft$  values for levels in ⁷¹Ge.

level	log ₁₀ ft (IBFM2)	$\log_{10} ft \ (EXP)$	level	log ₁₀ ft (IBFM2)	$\log_{10} ft (EXP)$
$3/2^{-}_{1}$	5.88	6.05 (2)	$3/2^{-}_{1}$	6.52	7.19(1)
3/22	7.90	7.21 (5)	$3/2^{-}_{2}$	7.79	
$3/2_{3}^{-}$	5.07	6.79 (4)	$3/2_{3}^{-}$	5.73	
$3/2_{4}^{-}$	6.46	6.71 (6)	$3/2_{4}^{-}$	5.21	6.33 (1)
$3/2_{5}^{-}$	6.73	7.02 (6)	$3/2_{5}^{-}$	7.34	6.94 (1)
$5/2^{-}_{1}$	4.26	5.49 (2)	$5/2_{1}^{-}$	4.60	5.85 (1)
5/22	6.65	6.94 (7)	$5/2^{-}_{2}$	6.08	
$5/2_{3}^{-}$	5.33	6.65 (5)	$5/2_{3}^{-}$	5.63	6.87 (2)
$5/2_{4}^{-}$	5.49	6.80 (6)	$5/2_{4}^{-}$	5.55	6.84 (2)
$7/2^{-}_{1}$	7.54	6.98 (5)	$7/2^{-}_{1}$	7.60	8.79 (25)
$7/2^{-}_{2}$	6.54	6.81 (5)	10.076		
$7/2_{3}^{-}$	5.96	6.20 (5)			

The ground states of parent ⁶⁹As and ⁷¹As nuclei are  $5/2_1^-$  levels. The hierarchy of values for transitions into different states of each angular momentum is reproduced for ⁶⁹Ge (except for the transition to the  $3/2_3^-$  level that is predicted to have a rather small  $\log_{10} ft$  value). The same is true for ⁷¹Ge. The theory predicts that the smallest  $\log_{10} ft$  value among all  $3/2^-$  levels in ⁷¹Ge has the  $3/2_4^-$  level. This result is in agreement with the experimental data. The only available experimental  $\log_{10} ft$  value in ⁷³Ge is for the  $1/2_1^-$  level ( $\log_{10} ft = 5.4$ ). The corresponding theoretical value (4.27) is the smallest calculated.

Systematic effect : For most decays the calculated values are smaller than the experimental values

#### a) Wave functions ?

If one takes the transition operators without normalization parameters, then the difference between the calculated and experimental values are caused by the transition matrix elements, that in this case have to be overestimated. This may indicate that other components are admixed in the wave functions (for example those involving intruder states), which would decrease the amplitudes of the present IBFM2 components, leading to an increase of the theoretical  $\log_{10} ft$ values.

# Accurate test of wave functions

#### b) Transfer operators ?

Normalization factors ?

- Additional terms ?
- Normalization factors + Additional terms ?









Sums of spectroscopic strengths



### ⁶²Ni (d, ³He) ⁶¹Co reaction



Sums of spectroscopic strengths





The effect of the additional term



is small

 $Log_{10}$  ft values of the  $\beta$ -decay from the As to the Ge isotopes. The symbol  $\bullet$  shows the experimental values with their errors, while the symbol x shows the results of calculations with the conventional operators. The symbol  $\circ$  shows the results of calculations with the additional d boson number conserving terms.

### CONCLUSIONS

The extensions of IBM with fermion degrees of freedom provide a consistent description of nuclear structure phenomena in:

- spherical nuclei
- deformed nuclei
- transitional nuclei

The structure results from a consistent calculation that includes interaction strengths obtained in the analysis of neighboring nuclei

All calculations are performed in the laboratory frame, and therefore the results can be directly compared with experimental data

The models can be related to the shell model

The symmetry approach can be applyed in special cases

There is a strong evidence that collective and single-particle degrees of freedom are closely related

# **Experimental nuclear structure:**

# F. G. Kondev

## Argonne National Laboratory, USA

E-mail: kondev@anl.gov












## So to summarize ... NUCLEAR PHYSICS IS A BIG CHALLENGE (because of complicated forces, energy scale, and sizes involved) The challenge of understanding how nucleon-nucleon interactions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motions build to create the mean field or how single-particle motion, and security. (intellectually, astrophysics, energy production, and security) (with new facilities just around the corner we have a chance to make major contributions to the knowledge - with advances in theory we have a great chance to understand it all - by compiling & evaluating data we have a chance to support various applications and to preserve the knowledge for future greater for the greater for the greater for the suport for for the suport for the superior. </t















PLUS Efficient, flexible, powerfulinexpensive.         MINUS Count-rate limited, Contaminant (Carbon etc, isotopic impurities) makes absolute identification of new nuclei difficult.         CROSS SECTION LOWER LIMIT ~100 μb       that is, ~10 ⁻⁴ Detection of Residues in Vacuum Mass Separator         PLUS True M/q, even true M measurement. With suitable focal plane detector can b         ULTRA sensitive. Suppresses contaminants.         MINUS Low Efficiency         CROSS SECTION LOWER LIMIT ~100 nb         that is ~10 ⁻⁷ Detection of Residues in Gas Filled Separator         Improves efficiency of vacuum separators, at cost of mass information and clenlines:         In some cases (heavy nuclei) focal plane counters clean up the data for good sensitive.	Detection of Light Charged Partic	les $(\alpha, p, n)$
MINUS Count-rate limited, Contaminant (Carbon etc, isotopic impurities) makes absolute identification of new nuclei difficult. CROSS SECTION LOWER LIMIT ~100 μb that is, ~10 ⁻⁴ Detection of Residues in Vacuum Mass Separator PLUS True M/q, even true M measurement. With suitable focal plane detector can b ULTRA sensitive. Suppresses contaminants. MINUS Low Efficiency CROSS SECTION LOWER LIMIT ~100 nb that is ~10 ⁻⁷ Detection of Residues in Gas Filled Separator Improves efficiency of vacuum separators, at cost of mass information and clenlines: In some cases (heavy nuclei) focal plane counters clean up the data for good sensitive.	PLUS Efficient, flexible, powerfulinexpensive.	
CROSS SECTION LOWER LIMIT ~100 μb that is, ~10 ⁻⁴ <u>Detection of Residues in Vacuum Mass Separator</u> PLUS True M/q, even true M measurement. With suitable focal plane detector can b ULTRA sensitive. Suppresses contaminants. MINUS Low Efficiency CROSS SECTION LOWER LIMIT ~100 nb that is ~10 ⁻⁷ <u>Detection of Residues in Gas Filled Separator</u> Improves efficiency of vacuum separators, at cost of mass information and clenliness In some cases (heavy nuclei) focal plane counters clean up the data for good sensitiv	MINUS Count-rate limited, Contaminant (Carbon etc, is absolute identification of new nuclei difficult.	otopic impurities) makes
Detection of Residues in Vacuum Mass Separator PLUS True M/q, even true M measurement. With suitable focal plane detector can b ULTRA sensitive. Suppresses contaminants. MINUS Low Efficiency CROSS SECTION LOWER LIMIT ~100 nb that is ~10 ⁻⁷ Detection of Residues in Gas Filled Separator Improves efficiency of vacuum separators, at cost of mass information and clenliness In some cases (heavy nuclei) focal plane counters clean up the data for good sensitiv	CROSS SECTION LOWER LIMIT ~100 µb	that is, $\sim 10^{-4}$
PLUS True M/q, even true M measurement. With suitable focal plane detector can b         ULTRA sensitive. Suppresses contaminants.         MINUS Low Efficiency         CROSS SECTION LOWER LIMIT ~100 nb         that is ~10 ⁻⁷ Detection of Residues in Gas Filled Separator         Improves efficiency of vacuum separators, at cost of mass information and clenlines:         In some cases (heavy nuclei) focal plane counters clean up the data for good sensitiv	Detection of Residues in Vacuum M	ass Separator
MINUS Low Efficiency CROSS SECTION LOWER LIMIT ~100 nb that is ~10 ⁻⁷ Detection of Residues in Gas Filled Separator Improves efficiency of vacuum separators, at cost of mass information and clenliness In some cases (heavy nuclei) focal plane counters clean up the data for good sensitiv	PLUS True M/q, even true M measurement. With suitable ULTRA sensitive. Suppresses contaminants.	focal plane detector can be
CROSS SECTION LOWER LIMIT ~100 nb that is ~10 ⁻⁷ <u>Detection of Residues in Gas Filled Separator</u> Improves efficiency of vacuum separators, at cost of mass information and clenliness In some cases (heavy nuclei) focal plane counters clean up the data for good sensitiv	MINUS Low Efficiency	
Detection of Residues in Gas Filled Separator Improves efficiency of vacuum separators, at cost of mass information and clenlines In some cases (heavy nuclei) focal plane counters clean up the data for good sensitiv	CROSS SECTION LOWER LIMIT ~100 nb	that is $\sim 10^{-7}$
Improves efficiency of vacuum separators, at cost of mass information and clenlines In some cases (heavy nuclei) focal plane counters clean up the data for good sensitiv	Detection of Residues in Gas Fille	d Separator
In some cases (heavy nuclei) focal plane counters clean up the data for good sensitiv	Improves efficiency of vacuum separators, at cost of mass	information and clenliness.
	In some cases (heavy nuclei) focal plane counters clean up	the data for good sensitivity



















_γ_ray D	есау			
<i>I</i> ,	$-I_{f} \mid \leq L \leq \mid I_{i} + I_{f}$	1	$I_i^{\pi_i}$	$E_i$
$\Delta \pi(EL) = ($ electric multi	$(-1)^L$ $\Delta \pi (ML)$	$L_{L} = (-1)^{L+1}$ ic multipole	$I_f^{\pi_f}$	$E_f$
dipole	quadrupole	octupole	hexadecapole $E_i = E_i -$	$-E_f$
E1:L=1,yes	E2:L=2,no	E3:L=3,yes	E4:L=4,no	E5:L=5,yes
M1:L=1,no	M2:L=2,yes	M3:L=3,no	M4:L=4,yes	M5:L=5,no
0+	2	8+	1-	+
0 + L = 0	$\begin{array}{c c} 0! & & \downarrow I \\ \hline & & I + & \downarrow I \\ \hline & & E1(M2.) \end{array}$	=1,2&3 $E3) \qquad 6+$ E2(1)	L = 214 M3, E4) 0.	L = 1
Pioneering Science and Technology				25 Office of Science U.S. Department of Energy



$F_{W(N)} = \frac{B(XL)_{Theory}}{B(XL)_{Exp}} = \frac{T_{1/2}^{\gamma}(XL)_{Exp}}{T_{1/2}^{\gamma}(XL)_{Theory}} \qquad \frac{\text{Hindrance Factor: Weisskopf (W): based on spherical shell model potential}}{\text{Nilsson (N): based on deformed Nilsson model potential}}$							
EL	$B(EL)_W, e^2 fm^{2L}$	$T_{1/2}^{\gamma}(EL)_{W}$ , sec	ML	$B(ML)_{W}, \mu_{N}^{2} fm^{2L-2}$	$T_{1/2}^{\gamma}(ML)_{W}$ , sec		
E1	0.06446A ^{2/3}	$6.762A^{-2/3}E_{\gamma}^{-3}\times10^{-15}$	M1	1.7905	$2.202 E_{\gamma}^{-3} \times 10^{-14}$		
E2	0.0594A ^{4/3}	$9.523A^{-4/3}E_{\gamma}^{-5} \times 10^{-9}$	M2	1.6501A ^{2/3}	$3.100 A^{-2/3} E_y^{-5} \times 10^{-8}$		
E3	$0.0594A^2$	$2.044A^{-2}E_{\gamma}^{-7} \times 10^{-2}$	МЗ	$1.6501A^{4/3}$	$6.655A^{-4/3}E_y^{-7}\times 10^{-2}$		
E4	0.06285A ^{8/3}	$6.499A^{-8/3}E_{y}^{-9}\times10^{4}$	M4	$1.7458A^2$	$2.116A^{-2}E_{\gamma}^{-9}\times 10^{5}$		
E5	0.06929A ^{10/3}	$2.893A^{-10/3}E_{y}^{-11} \times 10^{11}$	M5	1.9247 <i>A</i> ^{8/3}	$9.419 A^{-8/3} E_{\gamma}^{-11} \times 10^{11}$		




































































































## **Experimental nuclear structure:**

## F. G. Kondev

## **Argonne National Laboratory, USA**

E-mail: kondev@anl.gov

















































































## Gammasphere Spectrometer







































Science Objective       Isotopes and Lines (MeV)         Understand Type Ia SN explosion ⁵ Ni (0.158, 0.812,)         "Co (0.847, 1.238,)       "Co (0.122)         Understand Core Collapse SN       "SNi (0.158, 0.812,)         "Co (0.122)       "SNi (0.158, 0.812,)         Understand Core Collapse SN       "SNi (0.158, 0.812,)         "Co (0.122)       "SNi (0.158, 0.812,)         "Map the Galaxy in nucleosynthetic radioactivity       "Soi (0.847, 1.238,)         Map the Galaxy in nucleosynthetic radioactivity       "Soi (0.68, 0.078, 1.16)         Map Galactic positron annihilation (0.511)       "Al (1.809, 0.511)         "Map Galactic Positron annihilation (0.511)       "Soi (0.611)         "Map Galactic Novae       "Isotopes (0.511)         "Soi (0.477, 1.238)       "Soi (0.611)         "Map Galactic Novae       "Isotopes (0.511)         "Soi (0.62, 0.754), "Soi (1.577), "Soi (0.511)       "Soi (0.611)         "Soi (0.617, 1.238)       "Soi (1.779), "Soi (0.617, 2.75, 0.511)         Understand the dynamics of Galactic Novae       "Isotopes (0.617, 2.75, 0.511)         Cosmic Ray Interactions with the ISM       "Soi (0.877, 1.238)         Neutron Star Mass.Padius       p.p. (2.223)	Gamma Ray	Lines of the	Cosmos
Science ObjectiveIsotopes and Lines (MeV)Understand Type Ia SN explosion mechanism and dynamics5%Ni (0.158, 0.812,) 5%Co (0.847, 1.238,) 5%Co (0.122)Understand Core Collapse SN explosion mechanism and dynamics5%Ni (0.158, 0.812,) 5%Co (0.122)Map the Galaxy in nucleosynthetic radioactivity5%Co (0.847, 1.238,) 5%Co (0.122)Map the Galaxy in nucleosynthetic radioactivity2%Al (1.809, 0.511) 6%Fe, 6%Co (1.173, 1.332) 4%Ti (0.068, 0.078, 1.16)Map Galactic positron annihilation radiatione*-e*annihilation (0.511, 3 photon continuum) SN Ia 5%Co (0.511) 2%Al and 4%Ti positrons (0.511) 1%Al 10, 0%Fe positrons (0.511) 2%Al (1.275, 0.511)Understand the dynamics of Galactic Novae1%N, 1415O, 1%F positrons (0.511) 7Be (0.478, 2%Na (1.275, 0.511)) 7Be (0.477, 1.238)Neutron Star Mass-Radius Neutron Star Mass-Radiusp.p.0(2.22)			
Understand Type Ia SN explosion ^{5K} Ni (0.158, 0.812,)         SCO (0.847, 1.238,) ^{5C} Co (0.847, 1.238,)         Win (0.158, 0.812,) ^{5C} Co (0.122)         Understand Core Collapse SN ^{5C} Co (0.847, 1.238,)         wynamics ^{5C} Co (0.122)         Map the Galaxy in ^{5C} Co (0.122)         nucleosynthetic radioactivity ^{6D} Fe, ^{6O} Co (1.173, 1.332) ⁴⁴ Ti (0.068, 0.078, 1.16) ^{6D} Map Galactic positron         annihilation radiation       e [*] -e ⁻ annihilation (0.511)         SN Ia ^{5C} Co positrons (0.511) ^{SC} N Ia ^{5C} Co positrons (0.511) ^{SC} Al and ⁴⁴ Ti positrons (0.511) ^{SC} N Ia ^{6C} Co (1.173, 2.32) ^{VIII} Understand the dynamics of ¹³ N, ¹⁴¹⁵ O, ¹³ F positrons (0.511) ^{SC} Al (1.809, 0.511) ^{SC} N Ia ^(1.275, 0.511) ^{SC} Al (1.309, 2.754), ^{SC} Si(1.779), ^{SC} Fe(0.847, 1.238) ^{SC} Si(1.779), ^{SC} Fe(0.847, 1.238) ^{SC} Al (1.309, 2.754), ^{SC} Si(1.779), ^{SC} Ster (0.847		Science Objective	Isotopes and Lines (MeV)
Understand Core Collapse SN explosion mechanism and dynamics ⁵⁶ Ni (0.158, 0.812,)         Map the Galaxy in nucleosynthetic radioactivity ⁵⁷ Co (0.122), ²⁶ Al (1.809, 0.511) ⁶⁰ Fe, ⁶⁰ Co (1.173, 1.332) ⁶⁰ Fe, ⁶⁰ Co (1.173, 1.332) ⁶⁰ Ti (0.068, 0.078, 1.16) ⁶⁰ Fe, ⁶⁰ Co (1.173, 1.332) ⁶⁰ Te, ⁶⁰ Co (1.173, 1.332) ⁶¹ Ti (0.068, 0.078, 1.16) ⁶⁰ Te, ⁶⁰ Co (1.173, 1.332) ⁶¹ Ti (0.068, 0.078, 1.16) ⁶⁰ Te, ⁶⁰ Co (1.173, 1.332) ⁶¹ Ti (0.068, 0.078, 1.16) ⁶⁰ Te, ⁶⁰ Co (1.173, 1.332) ⁶¹ Ti (0.068, 0.078, 1.16) ⁶¹ Te, ⁶⁰ Co (1.173, 1.332) ⁶¹ Ti (0.068, 0.078, 1.16) ⁶¹ Te, ⁶⁰ Co (1.173, 1.332) ⁶¹ Ti (0.068, 0.078, 1.16) ⁶¹ Te, ⁶⁰ Co (1.173, 1.332) ⁶¹ Ti (0.068, 0.078, 1.16) ⁶¹ Te, ⁶⁰ Co (1.173, 1.332) ⁶¹ Ti (0.068, 0.078, 1.16) ⁶¹ Te, ⁶⁰ Co (1.173, 1.332) ⁶¹ Ti (0.068, 0.078, 1.16) ⁶¹ Te, ⁶⁰ Co (1.173, 1.332) ⁶¹ Ti positrons (0.511) ⁶² Galactic Novae ⁷¹ N ( ⁶¹ C) ⁷¹ S (6.178, ⁶² S (5.11) ⁷¹ Co (6.1, ²⁰ Ne(1.634), ²¹ Ma (1.275, 0.511) ⁷² Co (4.1, ¹⁶ O (6.1, ²⁰ Ne(1.634), ²¹ Ma (1.369, 2.754), ²⁵ Si(1.779), ⁵⁵ Fe(0.847, 1.238) ⁷¹ Neutron Star Mass-Radius ⁷¹ N ( ⁶² C (2.22))		Understand Type Ia SN explosion mechanism and dynamics	⁵⁶ Ni (0.158, <i>0.812</i> ,) ⁵⁶ Co ( <i>0.847</i> , <i>1.238</i> ,) ⁵⁷ Co (0.122)
Map the Galaxy in nucleosynthetic radioactivity       2 ⁶ Al (1.809, 0.511) ⁶⁰ Fe, ⁶⁰ Co (1.173, 1.332) ⁴⁴ Ti (0.068, 0.078, 1.16)         Map Galactic positron annihilation radiation       e*-e* annihilation (0.511, 3 photon continuum)         SN Ia ⁵⁶ Co positrons (0.511) ²⁶ Al (1.809, 0.511) ²⁶ Al (1.173, 1.332) ⁴⁴ Ti (0.068, 0.078, 1.16)         Map Galactic positron annihilation radiation       e*-e* annihilation (0.511, 3 photon continuum)         SN Ia ⁵⁶ Co positrons (0.511) ²⁶ Al and ⁴⁴ Ti positrons (0.511) ²⁶ Al (1.10, ¹⁸ F positrons (0.511) ²⁶ Al (1.275, 0.511) ²⁶ Co (1.173, 1.332) ⁴⁴ Ti (1.0, ¹⁸ F positrons (0.511) ²⁶ Al (1.10, ¹⁸ F positrons (0.511) ²⁶ Al (1.10, ¹⁸ F positrons (0.511) ²⁶ Co (1.173, 1.238) ²⁶ Al (1.275, 0.511) ²⁶ Co (1.173, 1.238) ²⁶ Al (1.275, 0.511) ²⁶ Al (1.10, ²⁰ F positrons (0.511) ²⁶ Al (1.309, ²⁷ F), ²⁶ F (0.847, 1.238) ²⁶ Co (1.173, 1.238) ²⁶ Al (1.309, ²⁷ F), ²⁶ F (0.847, 1.238) ²⁶ Al (1.309, ²⁷ Al (1.275, 0.511) ²⁶ Al (1.309, 2.754), ²⁸ Si(1.779), ⁵⁶ F (0.847, 1.238) ²⁶ Al (1.208, ²⁷ Al (1.208) <td rowspan="3"></td> <td>Understand Core Collapse SN explosion mechanism and dynamics</td> <td>³⁶Ni (0.158, <b>0.812</b>,) ⁵⁶Co (<b>0.847</b>, <b>1.238</b>,) ⁵⁷Co (0.122), ²⁶A1 (<b>1.809</b>, <b>0.511</b>)</td>		Understand Core Collapse SN explosion mechanism and dynamics	³⁶ Ni (0.158, <b>0.812</b> ,) ⁵⁶ Co ( <b>0.847</b> , <b>1.238</b> ,) ⁵⁷ Co (0.122), ²⁶ A1 ( <b>1.809</b> , <b>0.511</b> )
Map Galactic positron annihilation radiation       e*-e* annihilation (0.511, 3 photon continuum) SN Ia ⁵⁶ Co positrons (0.511)         Understand the dynamics of Galactic Novae ¹³ N, ¹⁴¹⁵ O, ¹³ F positrons (0.511)         Understand the dynamics of Galactic Novae ¹³ N, ¹⁴¹⁵ O, ¹³ F positrons (0.511)         Cosmic Ray Interactions with the ISM ¹² C (4.4), ¹⁶ O (6.1), ²⁰ Ne(1.634), ²⁴ Mg(1.369, 2.754), ²⁸ Si(1.779), ⁵⁶ Fe(0.847, 1.238)         Neutron Star Mass-Radius       npn(2.223)		Map the Galaxy in nucleosynthetic radioactivity	²⁶ A1 ( <i>1.809</i> , <i>0.511</i> ) ⁶⁰ Fe, ⁶⁰ Co ( <i>1.173</i> , <i>1.332</i> ) ⁴⁴ Ti (0.068, 0.078, <i>1.16</i> )
Understand the dynamics of Galactic Novae ¹³ N, ^{14,15} O, ¹⁸ F positrons (0.511)           'Be (0.478), ²² Na (1.275, 0.511)         'Be (0.478), ²² Na (1.275, 0.511)           Cosmic Ray Interactions with the ISM ¹² C (4.4), ¹⁶ O (6.1), ²⁰ Ne(1.634), ²⁴ Mg(1.369, 2.754), ²⁸ Si(1.779), ⁵⁶ Fe(0.847, 1.238)           Neutron Star Mass-Radius         npn(2.223)		Map Galactic positron annihilation radiation	e [*] -e ⁻ annihilation (0.511, 3 photon continuum) SN Ia ⁵⁶ Co positrons (0.511) ²⁶ Al and ⁴⁴ Ti positrons (0.511)
Cosmic Ray Interactions with the ISM ¹² C (4.4), ¹⁶ O (6.1), ²⁰ Ne(1.634), ²⁴ Mg(1.369, 2.754), ²⁸ Si(1.779), ⁵⁶ Fe(0.847, 1.238)           Neutron Star Mass-Radius         n=n(2.221)		Understand the dynamics of Galactic Novae	¹³ N, ^{14,15} O, ¹⁸ F positrons ( <b>0.511</b> ) ⁷ Be ( <b>0.478</b> ), ²² Na ( <b>1.275</b> , <b>0.511</b> )
Neutron Star Mass-Radius p.n. (2.223)		Cosmic Ray Interactions with the ISM	¹² C (4.4), ¹⁶ O (6.1), ²⁰ Ne( <i>1.634</i> ), ²⁴ Mg( <i>1.369</i> , 2.754), ²⁸ Si( <i>1.779</i> ), ⁵⁶ Fe( <i>0.847</i> , <i>1.238</i> )
Littud on blai mass-readins Printeree		Neutron Star Mass-Radius	p-n (2.223)


























Nuclear Data Section International Atomic Energy Agency P.O. Box 100 A-1400 Vienna Austria e-mail: services@iaeand.iaea.org fax: (43-1) 26007 cable: INATOM VIENNA telex: 1-12645 telephone: (43-1) 2600-21710 Web: http://www-nds.iaea.org