Global calculations with the interacting boson model

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Brief review of the interacting boson model

Global mass + spectra calculations

Discussion of NNDC

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Interacting boson model

- Describe the nucleus as a system of N interacting s and d bosons. Hamiltonian:

$$\hat{H}_{\text{IBM}} = \sum_{i=1}^{6} \varepsilon_i \hat{b}_i^+ \hat{b}_i + \sum_{i_1 i_2 i_3 i_4 = 1}^{6} \upsilon_{i_1 i_2 i_3 i_4} \hat{b}_{i_1}^+ \hat{b}_{i_2}^+ \hat{b}_{i_3} \hat{b}_{i_4} + \cdots$$

- Justification from
  - Shell model: s and d bosons are associated with S and D fermion (*Cooper*) pairs.
  - Geometric model: for large boson number the IBM reduces to a liquid-drop hamiltonian.

General IBM hamiltonian

- Most general rotationally invariant IBM hamiltonian:

$$\hat{H}_{\mathrm{IBM}} = E_0 + \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \cdots$$

$$\hat{H}_1 = \varepsilon_s \hat{n}_s + \varepsilon_d \hat{n}_d$$

$$\hat{H}_2 = \sum_{l_1 l_2 l'_1 l'_2, L} \tilde{\upsilon}^L_{l_1 l_2 l'_1 l'_2} \left(b^+_{l_1} \times b^+_{l_2}\right)^{(L)} \cdot \left(\tilde{b}_{l'_1} \times \tilde{b}_{l'_2}\right)^{(L)}$$

$$\hat{H}_3 = \sum_{l_1 l_2 l_3 l'_1 l'_2 l'_3, L} \tilde{\upsilon}^L_{l_1 l_2 l_3 l'_1 l'_2 l'_3} \left(b^+_{l_1} \times b^+_{l_2} \times b^+_{l_3}\right)^{(L)} \cdot \left(\tilde{b}_{l'_1} \times \tilde{b}_{l'_2} \times \tilde{b}_{l'_3}\right)^{(L)}$$

Parameters in the IBM hamiltonian

- Spectrum of a single nucleus: 0+1+5+10 parameters.
- Overall binding energy: 1+1+2+7 parameters.
- ⇒ A total of 27 parameters if all interactions up to three-body are included (cfr. 63 2-body *sd*-shell model matrix elements).

Order	Number of interactions		
	total	type I[a]	type II[b]
$n = 0$	1	1	0
$n = 1$	2	1	1
$n = 2$	7	2	5
$n = 3$	17	7	10

[a]Interaction energy is constant for all states with the same N.
[b]Interaction energy varies from state to state.

Classical limit

- Coherent state:

$$\left|N;\beta,\gamma\right\rangle \propto \left[s^{+} + \beta\cos\gamma\, d_0^{+} + \sqrt{\tfrac{1}{2}}\beta\sin\gamma\left(d_{-2}^{+} + d_{+2}^{+}\right)\right]^{N}\left|\mathrm{o}\right\rangle$$

- Generic form of the potential:

$$V(\beta,\gamma) \equiv \left\langle N;\beta,\gamma\right| E_0 + \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \cdots \left|N;\beta,\gamma\right\rangle$$

$$= E_0 + \sum_{n\geq 1}\frac{N(N-1)\cdots(N-n+1)}{\left(1+\beta^2\right)^n}\sum_{kl} a_{kl}^{(n)}\beta^{2k+3l}\cos^l 3\gamma$$

Coefficients up to third order

$$a_{00}^{(1)} = \varepsilon_s, \quad a_{10}^{(1)} = \varepsilon_d,$$

$$a_{00}^{(2)} = \tfrac{1}{2}\upsilon_{ssss}^{0}, \quad a_{10}^{(2)} = \sqrt{\tfrac{1}{5}}\upsilon_{ssdd}^{0} + \upsilon_{sdsd}^{2}, \quad a_{01}^{(2)} = -\sqrt{\tfrac{2}{7}}\upsilon_{sddd}^{2},$$

$$a_{20}^{(2)} = \tfrac{1}{10}\upsilon_{dddd}^{0} + \tfrac{1}{7}\upsilon_{dddd}^{2} + \tfrac{9}{35}\upsilon_{dddd}^{4},$$

$$a_{00}^{(3)} = \tfrac{1}{6}\upsilon_{ssssss}^{0}, \quad a_{10}^{(3)} = \sqrt{\tfrac{1}{15}}\upsilon_{ssssdd}^{0} + \tfrac{1}{2}\upsilon_{ssdssd}^{2},$$

$$a_{01}^{(3)} = -\tfrac{1}{3}\sqrt{\tfrac{2}{35}}\upsilon_{sssddd}^{0} - \sqrt{\tfrac{2}{7}}\upsilon_{ssdsdd}^{2},$$

$$a_{20}^{(3)} = \tfrac{1}{10}\upsilon_{sddsdd}^{0} + \sqrt{\tfrac{1}{7}}\upsilon_{ssdddd}^{2} + \tfrac{1}{7}\upsilon_{sddsdd}^{2} + \tfrac{9}{35}\upsilon_{sddsdd}^{4},$$

$$a_{11}^{(3)} = -\tfrac{1}{5}\sqrt{\tfrac{2}{21}}\upsilon_{sddddd}^{0} - \tfrac{\sqrt{2}}{7}\upsilon_{sddddd}^{2} - \tfrac{18}{35}\sqrt{\tfrac{2}{11}}\upsilon_{sddddd}^{4},$$

$$a_{30}^{(3)} = \tfrac{1}{14}\upsilon_{dddddd}^{2} + \tfrac{1}{30}\upsilon_{dddddd}^{3} + \tfrac{3}{154}\upsilon_{dddddd}^{4} + \tfrac{7}{165}\upsilon_{dddddd}^{6},$$

$$a_{02}^{(3)} = \tfrac{1}{105}\upsilon_{dddddd}^{0} - \tfrac{1}{30}\upsilon_{dddddd}^{3} + \tfrac{3}{110}\upsilon_{dddddd}^{4} - \tfrac{4}{1155}\upsilon_{dddddd}^{6}.$$

Application to SO(6)-like nuclei

- One- + two-body hamiltonian:

$$\hat{H}_{1+2} = \varepsilon \hat{n}_d + \kappa \hat{Q} \cdot \hat{Q} + \kappa' \hat{L} \cdot \hat{L} + \lambda \hat{n}_d^2$$

- Cubic interaction (usually *L*=3):

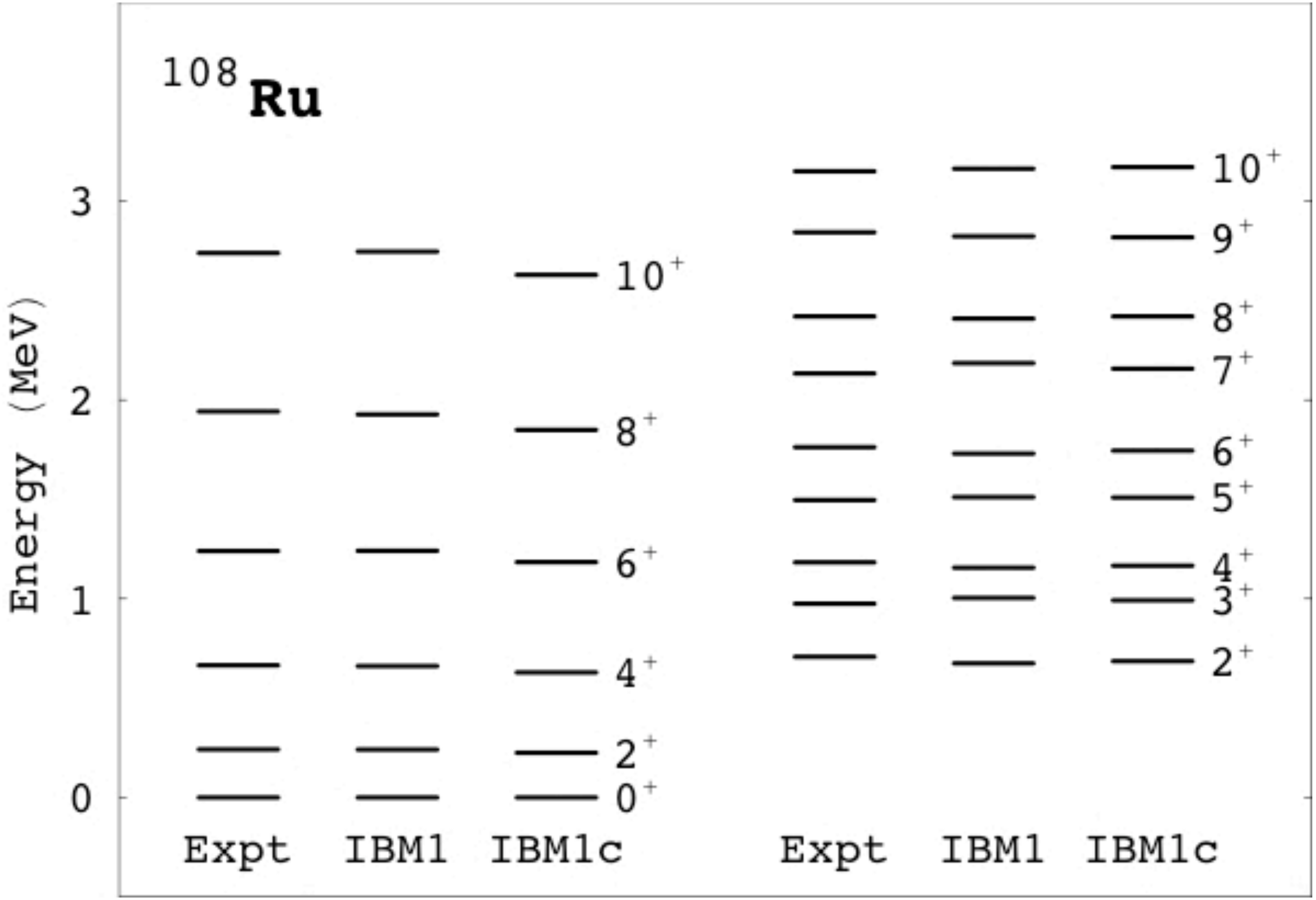
$$\hat{H}_3^d = \sum_L \upsilon_L \left(d^+ \times d^+ \times d^+\right)^{(L)} \cdot \left(\tilde{d} \times \tilde{d} \times \tilde{d}\right)^{(L)}$$

- Signature splitting $S(J)$ of γ band is sensitive to effects of cubic interaction (triaxiality)

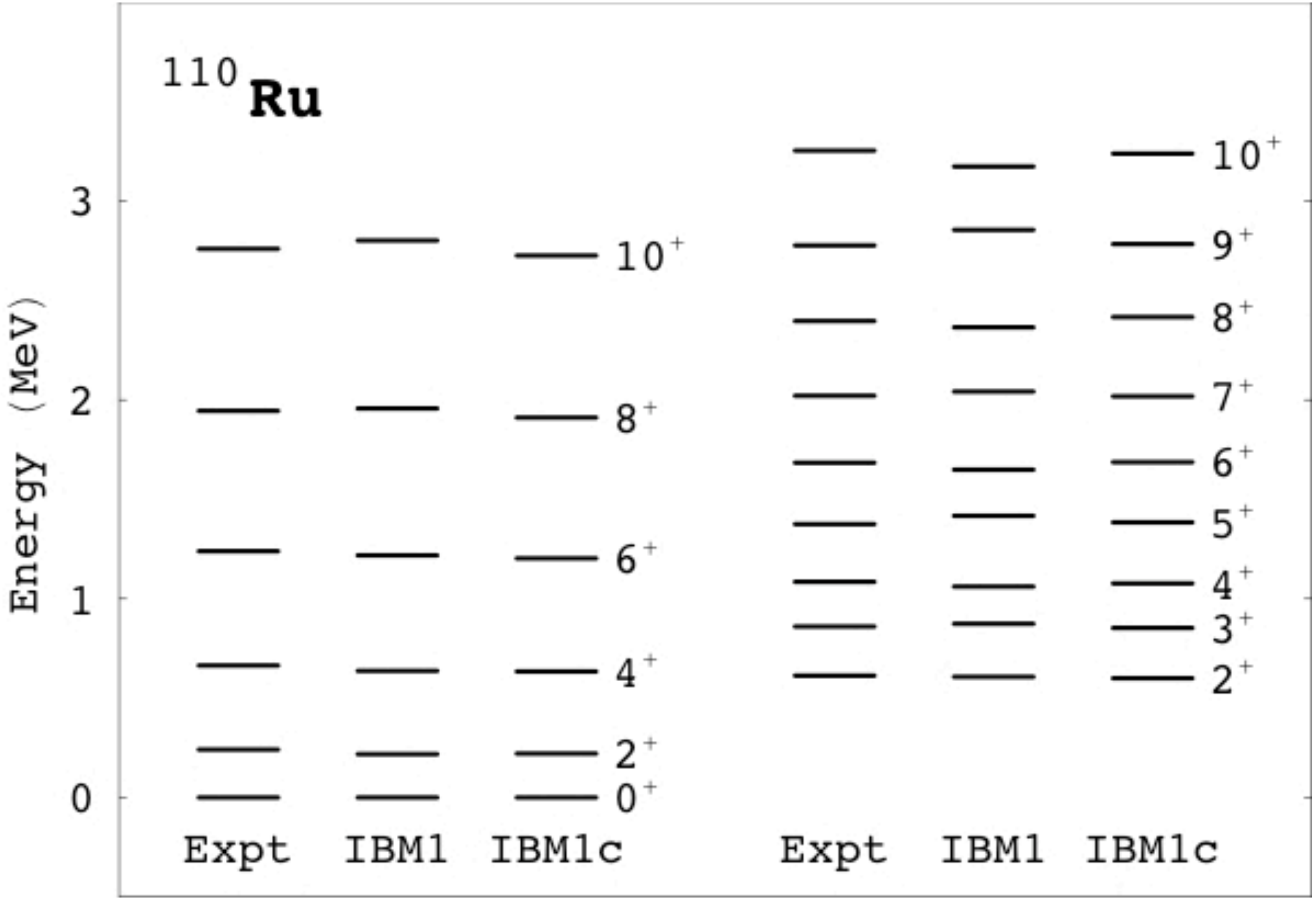
$$S(J) = \frac{E(J) - E(J-1)}{E(J) - E(J-2)} \cdot \frac{J(J+1) - (J-1)(J-2)}{J(J+1) - J(J-1)} - 1$$

I. Stefanescu *et al.*, Nucl. Phys. A **789** (2007) 125
B. Sorgunlu & P. Van Isacker, Nucl. Phys. A **808** (2008) 275

Spectra of ruthenium isotopes



Spectra of ruthenium isotopes



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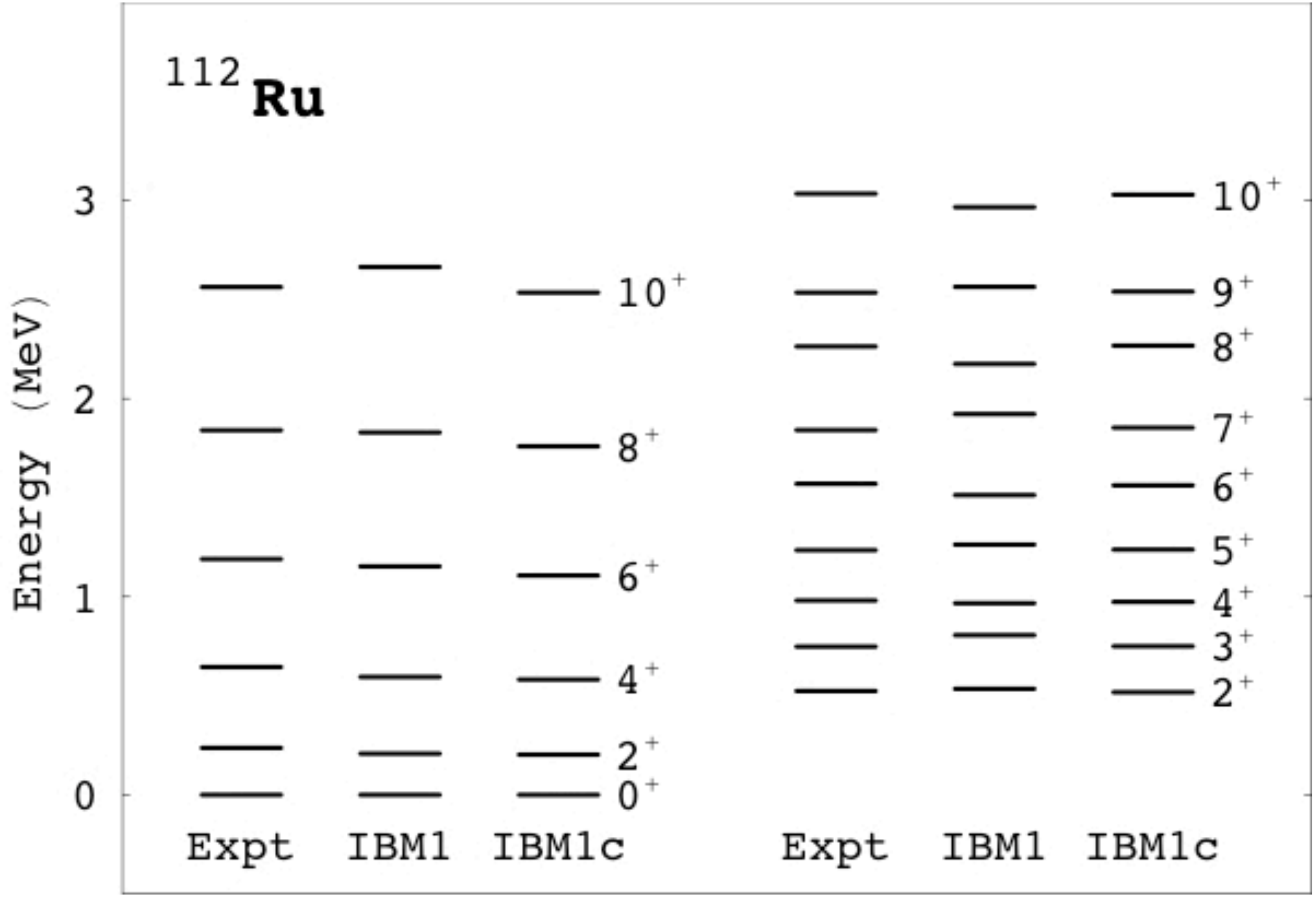
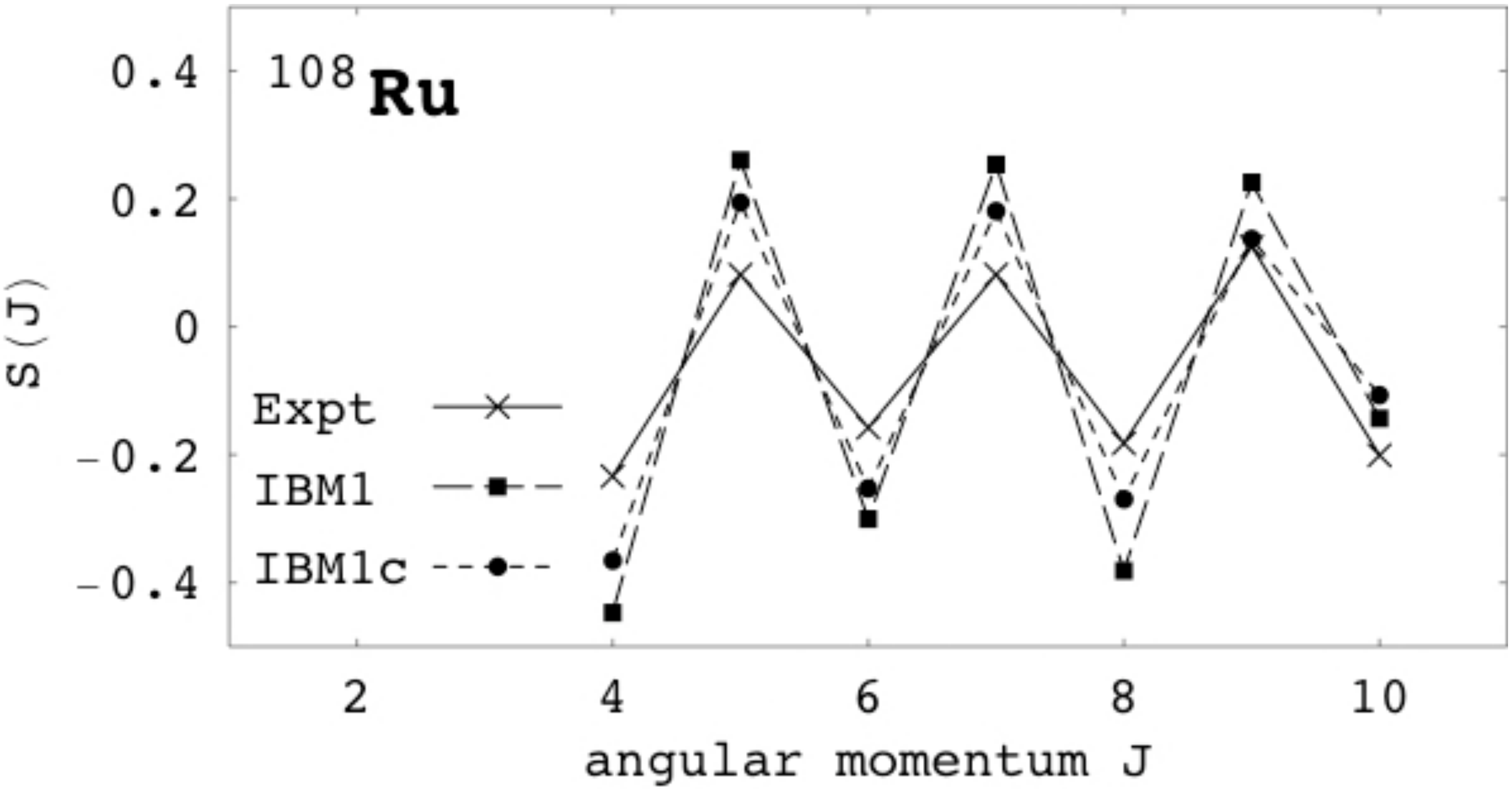
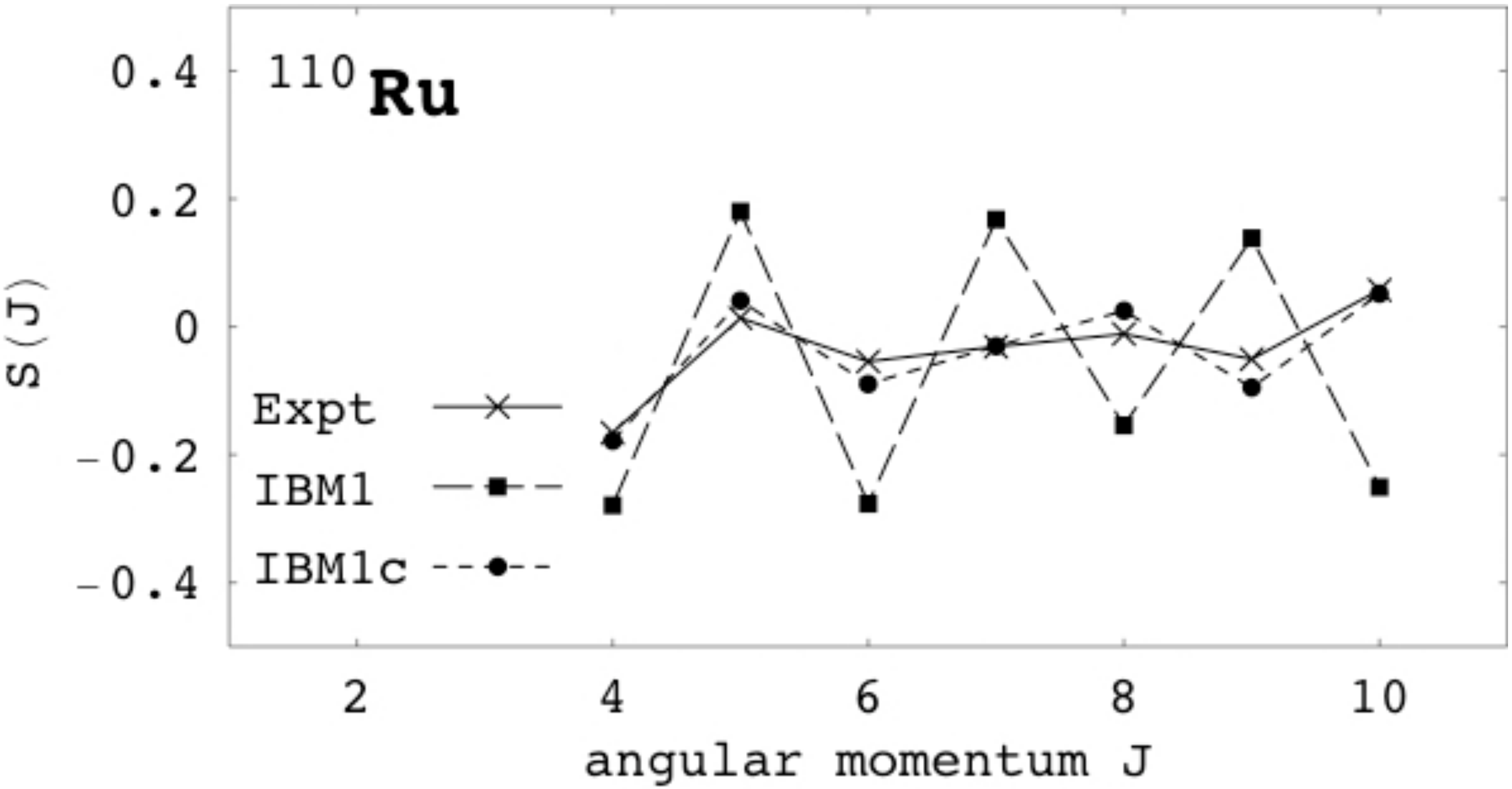


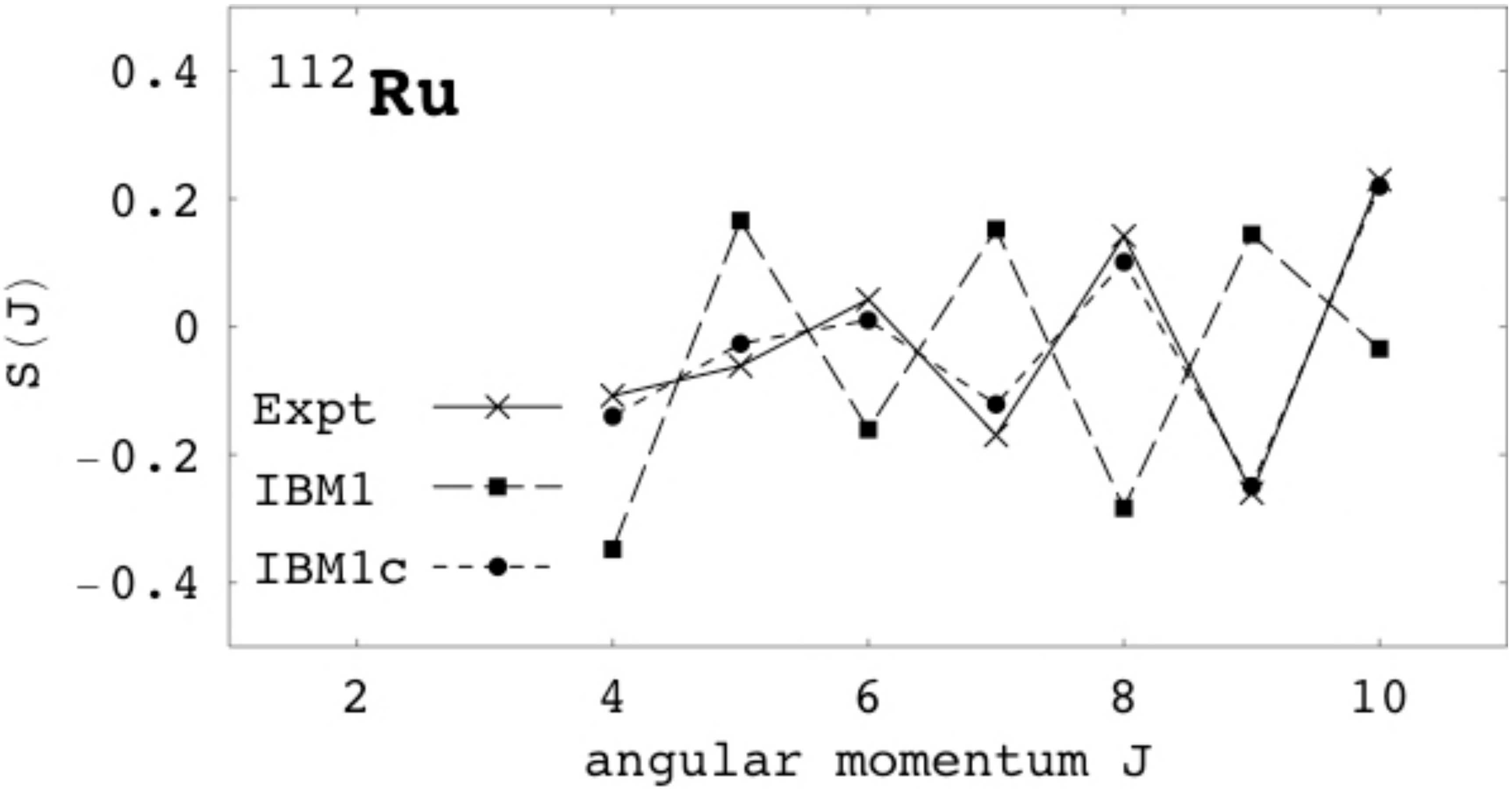
Signature splitting of γ band



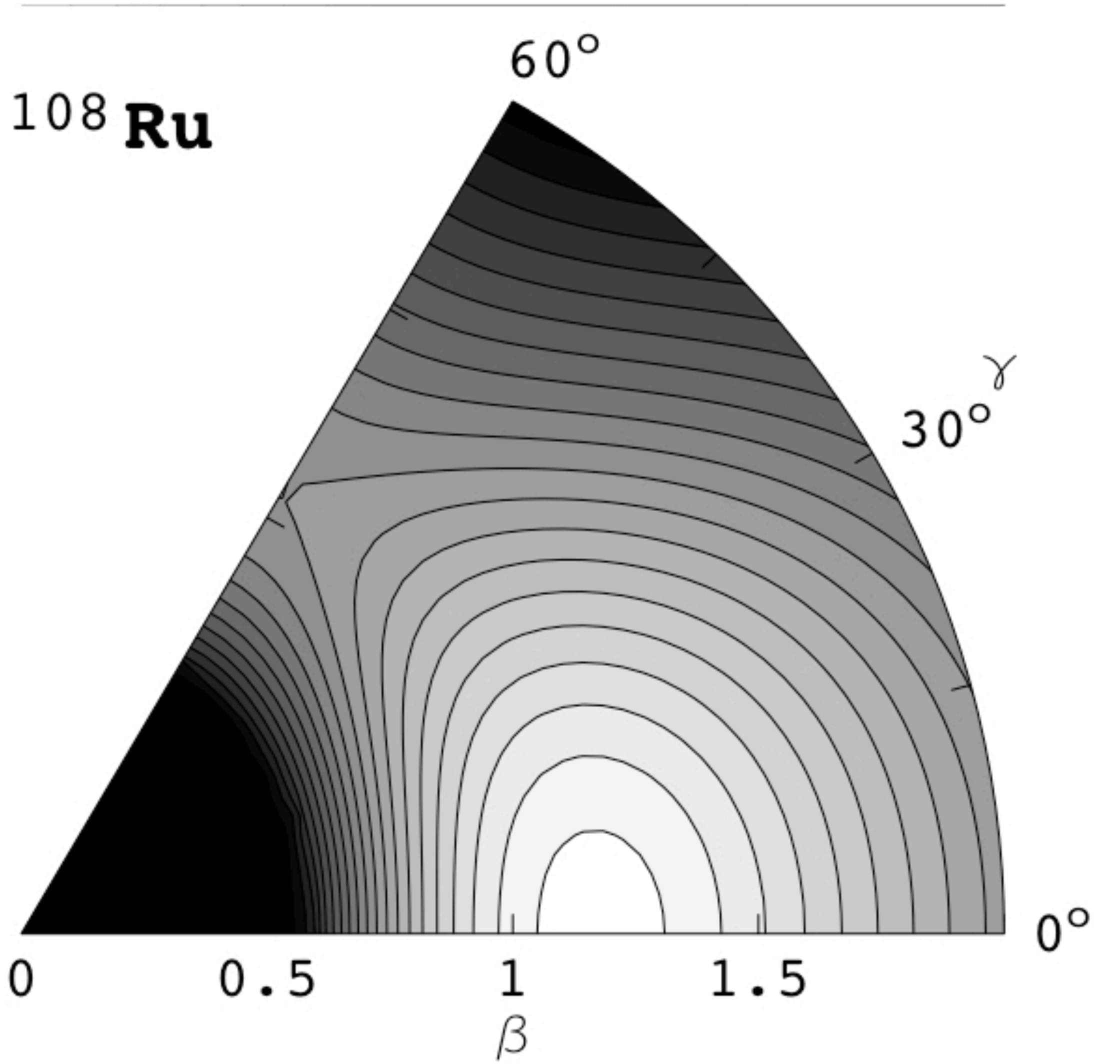
Signature splitting of γ band



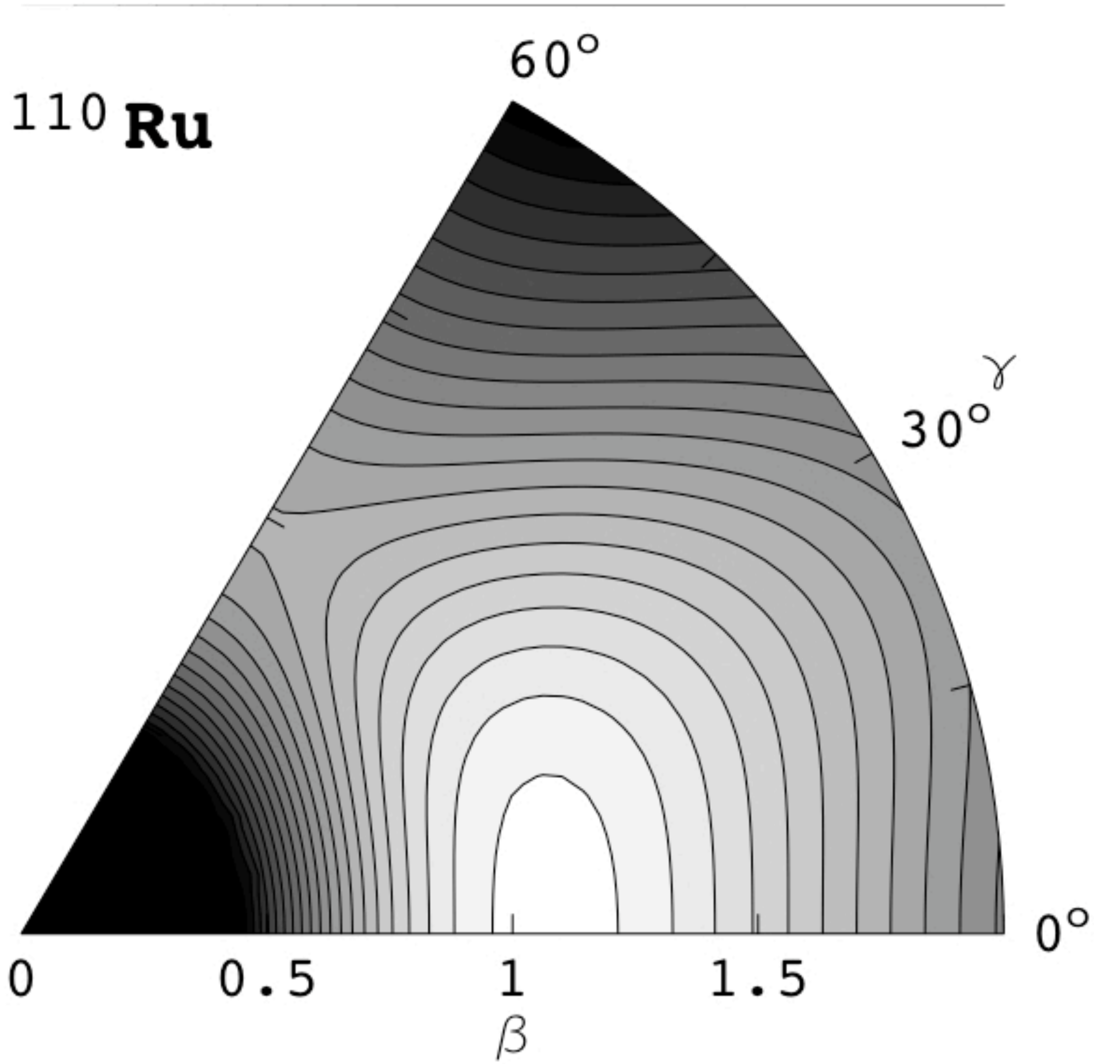
Signature splitting of γ band



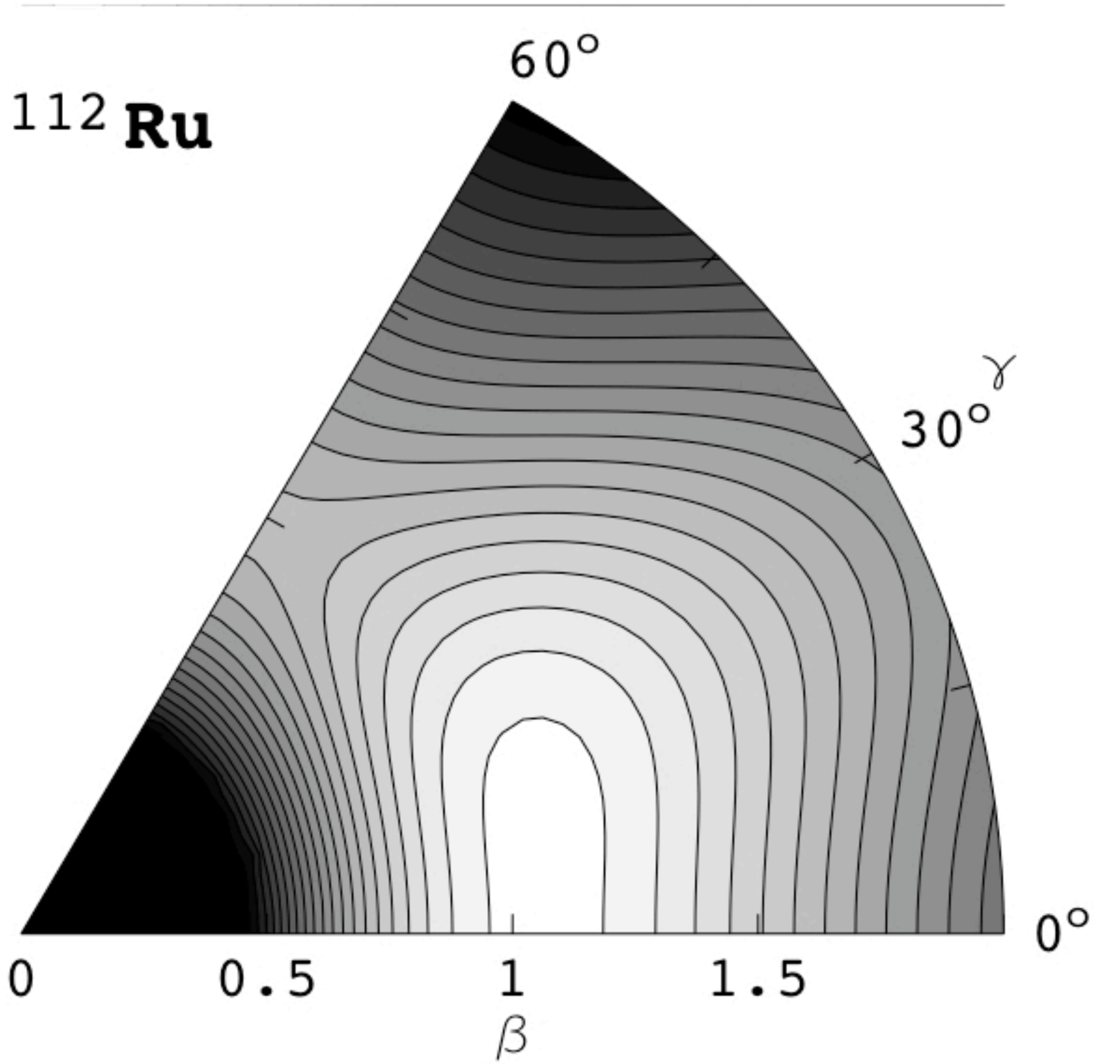
Shape of ^{108}Ru



Shape of ^{110}Ru



Shape of ^{112}Ru



Global IBM-1 calculations

- Consider an entire shell *e.g*. all even-even nuclei with $50 < Z < 82$ and $82 < N < 126$.
- Fit IBM-1 hamiltonian to all *known* nuclei in the shell and use this hamiltonian to *predict* properties of nuclei far from stability.
- Two strategies:
  - **Constant** hamiltonian for all nuclei.
  - Parameters depending on the **fractional filling** of the valence neutron and proton shells.
- Masses can be included in the analysis.

How to include masses?

- Relation between mass and binding energy:

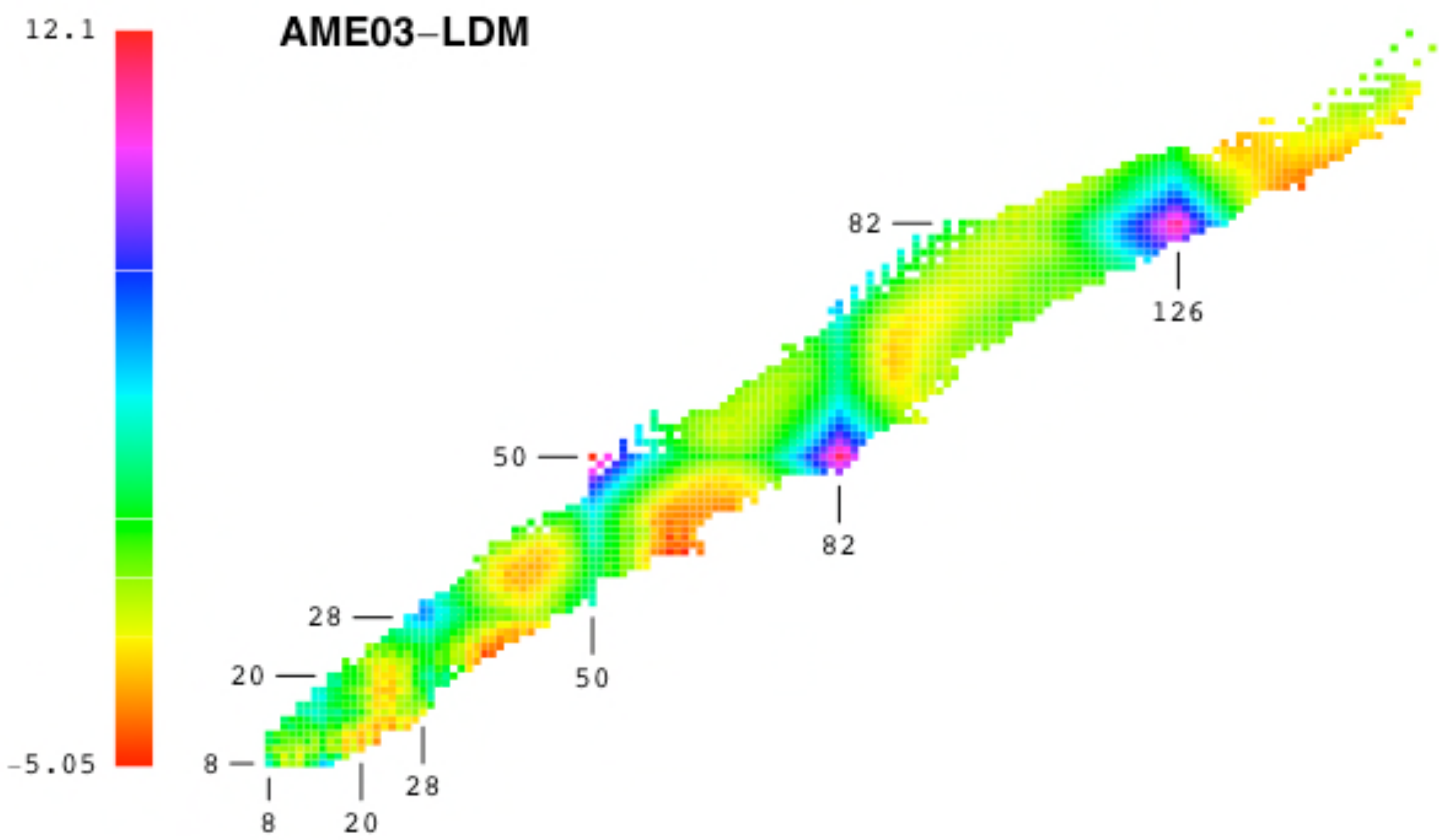
$$M(N,Z)c^2 = N\,m_\mathrm{n}c^2 + Z\,m_\mathrm{p}c^2 - B(N,Z)$$

- Liquid drop mass formula (von Weizsäcker):

$$B(N,Z) = a_\mathrm{vol}A - a_\mathrm{sur}A^{2/3} - a_\mathrm{cou}\frac{Z(Z-1)}{A^{1/3}} + a_\mathrm{pai}\frac{\delta(N,Z)}{A^{1/2}} - \frac{S_\mathrm{v}}{1 + S_\mathrm{v}A^{-1/3}/S_\mathrm{s}}\frac{T(T+1)}{A}$$

- Fit to nuclear masses in AME03: $\sigma \approx 2.4$ MeV.

The ‘unfolding’ of the mass surface



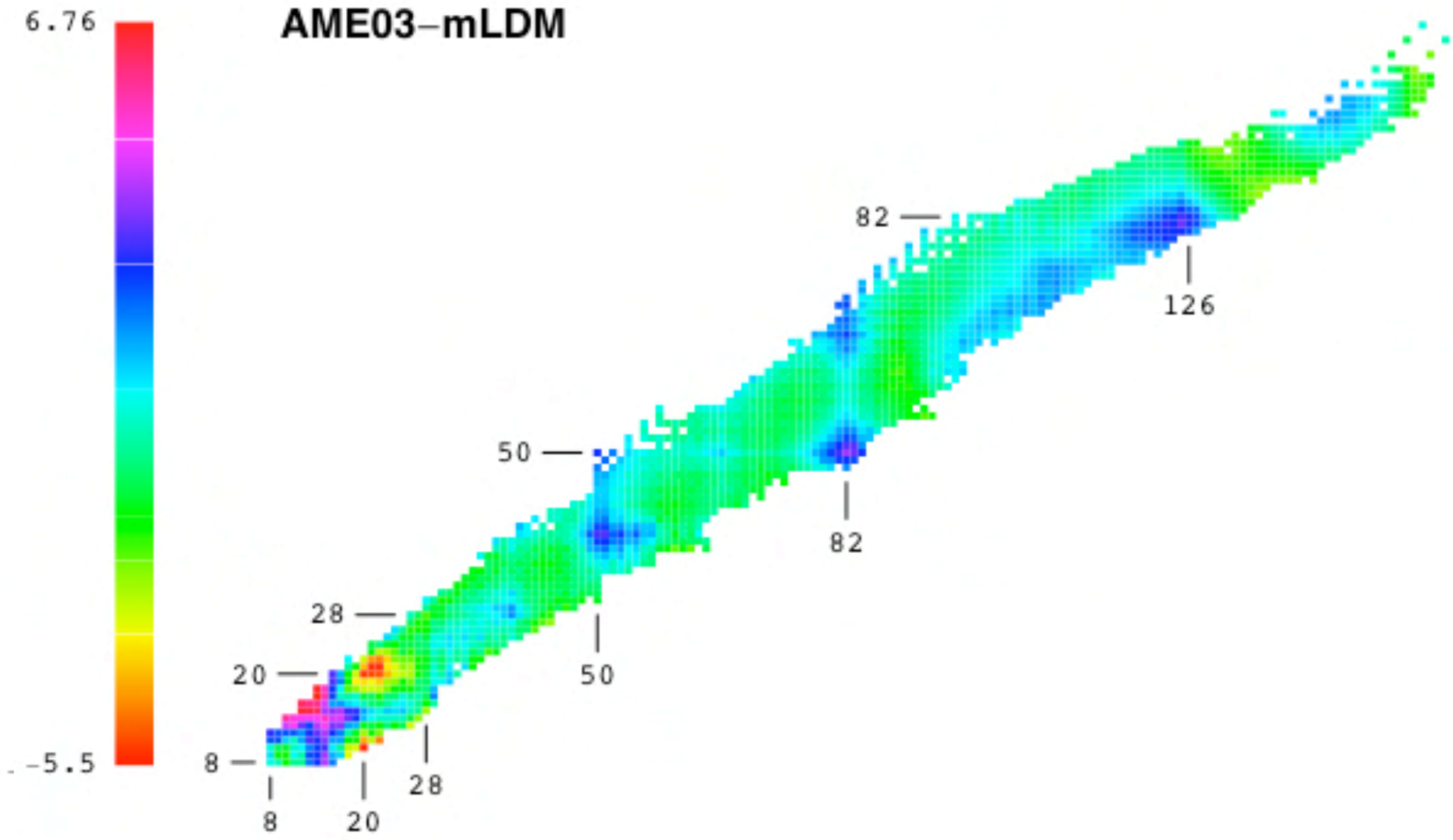
Shell corrections

- Observed deviations suggest shell corrections depending on $n_\nu + n_\pi$, the total number of valence neutrons + protons (particles or holes).
- A simple parametrisation consists of two terms, linear and quadratic in $F_{\mathrm{max}} = (n_\nu + n_\pi)/2$.

$$B(N,Z) = a_{\mathrm{vol}} A - a_{\mathrm{sur}} A^{2/3} - a_{\mathrm{cou}} \frac{Z(Z-1)}{A^{1/3}} + a_{\mathrm{pai}} \frac{\delta(N,Z)}{A^{1/2}}$$

$$- \frac{S_{\mathrm{v}}}{1 + S_{\mathrm{v}} A^{-1/3} / S_{\mathrm{s}}} \frac{T(T+1)}{A} + a_F F_{\mathrm{max}} + a_{FF} F_{\mathrm{max}}^2$$

Shell-corrected LDM



Global mass + spectra calculations

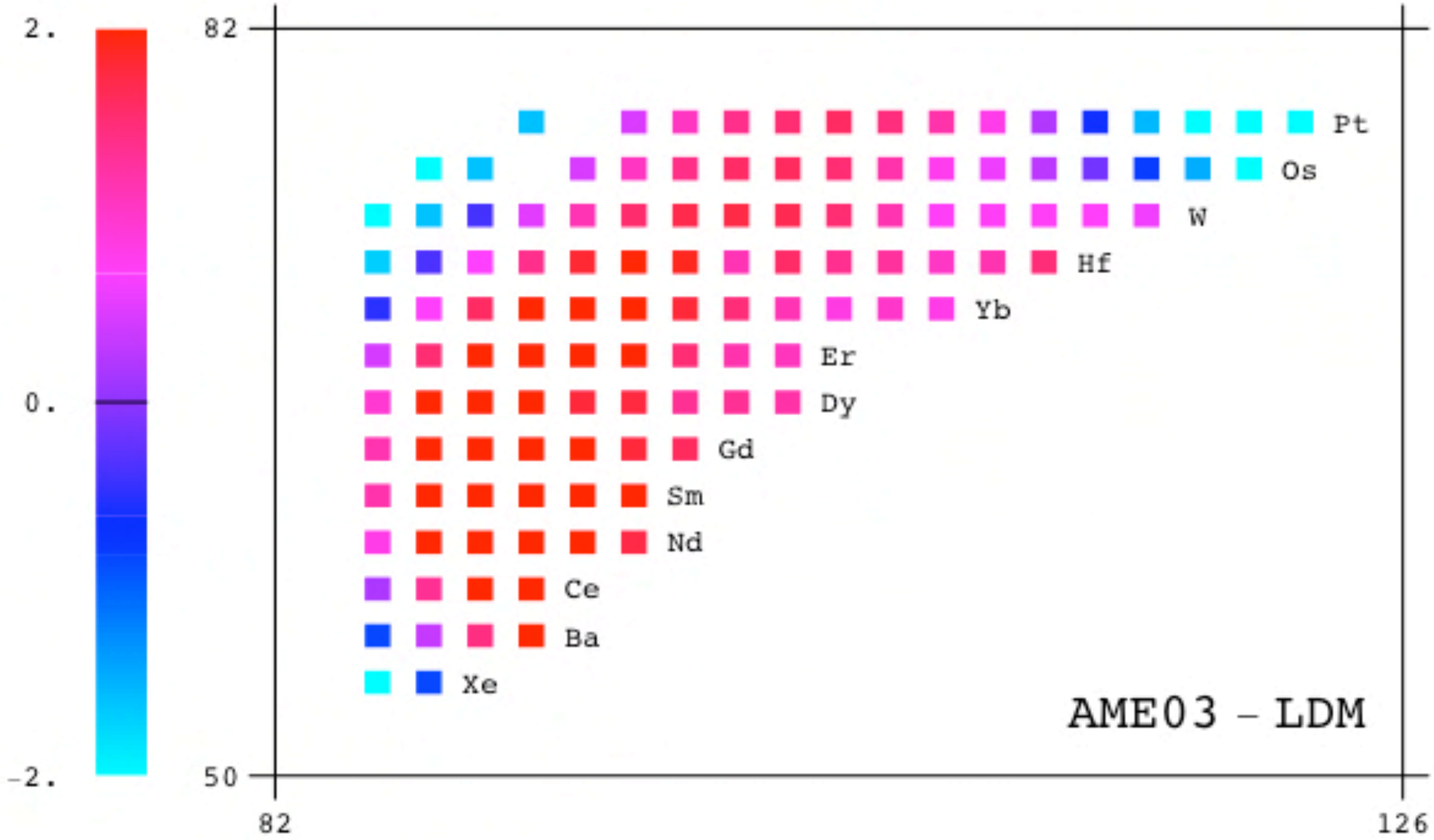
- Masses:

$$B(N,Z) = a_{\text{vol}} A - a_{\text{sur}} A^{2/3} - a_{\text{cou}} \frac{Z(Z-1)}{A^{1/3}} + a_{\text{pai}} \frac{\delta(N,Z)}{A^{1/2}}$$

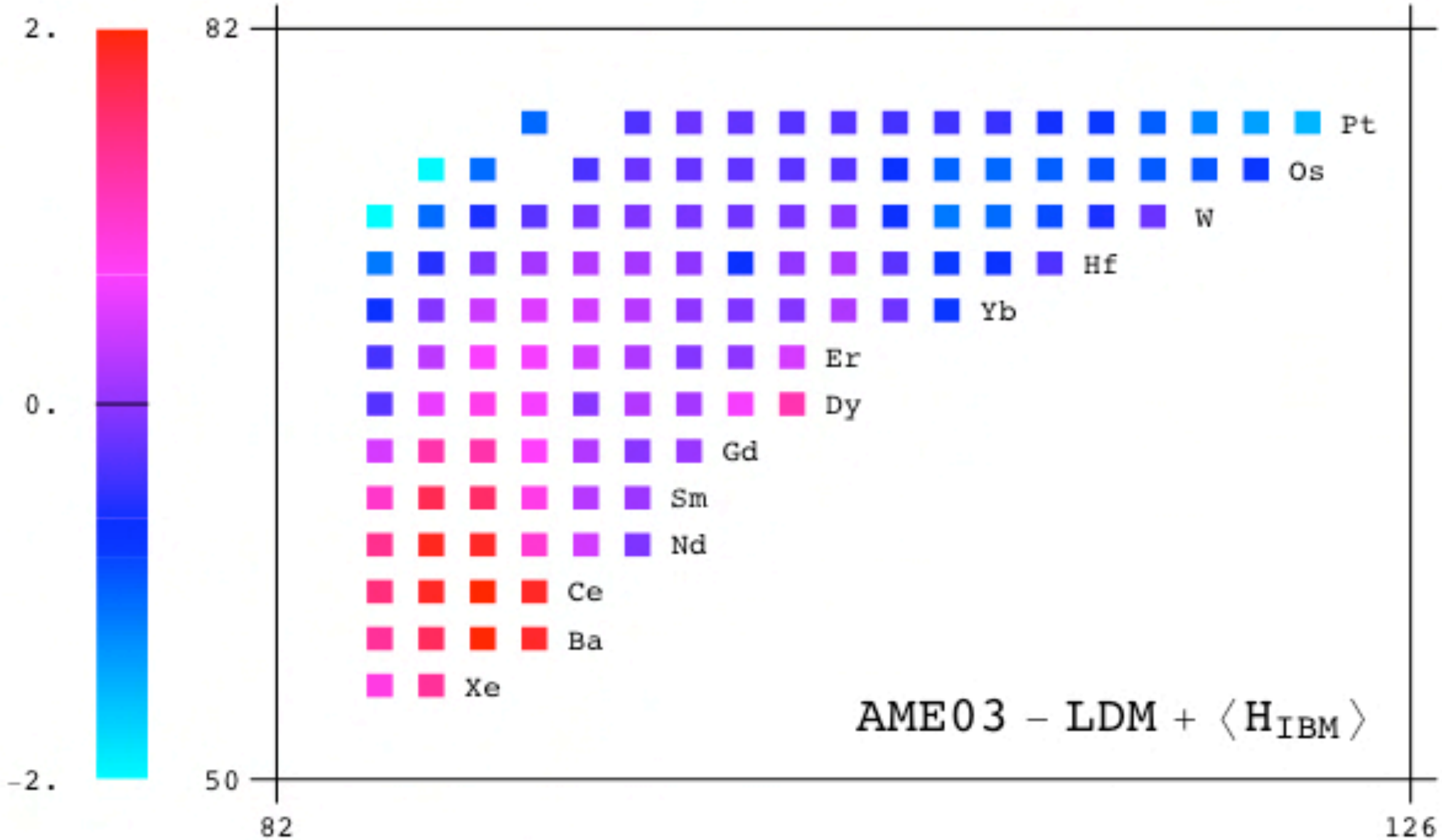
$$- \frac{S_{\text{v}}}{1 + S_{\text{v}} A^{-1/3} / S_{\text{s}}} \frac{T(T+1)}{A} - \langle 0_1^+ | \hat{H}_{\text{IBM}} | 0_1^+ \rangle$$

- Energy spectra from diagonalization of H_{IBM}.
- Problem: Initial choice of parameters.
- Criticism: H_{IBM} depends on N and not on N_ν and N_π separately $\Rightarrow$ spectra of $N_\nu+N_\pi = \text{c}^{\text{te}}$ nuclei are identical (F-spin multiplets).

AME03-LDM



AME03-LDM+$\langle gs | H_{IBM} | gs \rangle$



Fractional-filling dependence

- One- + two-body hamiltonian:

$$\hat{H}_{1+2} = \varepsilon \hat{n}_d + \kappa \hat{Q} \cdot \hat{Q} + \kappa' \hat{L} \cdot \hat{L} + \kappa'' \hat{P}_+ \hat{P}_- + \lambda \hat{n}_d^2$$

- Dependence of parameters. For example:

$$x = \sum_{ij} x_{ij} \left(f_\nu\right)^i \left(f_\pi\right)^j, \quad f_\rho = \frac{n_\rho}{\Omega_\rho}$$

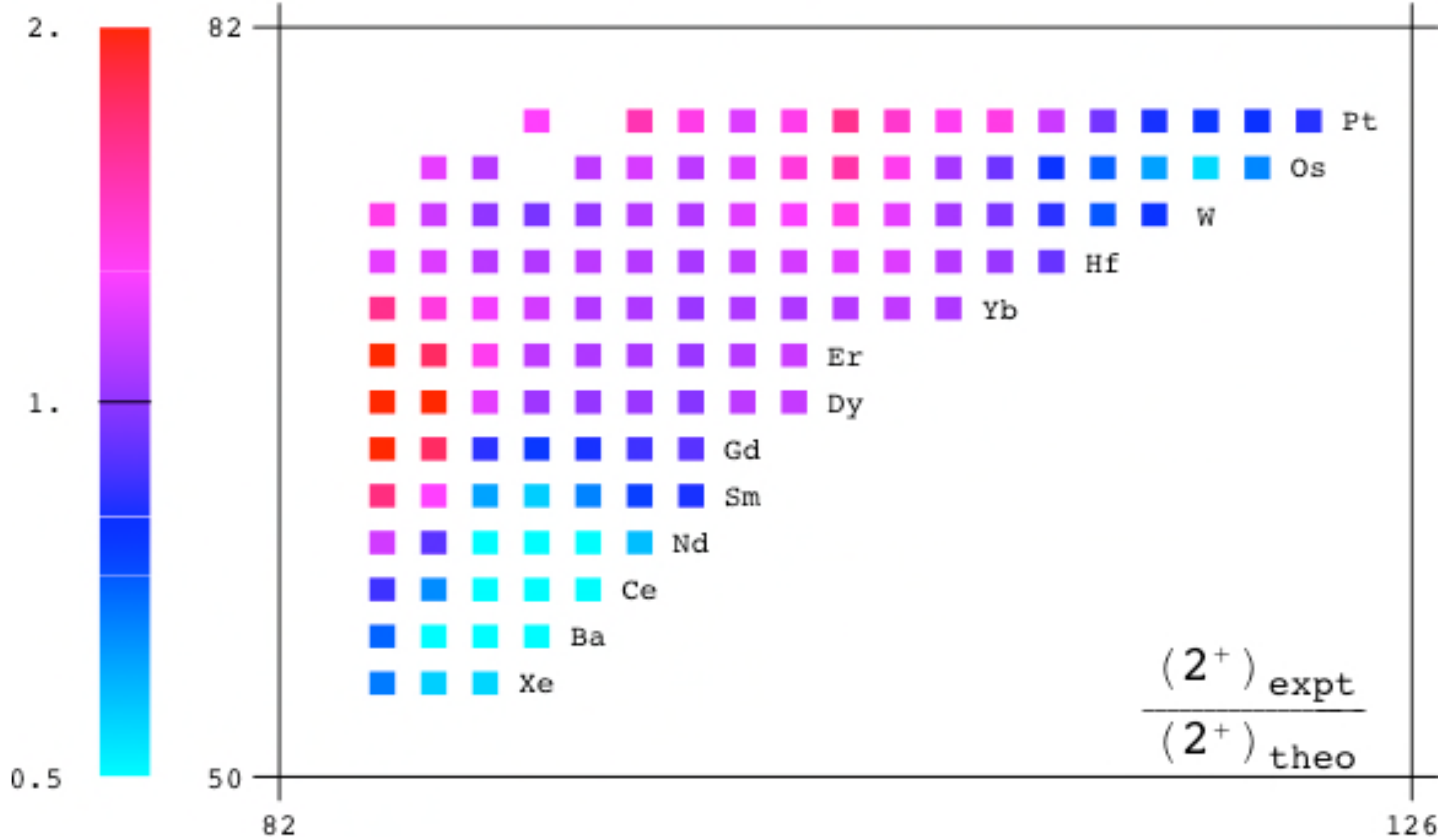
$$x = \sum_{ij} x_{ij} \left(F_\nu\right)^i \left(F_\pi\right)^j, \quad F_\rho = \frac{N_\rho}{\Omega_\rho}$$

$$x = \sum_{i} x_i \left(P\right)^i, \quad P = \frac{N_\nu N_\pi}{N_\nu + N_\pi}$$

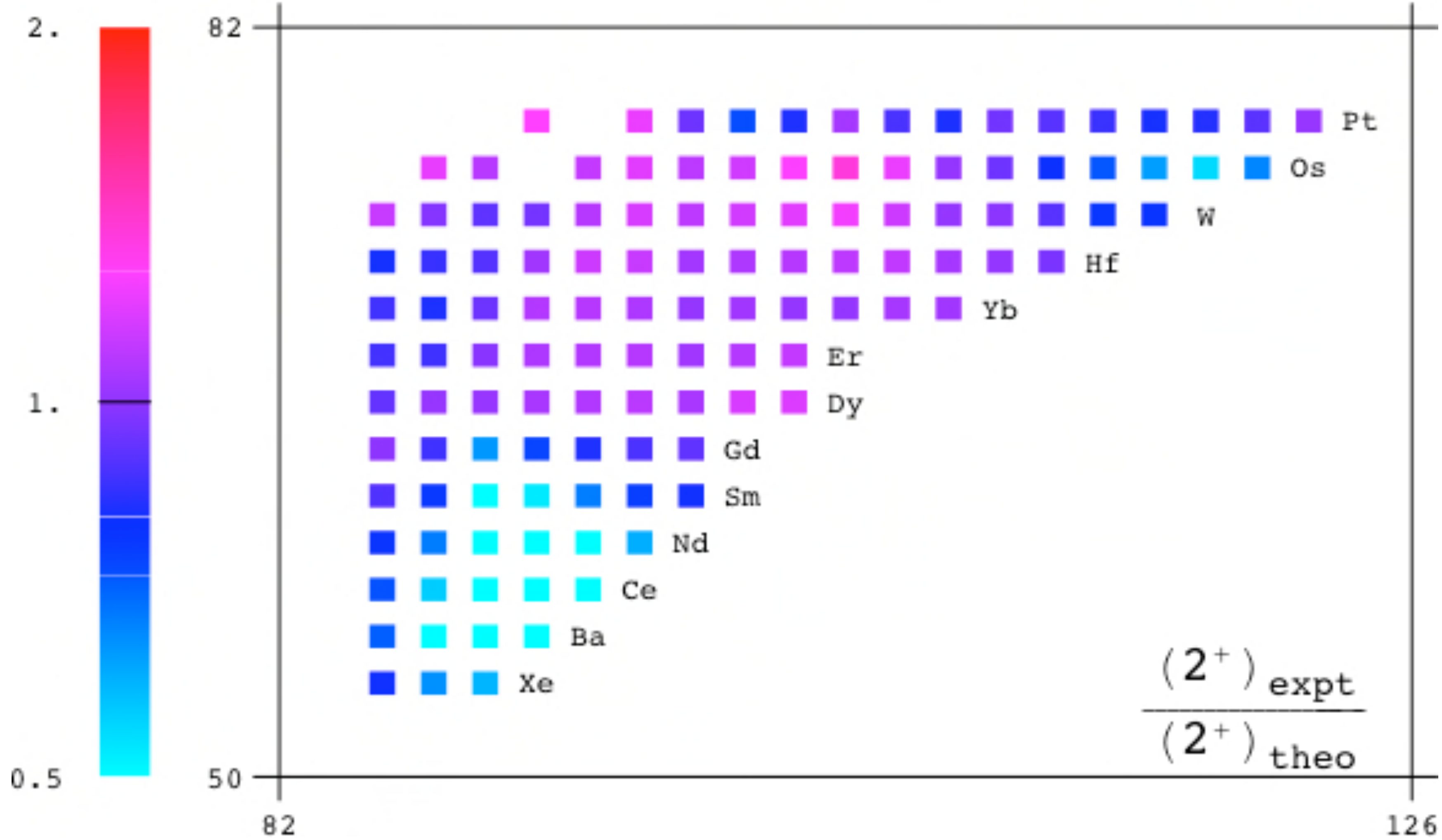
Application to rare-earth nuclei

- Application to even-even rare-earth nuclei with $84 < N < 124$ & $52 < Z < 80$ (1280 levels in 123 nuclei).
- Parameters ε and λ linear in F_ν and F_π; κ, κ' and κ'' linear in P.
- Root-mean-square deviation of 179 keV.

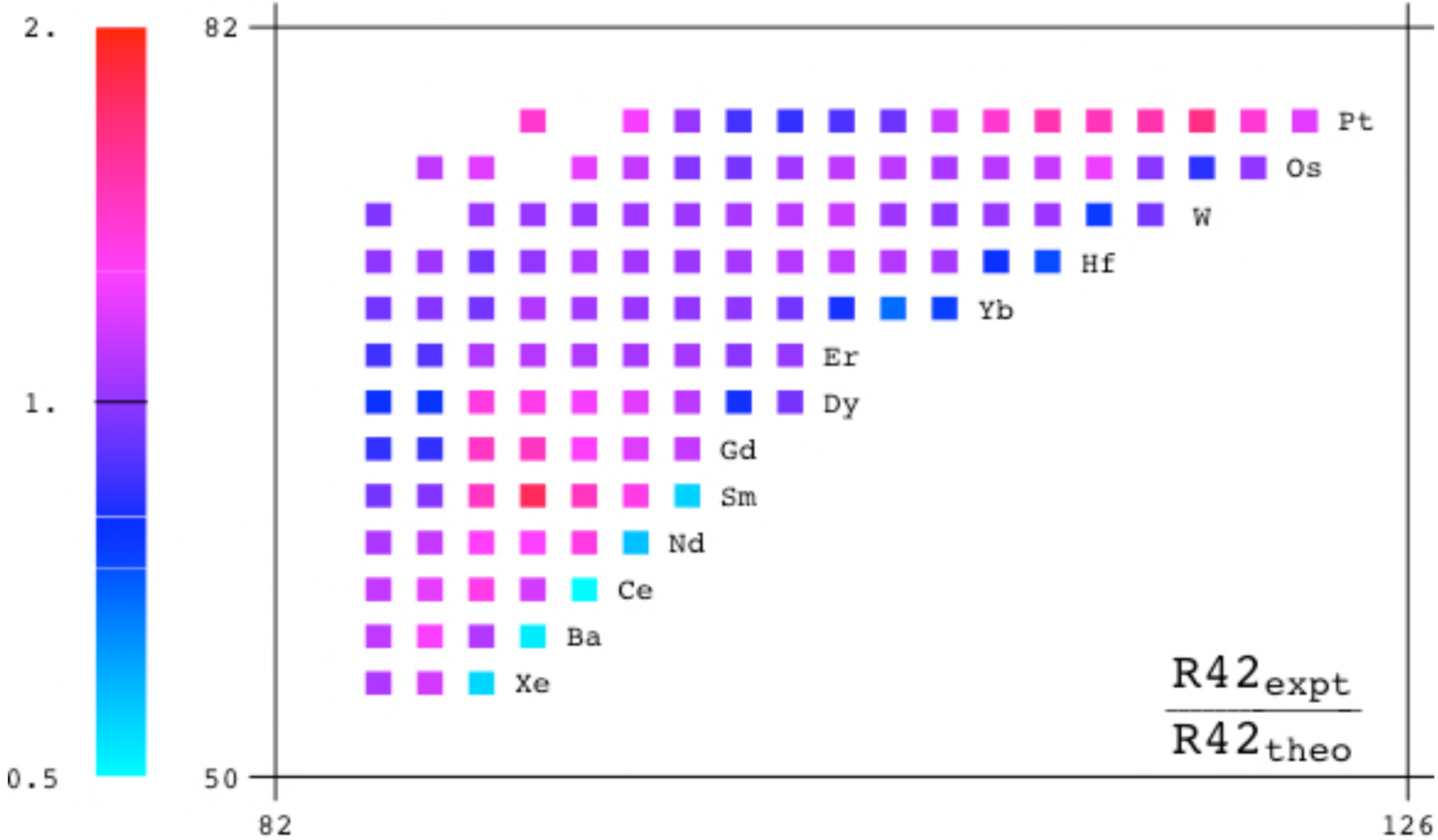
Constant IBM hamiltonian



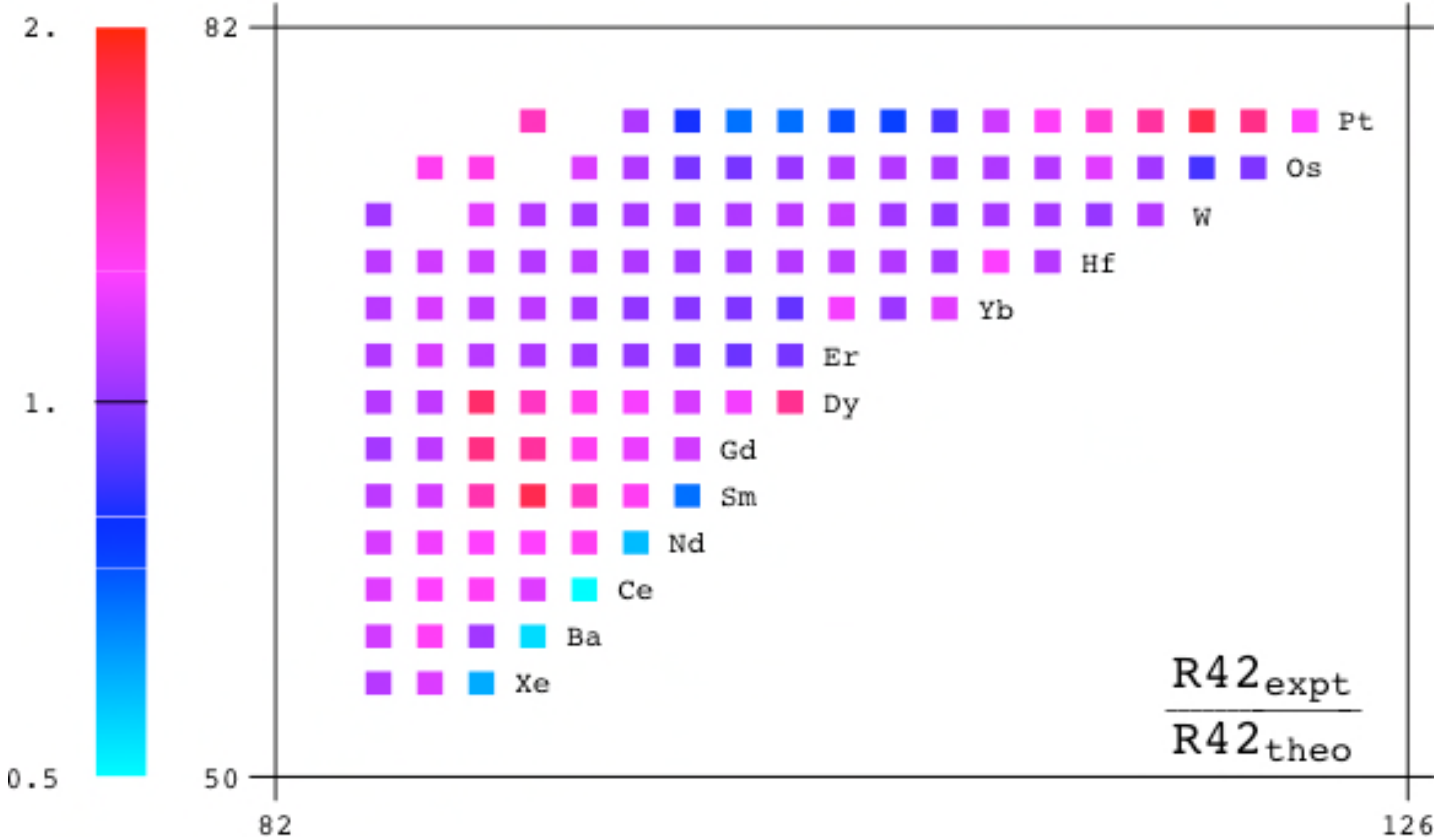
Variable IBM hamiltonian



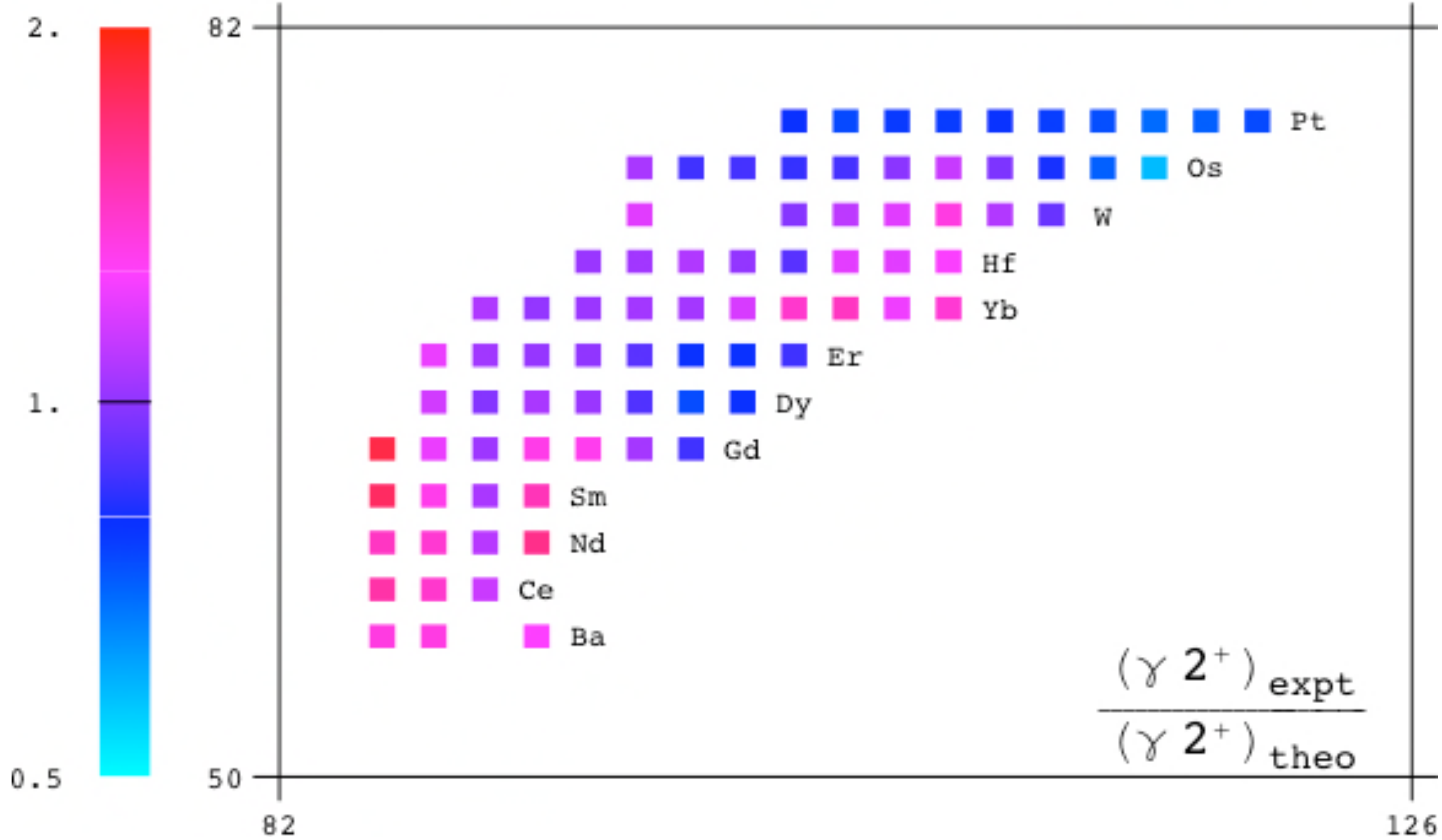
Constant IBM hamiltonian



Variable IBM hamiltonian



Constant IBM hamiltonian



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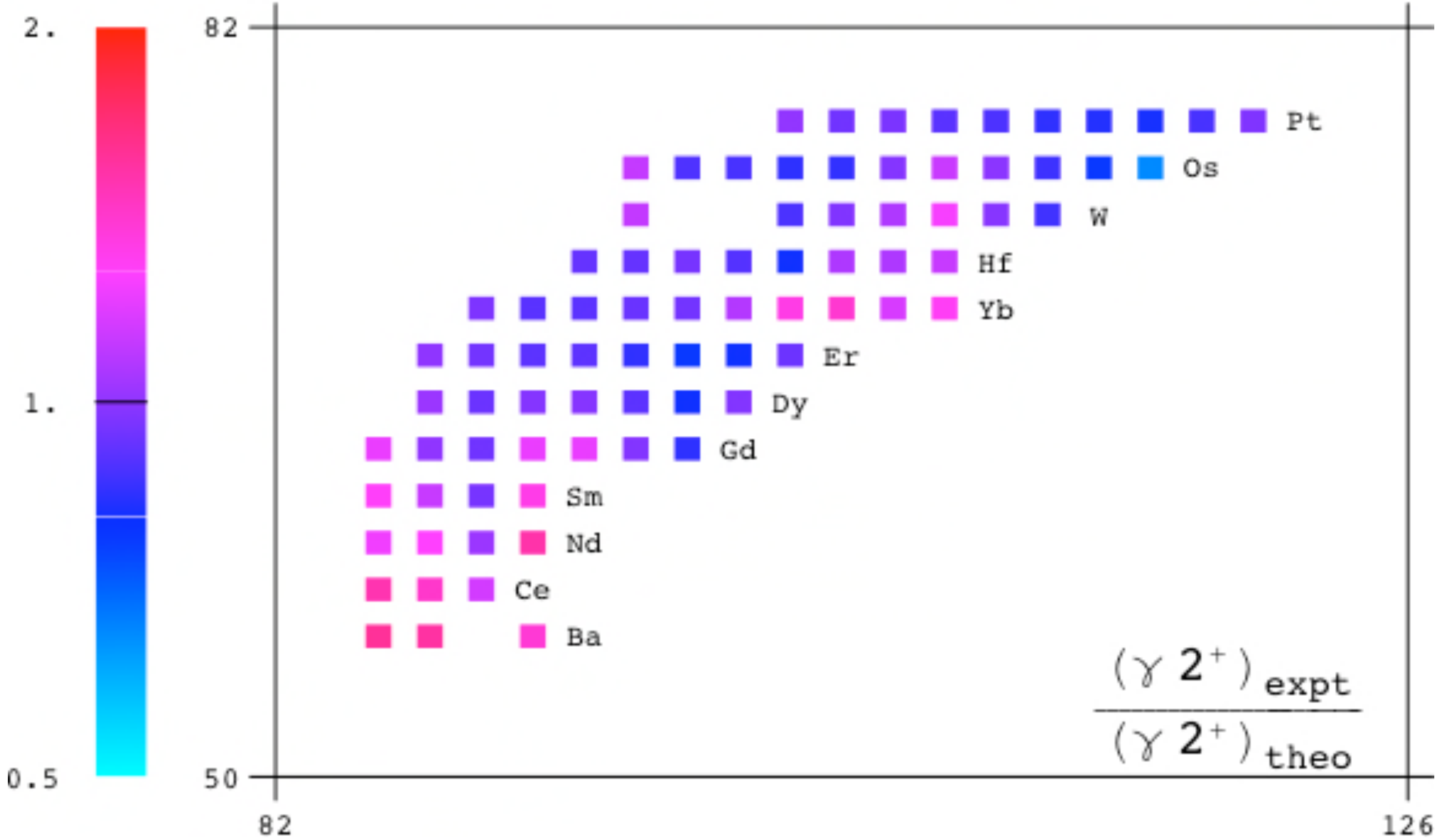
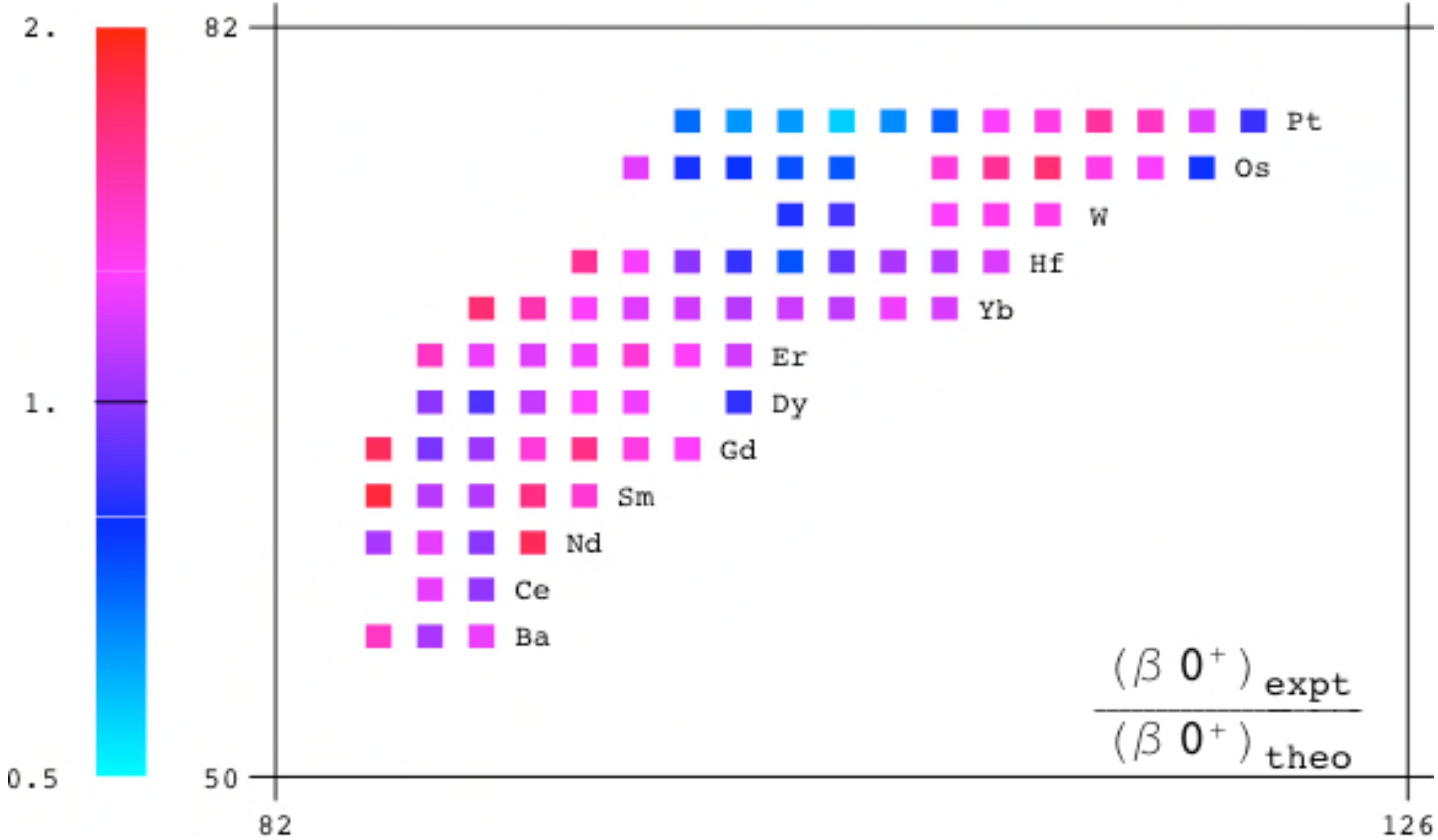


Constant IBM hamiltonian



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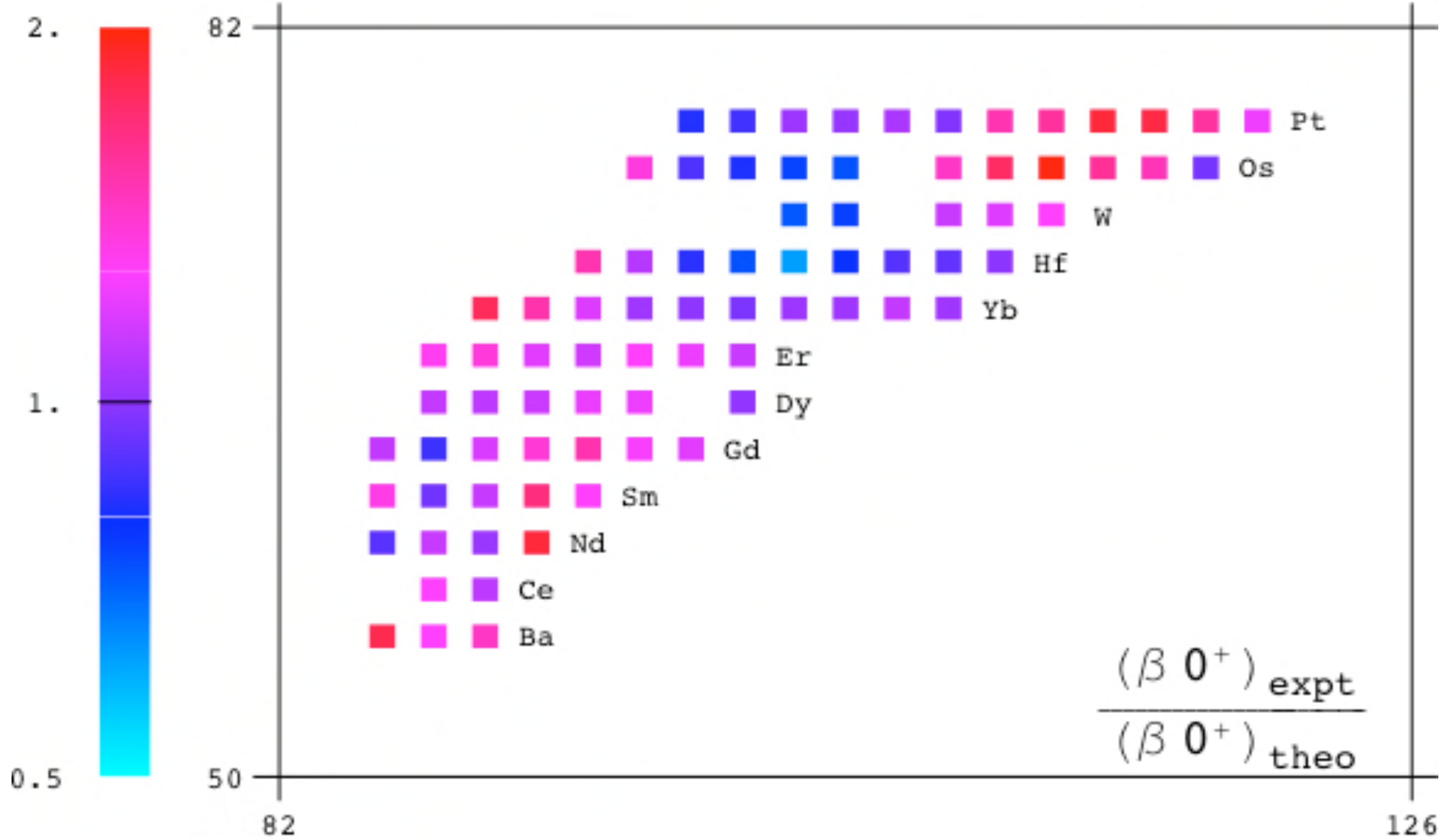

Conclusions

- Simple application of well-established model.
- IBM is valence-nucleon model $\Rightarrow$ predictive power depends on an assumed shell structure.
- Geometry is derived from data in an unbiased manner.
- For further study:
  - Effective minimization procedure in a multi-dimensional parameter space.
  - Dependence on fractional filling.

Comments on the use of NNDC

- Nuclear masses:
  - From Audi, Wapstra and Thibault (2003).
  - But: No update since 2003. Use of isolated compilations (e.g. Jyväskylä) is difficult and perhaps dangerous.
- Nuclear radii:
  - Several compilations exist.
  - But: Do they have the same reliability as AME? Which one to use? Regular updates?

Comments on the use of NNDC

- Nuclear spectra:
  - Systematic use of band-plotting option in NuDat
    $\Rightarrow$ important for identification of collective bands.
  - Occasional problems:
    - Gamma-band structure in ^{160}Dy and ^{170}Yb??
    - Gamma band cut in two in ^{180}W??
    - 2^+ @ 691 keV in ^{174}Os should be in beta band??
    - Beta band cut in two in ^{180}Os??
  - In general: confusion concerning beta band in deformed nuclei. What are the correct criteria to label a $K^\pi = 0^+$ band as a collective beta band?