NEW APPROACH FOR THE DETERMINATION OF THE NUCLEAR LEVEL DENSITY PARAMETERS

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ABSTRACT

The nuclear level density parameters of the stable nuclides have been determined from the neutron resonance data on the basis of new values for the spin dispersion parameter. The latter parameter is deduced from the spin dependent average level spacings of s-wave neutron resonances of odd target nuclides. In addition, the temperature dependence of the level density parameter is studied and an analytical relation for it is presented.

I. INTRODUCTION

The nuclear level density plays a major role in statistical calculations in reactor physics, nuclear model calculations, astrophysics, intermediate-energy heavy-ion collision investigations, and spallation neutron calculations. However, up until very recently[1], little attention was paid into a critical examination of the derivation of the level density parameter. This is largely due to a lack of a detailed knowledge of the spin dispersion parameter. Previous analysis [2,3,4] relied on two theoretical expressions for this parameter, one due to Gilbert and Cameron, [5] the other to Bohr and Mottelson [6] . The validity of the theoretical values of the spin dispersion parameter were not verified previously, particularly over an extended atomic mass range.

II. ANALYSIS AND RESULTS

The spin dispersion parameter was determined from the ratio of the average level spacings of s-wave neutron resonances[7, 8] for the two spin states I + 0.5, and I - 0.5 , where I is the spin of the target nucleus, and on the basis of the Bethe level density formula for a single parity state:

$$\rho(U,J) = \frac{(2J+1)}{24U^{5/4}2^{1/2}\sigma^3} e^{2(aU)^{1/2}} \; e^{\frac{-(J+1/2)^2}{2\sigma^2}} \qquad \textbf{(1)}$$

In his relation, U is the effective excitation energy, which is calculated by the relation $U = B_n + 0.5E_{max}$ - Pair, (B_n is the neutron separation energy , E_{max} is the maximum energy of the known resonance parameters, Pair is the pairing energy.) J is the spin of the level (neutron resonance, in the present case), a is the nuclear level density parameter, and σ is the spin dispersion parameter.

To obtain reliable estimates of the average level spacings and correct for unobserved levels or spin-unassigned levels, the Porter-Thomas and Dyson - Mehta statistics were applied. With the aid of Eqn. 1, the spin dispersion parameters for nuclides in the mass range from ^{46}Sc to ^{210}Bi were deduced readily. A least-squares fitting procedure of the resulting spin dispersion parameters in terms of the Gilbert-Cameron relation gave the following result:

$$\sigma^2 = 0.0494aTA^{2/3} \qquad \textbf{(2)}$$

where T= nuclear temperature = $(U/a)^{1/2}$

The coefficient in the above relation is about half of the theoretical value, 0.0888 [4.] This reduction of the spin dispersion parameter can be accounted for on the basis of the influence of nucleon pairing correlations[6,9].

With the aid of equations (1) and (2), the nuclear level density parameter is calculated on the basis of the recommended average level spacings of s-wave neutron resonances [7, 8] for nuclides in the mass range from ^{20}F to ^{250}Cf. The results are included in Table 1.

Table 1. The level density parameters as determined in the present analysis. The binding energy, B_n , and the pairing energy, Pair, are in MeV units, while the level density parameter is in MeV^{-1} units.

Z	A	I	B_n	Pair	a
9	20	0.5	6.601	0.00	3.79
11	24	1.5	6.959	0.00	3.72
12	25	0.5	7.332	2.46	3.03
12	26	2.5	11.093	4.26	4.02
13	27	2.5	7.725	0.00	3.83
14	29	0.0	8.474	2.09	3.20
15	32	0.5	7.937	0.00	3.81
16	33	0.0	8.646	1.62	3.50
16	34	1.5	11.417	3.48	3.66
16	35	0.0	6.986	1.62	4.27
17	36	1.5	8.580	0.00	4.25
17	38	1.5	6.108	0.00	5.94
18	41	0.0	6.099	1.62	6.22
19	40	1.5	7.801	0.00	5.21
19	42	1.5	7.535	0.00	5.62
20	41	0.0	8.363	1.83	5.20
20	43	0.0	7.933	1.83	6.34
20	44	3.5	11.132	3.27	6.64
20	45	0.0	7.415	1.83	6.66
20	49	0.0	5.142	1.83	4.40
21	46	3.5	8.761	0.00	6.82
22	47	0.0	8.880	1.73	6.05
22	48	2.5	11.628	3.27	6.20
22	49	0.0	8.143	1.73	6.27
22	50	3.5	10.945	3.03	6.34
22	51	0.0	6.372	1.73	6.14
23	50	6.0	11.052	1.30	6.97
23	52	3.5	7.311	0.00	6.40
24	51	0.0	9.262	1.35	5.35
24	53	0.0	7.940	1.35	5.58
24	54	1.5	9.720	2.62	5.47
24	55	0.0	6.246	1.35	7.10
25	56	2.5	7.270	0.00	7.10
26	55	0.0	9.298	1.54	5.22
26	57	0.0	7.646	1.54	6.17
26	58	0.5	10.043	2.83	5.62
26	59	0.0	6.587	1.54	7.15
27	60	3.5	7.492	0.00	7.59
28	59	0.0	9.000	1.20	5.44
28	60	1.5	11.389	2.49	5.81
28	61	0.0	7.820	1.20	6.51
28	62	1.5	10.598	2.61	6.33
28	63	0.0	6.839	1.20	6.91
28	65	0.0	6.098	1.20	7.39
29	64	1.5	7.916	0.00	8.31
29	66	1.5	7.067	0.00	8.52
30	65	0.0	7.979	1.06	8.39
30	67	0.0	7.052	1.06	8.74
30	69	0.0	6.482	1.06	9.50
30	71	0.0	5.835	1.06	9.94
31	70	1.5	7.655	0.00	9.64
31	72	1.5	6.521	0.00	11.25
32	71	0.0	7.415	1.36	11.49
32	73	0.0	6.780	1.36	10.92
32	74	4.5	10.200	3.24	12.09
32	75	0.0	6.505	1.36	10.52
32	77	0.0	6.071	1.36	10.93
33	76	1.5	7.328	0.00	11.79
34	75	0.0	8.028	1.43	11.76
34	77	0.0	7.418	1.43	11.65
34	78	0.5	10.497	2.90	10.37
34	79	0.0	6.961	1.43	11.07
34	81	0.0	6.701	1.43	11.44
34	83	0.0	5.896	1.43	10.95
35	80	1.5	7.892	0.00	11.82
35	82	1.5	7.593	0.00	10.98
37	86	2.5	8.650	0.00	9.17
37	88	1.5	6.078	0.00	9.34
38	85	0.0	8.524	1.24	11.10
38	87	0.0	8.428	1.24	9.12
38	88	4.5	11.113	2.17	8.41
38	89	0.0	6.364	1.24	7.59
39	90	0.5	6.857	0.00	8.33
40	91	0.0	7.200	1.20	8.06
40	92	2.5	8.635	1.92	8.68
40	93	0.0	6.732	1.20	10.01
40	95	0.0	6.471	1.20	10.63
40	97	0.0	5.581	1.20	10.54
41	94	4.5	7.230	0.00	12.01
42	93	0.0	8.067	1.28	8.85
42	9 5	0.0	7.375	1.28	10.38
42	96	2.5	9.154	2.40	10.80
42	97	0.0	6.821	1.28	12.06
42	98	2.5	8.642	2.57	12.45
42	99	0.0	5.926	1.28	14.22
42	101	0.0	5.398	1.28	16.49
43	100	4.5	6.764	0.00	15.11
44	100	2.5	9.673	2.57	12.48
44	102	2.5	9.220	2.22	13.14
44	103	0.0	6.232	1.28	15.47
44	105	0.0	5.908	1.28	16.97
45	104	0.5	6.999	0.00	14.69

Table 1 (continuation)

Z	A	I	B_n	Pair	a
46	105	0.0	7.094	1.35	14.02
46	106	2.5	9.562	2.59	14.02
46	107	0.0	6.530	1.35	15.36
46	108	2.5	9.223	2.50	14.28
46	109	0.0	6.154	1.35	16.69
46	111	0.0	5.760	1.35	1 .67
47	108	0.5	7.269	0.00	14.85
47	110	0.5	6.806	0.00	16.15
48	107	0.0	7.928	1.36	13.92
48	109	0.0	7.360	1.36	14.97
48	111	0.0	6.976	1.36	15.02
48	112	0.5	9.395	2.50	13.60
48	113	0.0	6.544	1.36	15.97
48	114	0.5	9.041	2.68	14.28
48	115	0.0	6.145	1.36	16.37
48	117	0.0	5.770	1.36	17.19
49	114	4.5	7,275	0.00	15.29
49	116	4.5	6.784	0.00	15.95
50	113	0.0	7.744	1.19	13.43
50	115	0.0	7.546	1.19	13.06
50	116	0.5	9.562	2.51	12.48
50	117	0.0	6.944	1.19	13.43
50	118	0.5	9.327	2.34	12.61
50	119	0.0	6.485	1.19	13.48
50	120	0.5	9.106	2.43	12.54
50	121	0.0	6.176	1.19	12.69
50	123	0.0	5.948	1.19	12.88
50	125	0.0	5.732	1.19	12.83
51	122	2.5	6.806	0.00	15.51
51	124	3.5	6.467	0.00	14.94
52	123	0.0	6.933	1.14	15.41
52	124	0.5	9.423	2.57	14.13
52	125	0.0	6.578	1.14	15.13
52	126	0.5	9.120	2.23	12.80
52	127	0.0	6.290	1.14	13.61
52	129	0.0	6.085	1.14	13.17
52	131	0.0	5.924	1.14	12.12
53	128	2.5	6.826	0.00	15.28
53	130	3.5	6.463	0.00	15.12
54	125	0.0	7.730	1.12	14.94
54	129	0.0	6.908	1.12	14.04
54	130	0.5	9.254	2.32	13.28
54	131	0.0	6.612	1.12	14.73
54	132	1.5	8.936	2.16	10.58
54	133	0.0	6.448	1.12	12.64
55	134	3.5	6.891	0.00	14.20
55	135	4.0	8.828	0.00	12.71
56	131	0.0	7.494	1.15	15.72
56	135	0.0	6.975	1.15	13.78
56	136	1.5	9.107	1.85	11.45
56	137	0.0	6.898	1.15	10.42
56	138	1.5	8.611	2.00	10.00

Table 1 (continuation)

Z	A	I	B_n	Pair	a
56	139	0.0	4.723	1.15	12.27
57	139	5.0	8.778	0.85	12.71
57	140	3.5	5.161	0.00	13.65
58	137	0.0	7.490	1.17	14.61
58	141	0.0	5.429	1.17	12.35
59	142	2.5	5.843	0.00	13.73
60	143	0.0	6.123	1.18	13.49
60	144	3.5	7.817	1.94	14.44
60	145	0.0	5.756	1.18	15.87
60	146	3.5	7.565	2.10	16.98
60	147	0.0	5.292	1.18	18.11
60	148	2.5	7.334	2.17	20.96
60	149	0.0	5.093	1.18	20.49
60	151	0.0	5.334	1.18	19.79
61	148	3.5	5.902	0.00	19.09
62	148	3.5	8.140	2.14	17.50
62	150	3.5	7.985	2.21	19.74
62	151	0.0	5.596	1.22	21.76
62	152	3.5	8.258	2.32	21.43
62	153	0.0	5.867	1.22	20.60
62	155	0.0	5.814	1.22	18.82
63	152	2.5	6.305	0.00	22.12
63	154	2.5	6.435	0.00	20.99
64	153	0.0	6.407	0.97	21.18
64	155	0.0	6.438	0.97	20.83
64	156	1.5	8.536	1.89	18.38
64	157	0.0	6.360	0.97	18.79
64	158	1.5	7.937	1.70	17.40
64	159	0.0	5.942	0.97	18.47
64	161	0.0	5.635	0.97	17.68
65	160	1.5	6.375	0.00	19.29
66	157	0.0	6.975	0.92	21.18
66	161	0.0	6.454	0.92	19.48
66	162	2.5	8.196	1.62	17.93
66	163	0.0	6.271	0.92	18.40
66	164	2.5	7.656	1.79	17.42
66	165	0.0	5.715	0.92	17.31
67	166	3.5	6.243	0.00	18.80
68	163	0.0	6.905	0.62	19.53
68	165	0.0	6.650	0.62	18.93
68	167	0.0	6.436	0.62	17.75
68	168	3.5	7.771	1.23	16.87
68	169	0.0	6.003	0.62	17.37
68	171	0.0	5.681	0.62	16.88
69	170	0.5	6.593	0.00	18.87
70	171	0.0	6.414	0.68	19.10
70	172	0.5	8.020	1.37	16.92
70	173	0.0	6.367	0.68	17.40
70	174	2.5	7.465	1.23	16.40
70	175	0.0	5.822	0.68	16.93
70	177	0.0	5.566	0.68	17.34
71	176	3.5	6.293	0.00	19.97

Table 1 (continuation)

Z	A	I	B_n	Pair	a
71	177	7.0	7.072	0.40	20.55
72	175	0.0	6.790	0.64	18.83
72	177	0.0	6.383	0.64	19.16
72	178	3.5	7.266	1.04	18.69
72	179	0.0	6.099	0.64	18.38
72	180	4.5	7.388	1.37	18.95
72	181	0.0	5.694	0.64	18.22
73	182	3.5	6.063	0.00	19.20
73	183	3.0	6.934	0.58	18.37
74	181	0.0	6.685	0.72	19.15
74	183	0.0	6.191	0.72	17.90
74	184	0.5	7.411	1.30	17.05
74	185	0.0	5.754	0.72	18.54
74	187	0.0	5.466	0.72	19.13
75	186	2.5	6.179	0.00	19.84
75	188	2.5	5.872	0.00	20.40
76	190	1.5	7.793	1.88	18.91
77	192	1.5	6.199	0.00	20.82
77	194	1.5	6.066	0.00	19.36
78	193	0.0	6.247	0.71	19.78
78	195	0.0	6.169	0.71	17.70
78	196	0.5	7.921	1.50	16.07
79	198	1.5	6.513	0.00	16.08
80	199	0.5	6.665	0.87	15.76
80	200	0.5	8.028	1.69	12.59
80	201	0.0	6.230	0.87	12.92
80	202	1.5	7.755	1.58	12.29
81	204	0.5	6.655	0.00	10.95
81	206	0.5	6.503	0.00	7.10
82	205	0.0	6.734	0.83	9.68
82	207	0.0	6.740	0.83	5.57
82	208	0.5	7.368	1.21	4.12
82	209	0.0	3.936	0.83	4.16
83	210	4.5	4.604	0.00	10.25
88	227	0.5	4.552	0.89	27.42
90	230	2.5	6.791	1.38	25.15
90	231	0.0	5.120	0.78	27.80
90	233	0.0	4.786	0.78	27.71
91	232	1.5	5.561	0.00	27.02
91	234	1.5	5.209	0.00	28.26
92	233	0.0	5.753	0.69	25.83
92	234	2.5	6.843	1.26	24.10
92	235	0.0	5.297	0.69	25.55
92	236	3.5	6.545	1.18	25.00
92	237	0.0	5.124	0.69	26.12
92	239	.0	4.806	0.69	26.70
93	238	2.5	5.488	0.00	26.17
94	239	0.0	5.647	0.61	24.41
94	240	0.5	6.534	1.04	23.42
94	241	0.0	5.240	0.61	25.64
94	242	2.5	6.309	1.11	24.35
94	243	0.0	5.037	0.61	25.68

Table 1. (Continuation)

Z	A	I	B_n	Pair	a
95	242	2.5	5.541	0.00	26.25
95	243	1.0	6.363	0.50	26.16
95	244	2.5	5.362	0.00	26.72
96	243	0.0	5.695	0.72	24.09
96	244	2.5	6.799	1.22	23.61
96	245	0.0	5.519	0.72	24.90
96	246	0.5	6.456	1.11	24.26
96	247	0.0	5.157	0.72	24.16
96	248	4.5	6.212	1.11	24.66
96	249	0.0	4.713	0.72	26.32
97	250	3.5	4.970	0.00	27.12
98	250	4.5	6.623	1.16	23.76

To find the temperature or excitation energy dependence of the level density parameter, a, the liquid drop model value of this parameter is expressed [10] in the following form:

$$a_{ldm} = a_v A + a_s A^{2/3} + a_c A^{1/3} \qquad \mathbf{(3)}$$

where a_v, a_s, and a_c are the volume, surface, and curvature components of the level density parameter at excitation energies where shell effects vanished i.e. at nuclear temperatures larger than about 2 MeV. The shell effects are taken into consideration by adopting the formalism described in reference 11. This is achieved by multiplying Eq. 3 by the following relation:[11]

$$SC = [1 - \frac{\Delta_s}{U}(1 - e^{\frac{-\gamma U}{A^{1/3}}})] \qquad \mathbf{(4)}$$

where SC stands for the shell correction of the level density parameter, Δ_s is the liquid drop model shell energy [12] and γ is a damping factor, which will be determined in the present study.

A least-squares fitting procedure for the data of Table 1 in terms of Eqns. (3) and (4), yielded the following results, $a_v = 0.076 \pm 0.009$ MeV$^{-1,}$ $a_s = 0.180 \pm 0.047$ MeV^{-1}, and $\gamma = 0.47 \pm 0.04$ MeV^{-1} The errors are due only to the fitting procedure and do not include the errors of the level density parameters. Since it is difficult to determine the curvature term a_c from the present data, a value of 0.157 MeV^{-1}, based on theoretical computations,[13,14] was assumed in the analysis.

The temperature dependence of the level density parameter, which was deduced from heavy ion collision measurements in the nuclear mass region around A= 160, was investigated by Shlomo and Natowitz in the framework of Thomas-Fermi approximation with corrections for continuum effects.[15] The authors presented their results, the inverse level density parameter, A/a, for nuclear masses 40, 60, 110, 160, 210 in graphical forms. However, it is very useful to obtain a parameterization of these results for application in statistical calculations in the fields of intermediate-energy heavy-ion collisions, spallation neutron studies, and photonuclear investigations at high excitation energies. In the present study, a description of the variation of the level density parameter with temperature on the basis of the temperature dependence of the effective nucleon mass[16,17] is readily achieved. The details will be published elsewhere. However, we present here the final result:

$$a(T) = a(0)[0.7143(1+0.4e^{-(\frac{TA^{1/3}}{21})^{2}})]^{1.7} \quad \mathbf{(5)}$$

where $a(0)= SC.a_{ldm.}$ The factor 0.4 is an average value over the nuclear mass range 40-210. On the other hand, for specific nuclear masses, such as 40, 60, 110, 160, and 210 the following respective values [15] 0.37, 0.39, 0.43, 0.45, 0.47 can be substituted for 0.4, with the appropriate changes for the normalization coefficient in Eqn. (5).

III. DISCUSSION AND CONCLUSIONS

The spin dispersion parameter, σ, was determined from the average level spacings of s-wave resonances for target nuclides with non-zero spin. The present results of σ are found to be consistently lower than the Gilbert-Cameron theoretical estimate, as well as the Bohr-Mottelson relation, which is expressed in terms of a rigid body moment of inertia. This finding implies that the moment of inertia is reduced by a factor of two. In the rare earth region this interpreted in terms of the influence of pairing correlations[6].

On the basis of the present results for the spin dispersion parameter, the nuclear level density parameters for the various stable nuclides were then calculated on the basis of the Bethe Fermi-gas model and found to be generally low by about 10%-20% when compared with other determinations. The present results have an impact on the calculations of neutron yields per proton in spallation neutron investigations and design studies.[18]

The temperature, or alternatively the excitation-energy, dependence of the level density parameter is investigated and an analytical relation (Eqn. 5) in terms of the effective nucleon mass successfully reproduced the theoretical estimates[15] to better than 5% over the temperature range 0-10 MeV. The possible influence of the temperature dependence of the level density parameter on spallation neutron and gamma-ray yields are presently under investigation.

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