HOW TO DRAW A LEVEL SCHEME ?

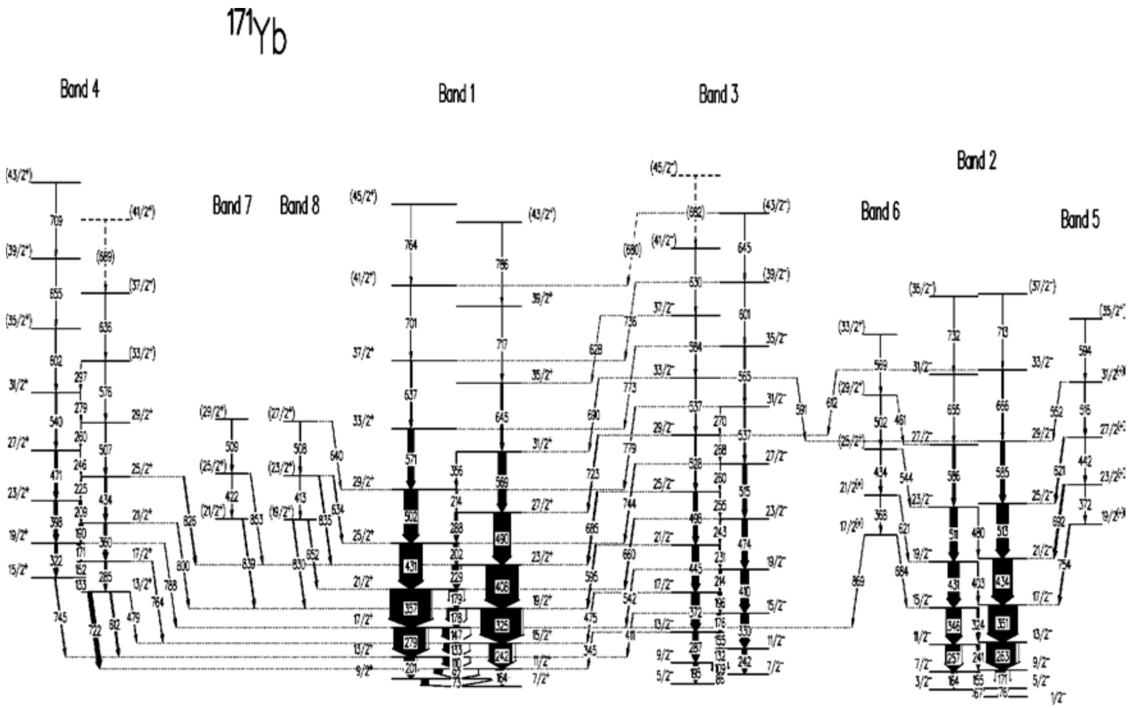
N. NICA

TEXAS A&M UNIVERSITY

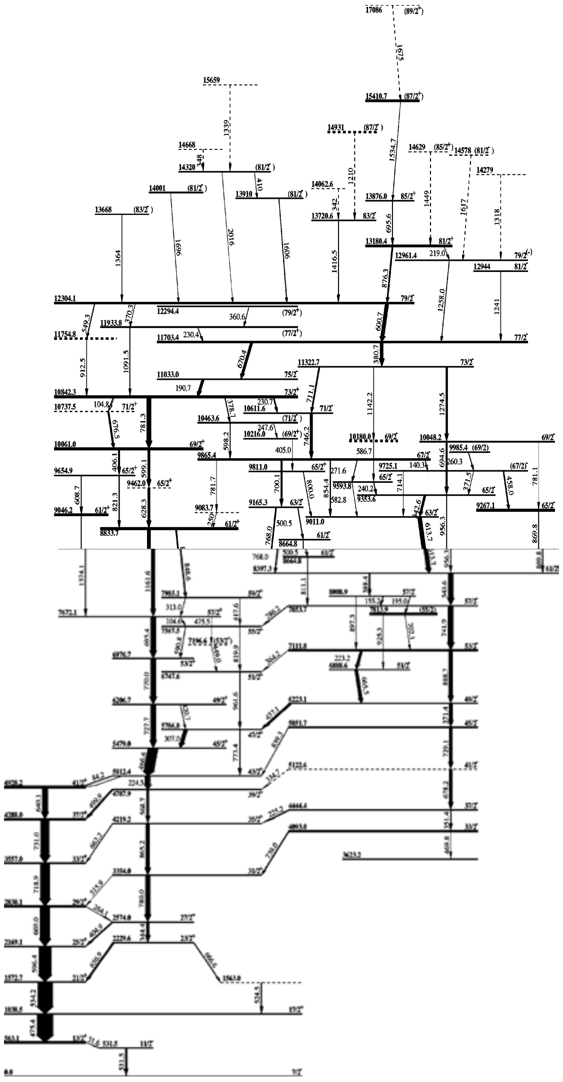
? ? ?

Level Schemes

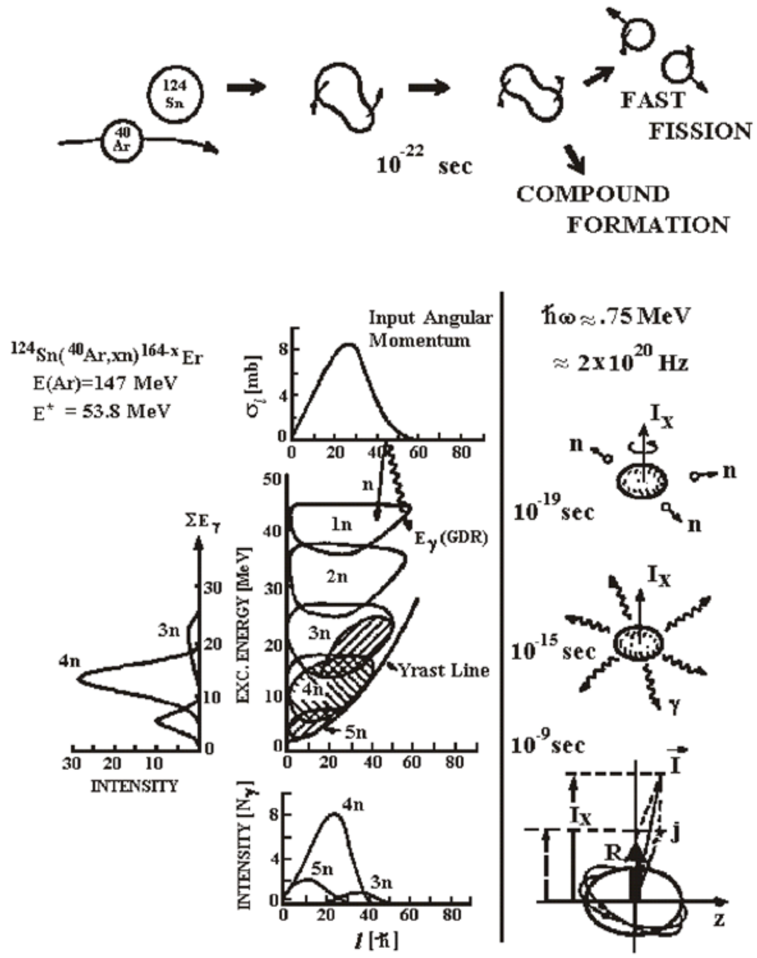
^{171}Yb



^{155}Er



1. Introduction



Population of the entry states

Moments of Inertia

1. **Collective rotations** – ***"I(I+1) rule" directly*** – **generating *rotational bands* based on particular *intrinsic configurations* (band heads)**

$$E(I) = \frac{\hbar^2}{2\Im} I(I+1)$$

where $\Im$ is the *moment of inertia* of the *deformed core* of nuclei in between closed shells , and I is the (total) nuclear spin

$$I = R + i$$

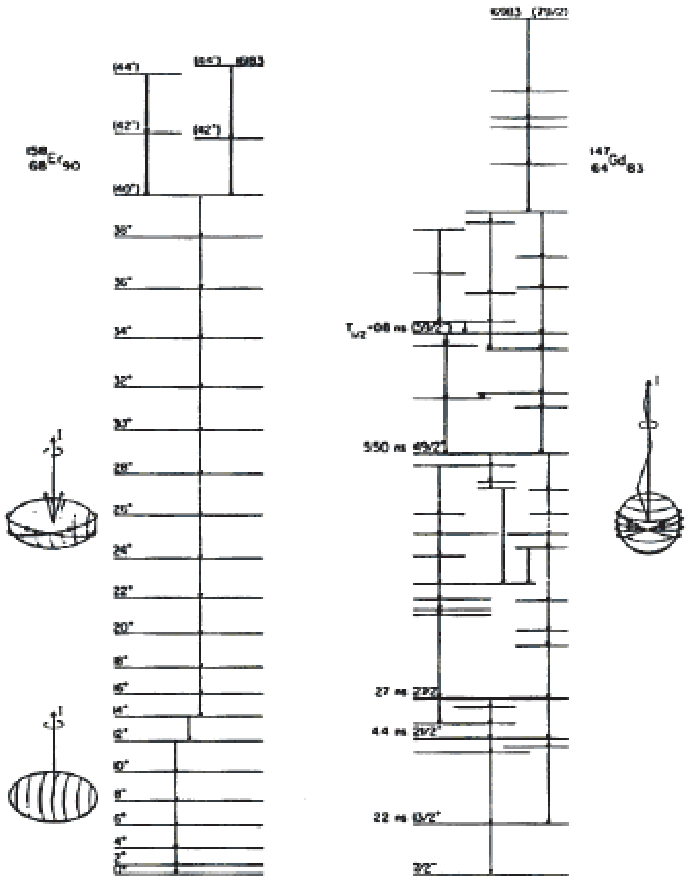
R **is the angular momentum of the rotating core**

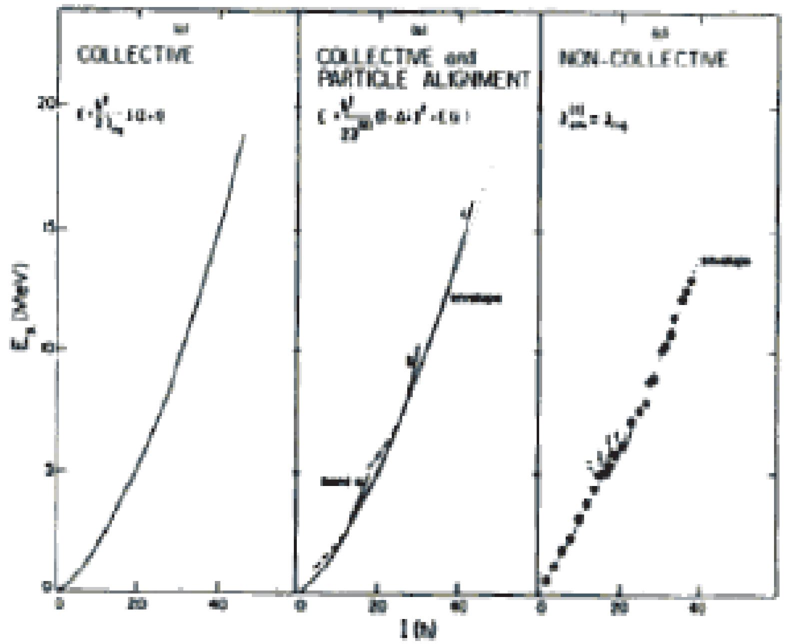
i **is the intrinsic single-particle angular momentum**

2. **Non-collective rotations** – **of *spherical* nuclei at closed shells, where the nuclear spin results from successive single-particle alignments and the *"I(I+1) rule"* is satisfied "on average"**

Moments of Inertia: *Band* ($\Im_{band}$) and *Effective* ($\Im_{eff}$)

Collective **Non-collective**

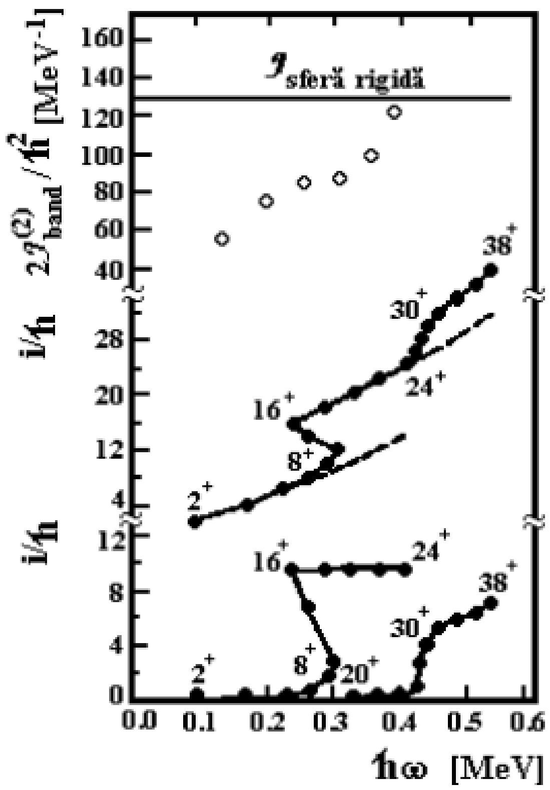




Consequences of "I(I+1)" Rule

E2 γ-ray energy:

$$E_\gamma = E(I) - E(I-2) = \frac{\hbar^2}{2\Im}(4I-2) = 2c(2I-1)$$



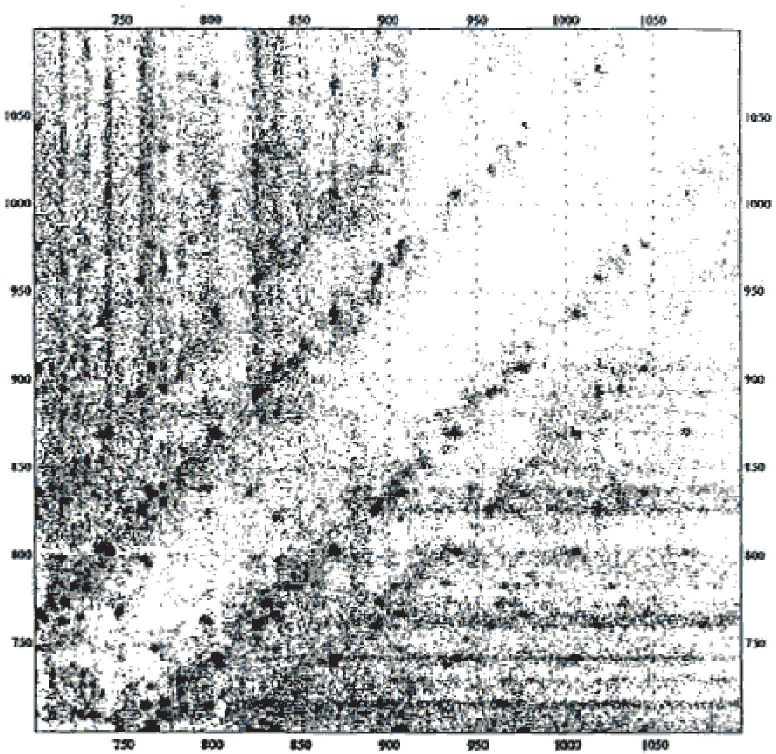
Rotational parameter:

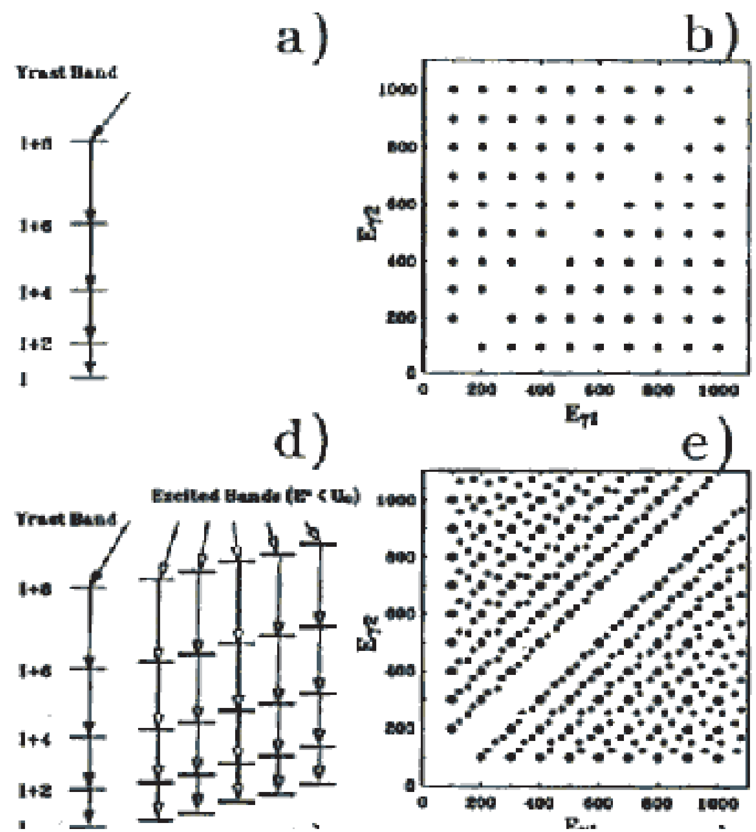
$$c = \frac{\hbar^2}{2\Im}$$

γ-ray energy difference

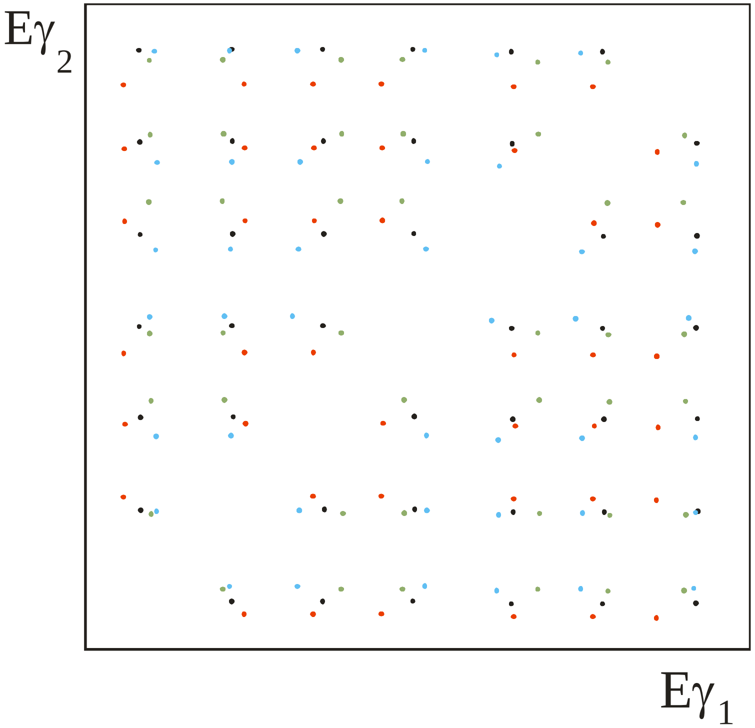
$$\Delta E_\gamma = E_\gamma(I) - E_\gamma(I-2) = 8\frac{\hbar^2}{2\Im} = 8c$$

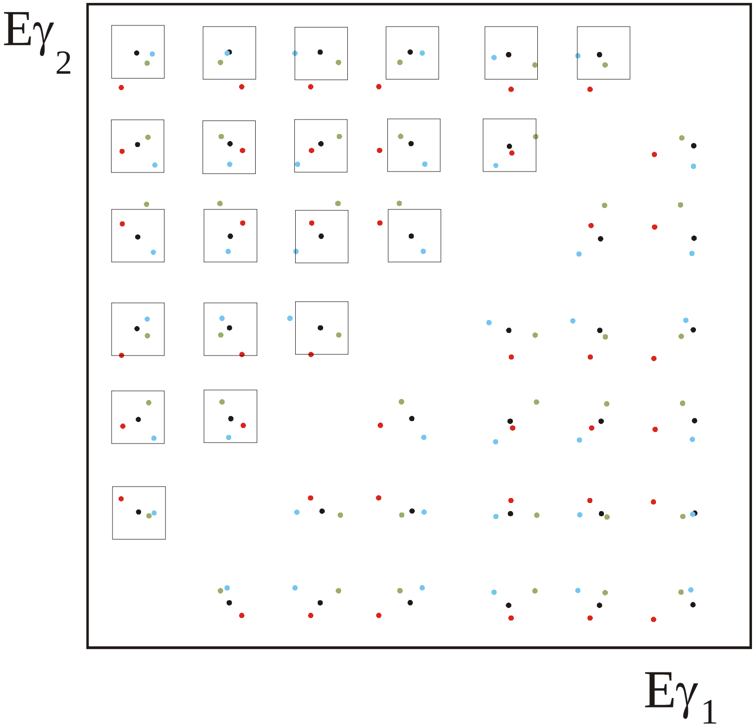
γ–γ Coincidence Matrix





γ–γ Coincidence Matrix

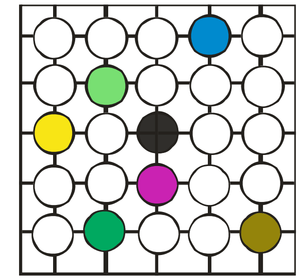




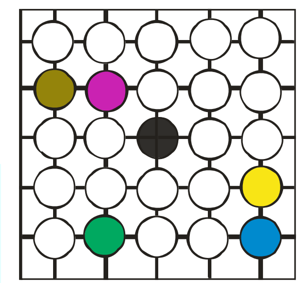
Repeatability 1: Study of distributions of differences of γ-ray coincidence energies

REPEATABILITY:

- Repeated appearance of satellite peaks relative to the coincidence peaks of a reference rotational band at same location.



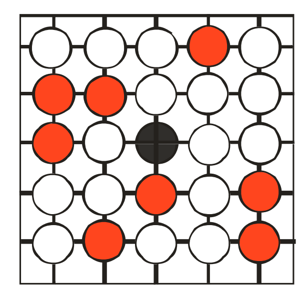
*- The repeatability peaks are situated on a regular grid with characteristic distance **d_{grid}**:*



$$(\Delta E_{\gamma_1}, \Delta E_{\gamma_2}) =$$

$$= (E^r_{\gamma_1}, E^r_{\gamma_2}) - (E^s_{\gamma_1}, E^s_{\gamma_2}) = (m \cdot d_{grid}, n \cdot d_{grid}), \quad m, n \in Z$$

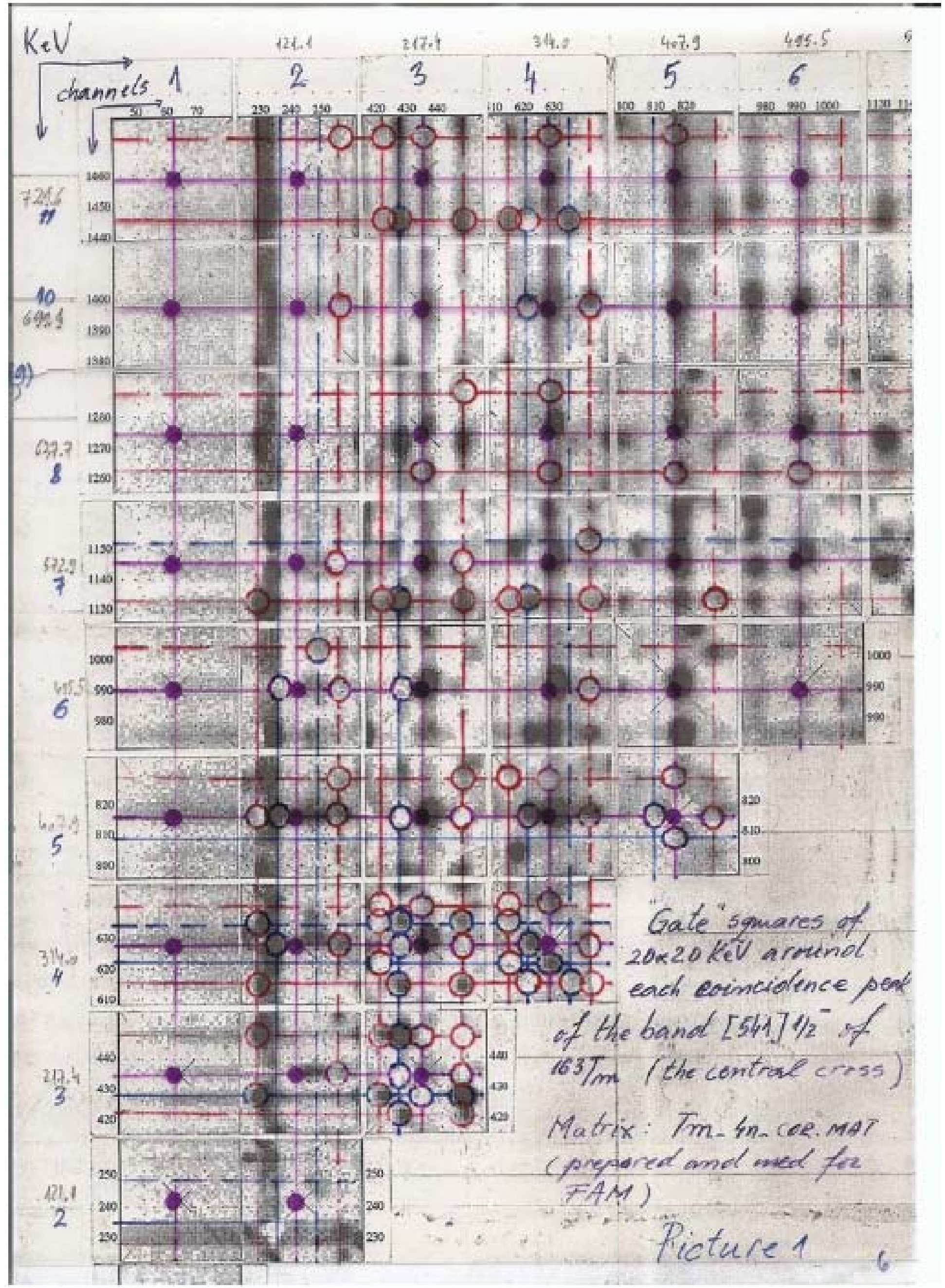
$\Sigma =$



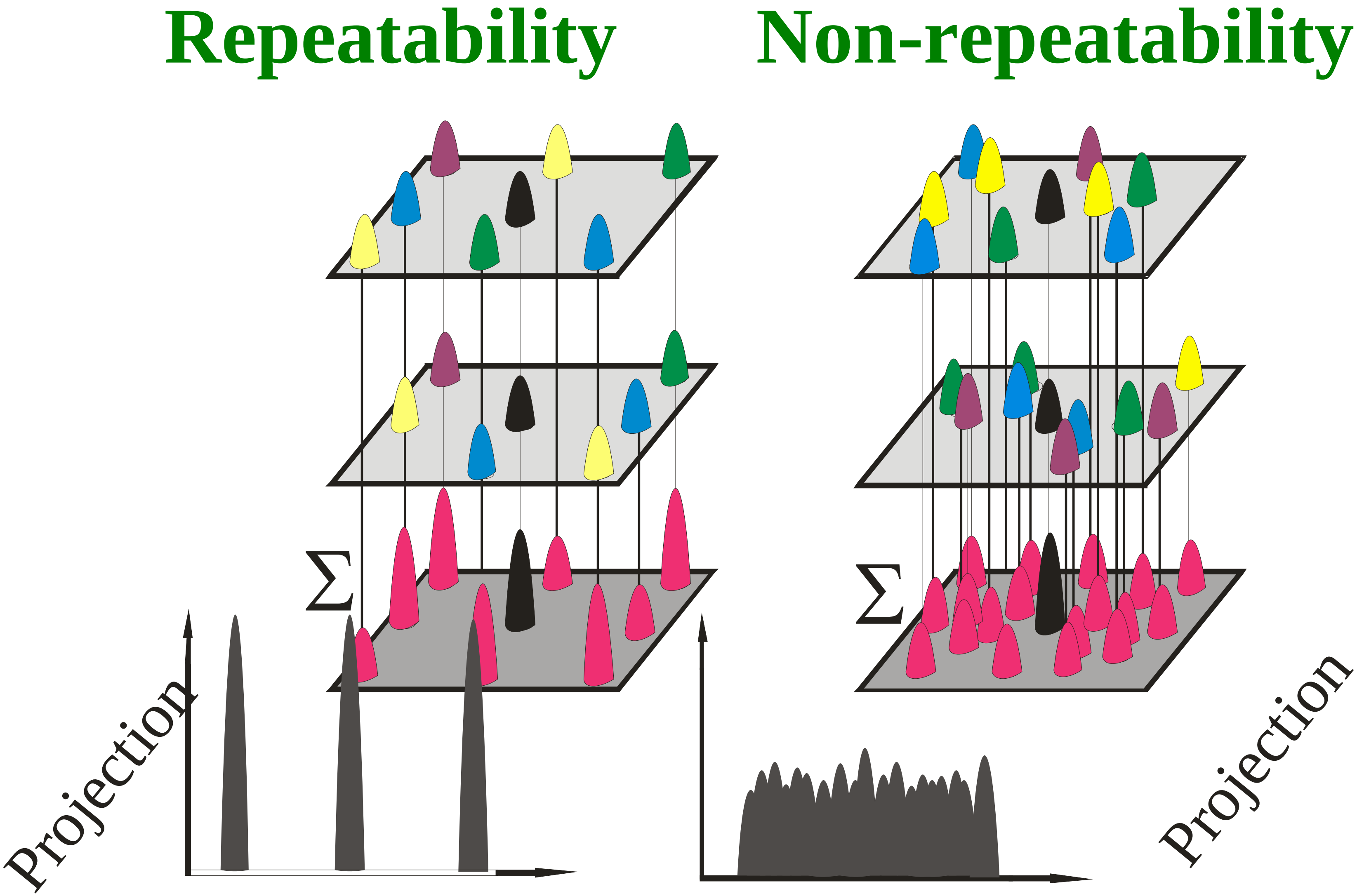
- The repeatability peaks appears "statistically" at a number of repeatability positions, including the windows situated on the diagonal of the central valley.

Sample of repeatability around reference band [541]1/2$^-$ of ^{163}Tm

Repeatable satellite peaks on the regular grid with d_{grid} = 3.2 keV

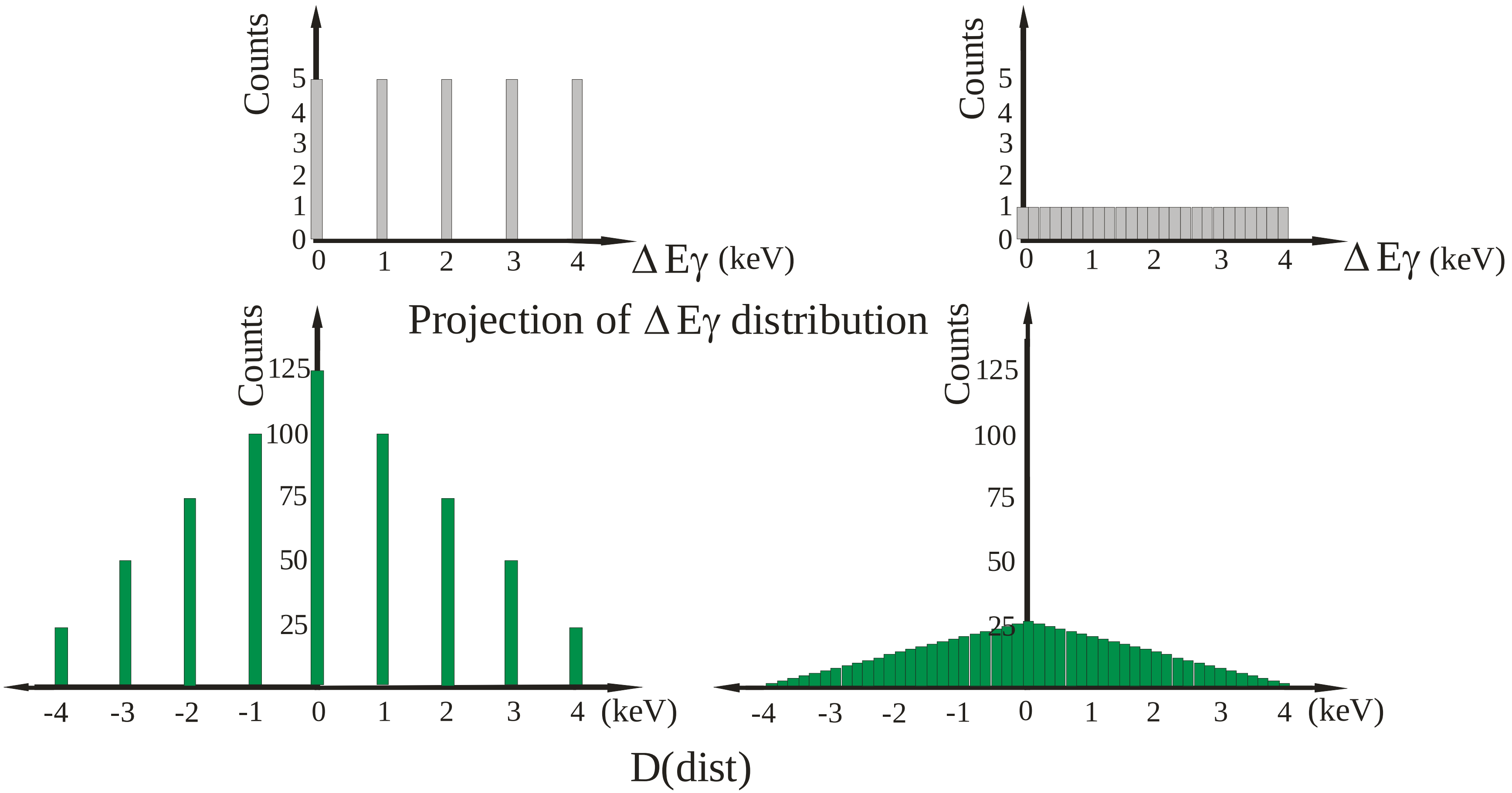


ΔEγ Distribution

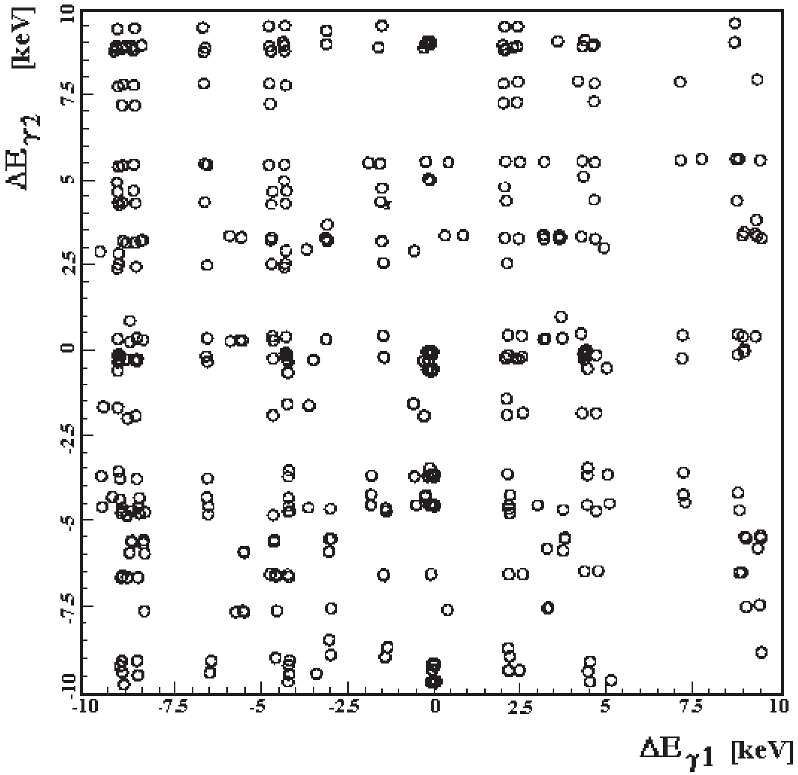


Distribution of distances *D(dist)* of ΔEγ distribution

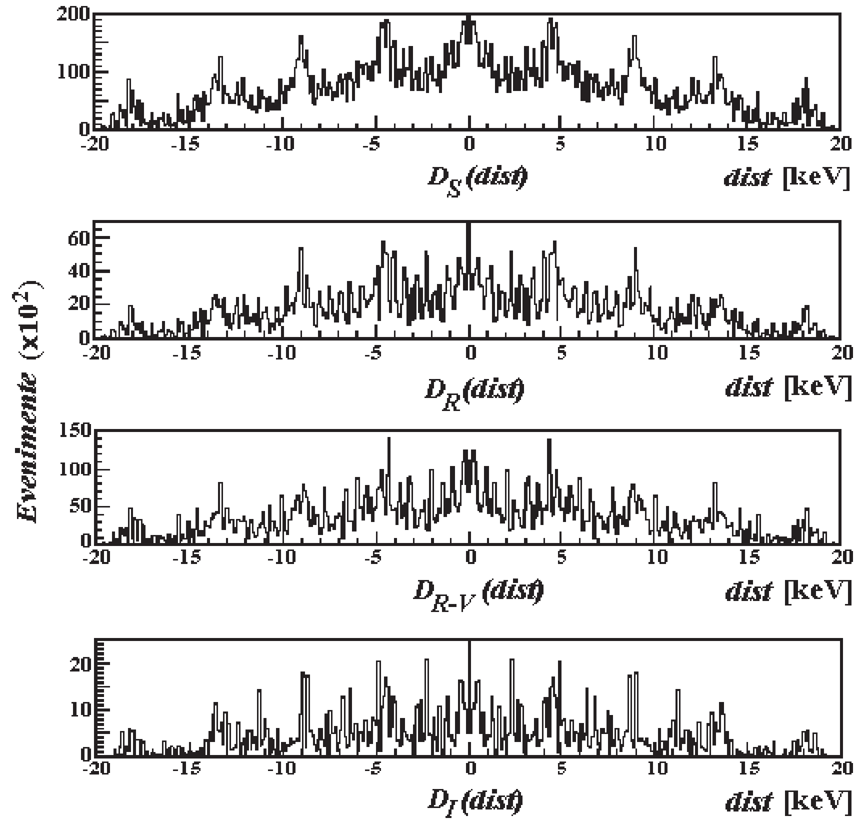
Repeatability — Non-repeatability



Repeatability of [411]1/2+ band in ^{163}Tm



ΔE_{γ} from upper half of coinc. matrix (case "I"): d_{grid} = 4.5 keV



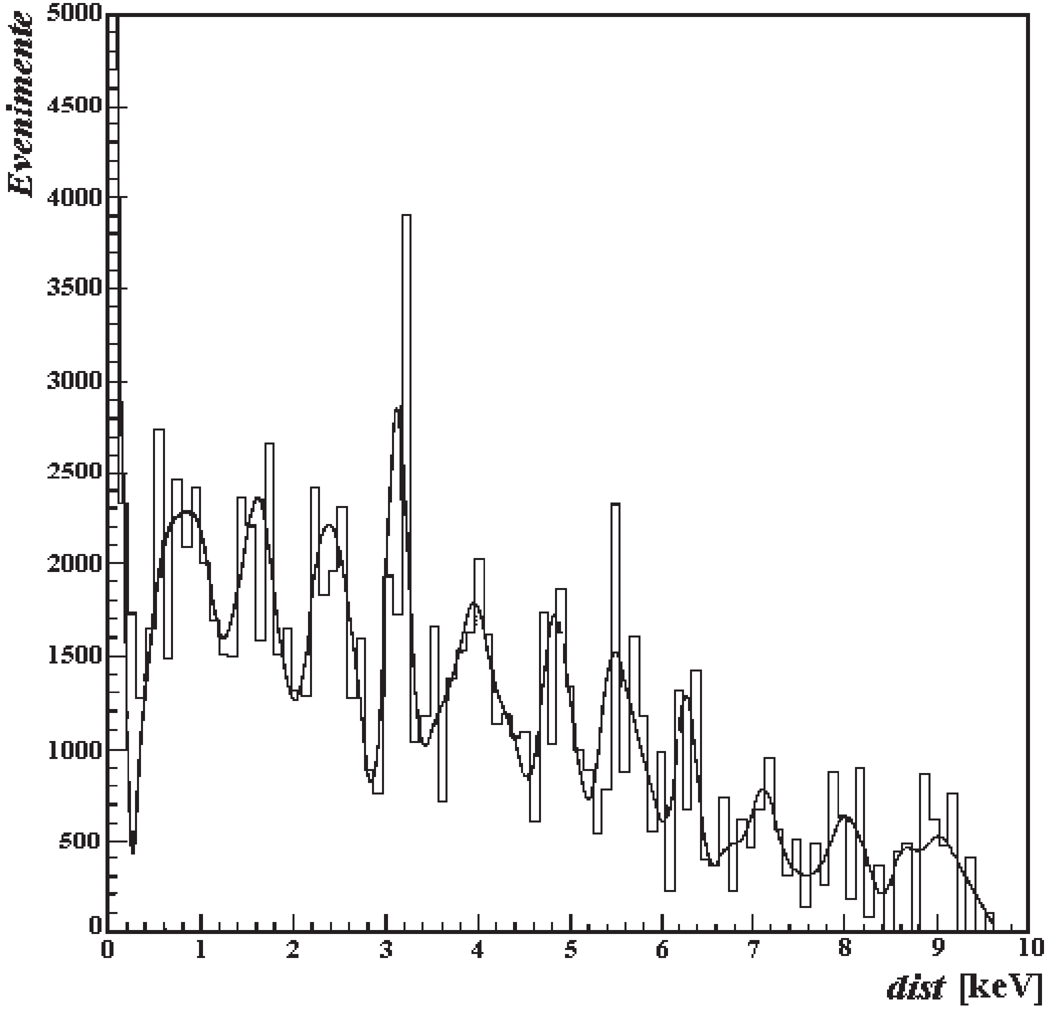
Distributions $D(dist)$: d_{grid} = 4.5 keV

IMP!: Fractal-like structure of hierarchized maxima!

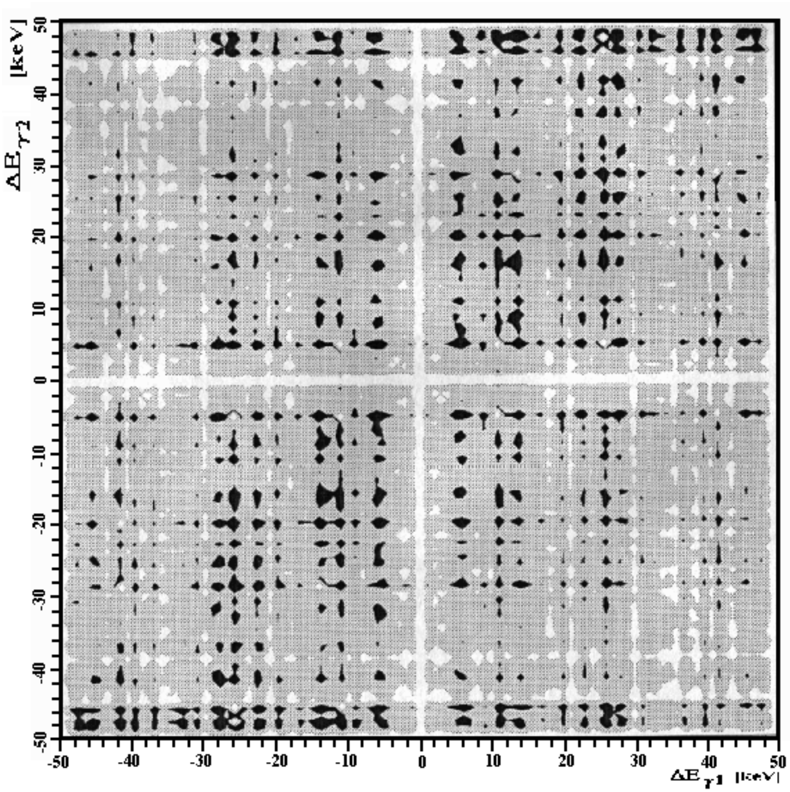
Repeatability of [541]1/2$^-$ band in ^{163}Tm

$D_S(dist)$ distribution

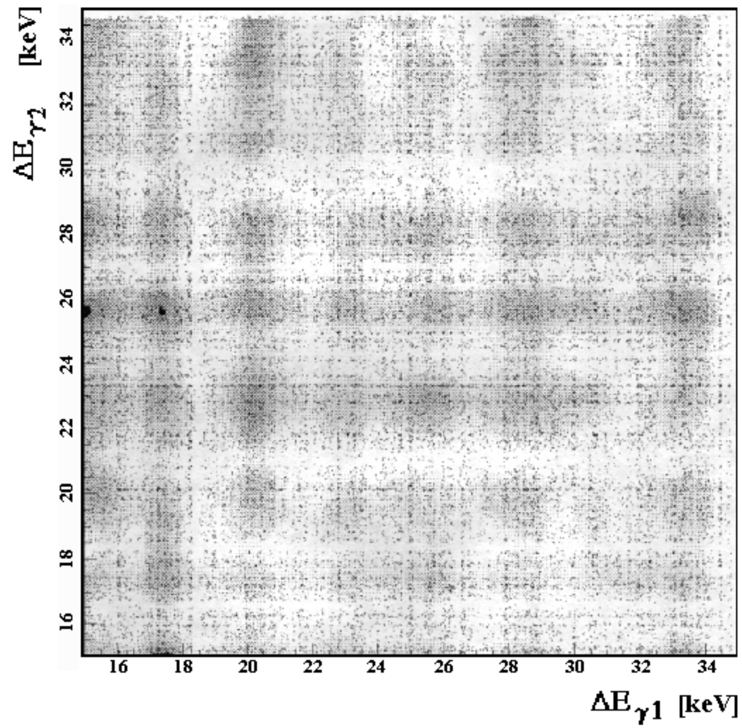
D_{grid} = 0.8 keV



Repeatability in ^{163}Tm (all-bands reference, “total reference”)

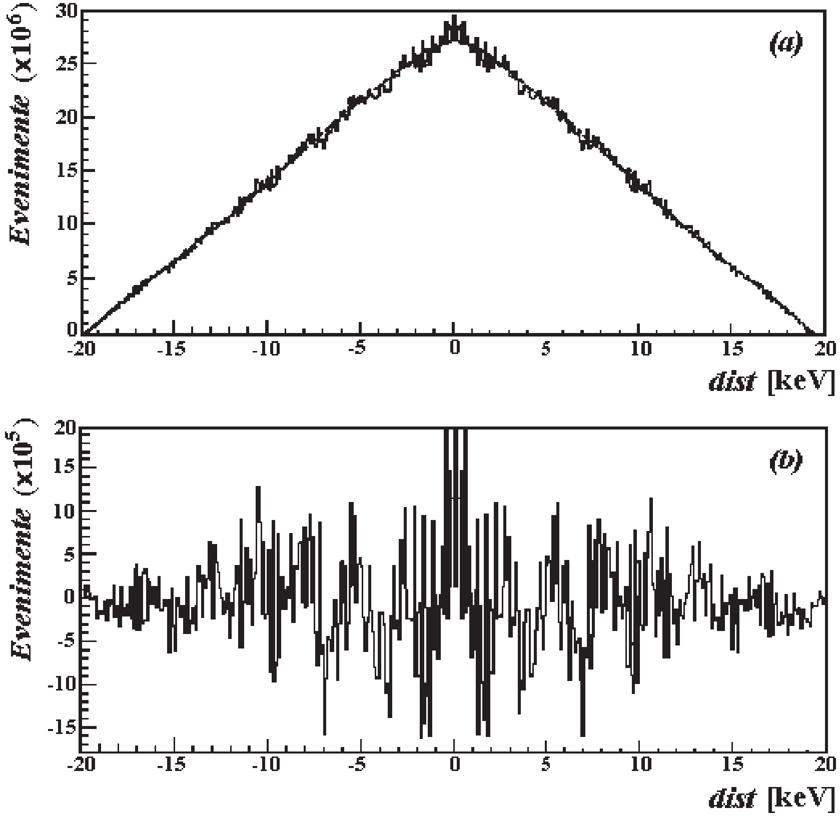


ΔE_{γ} distribution (1 kev/ch) reveal large scale repeatability pattern with $d_{grid} \approx 2.7$ keV

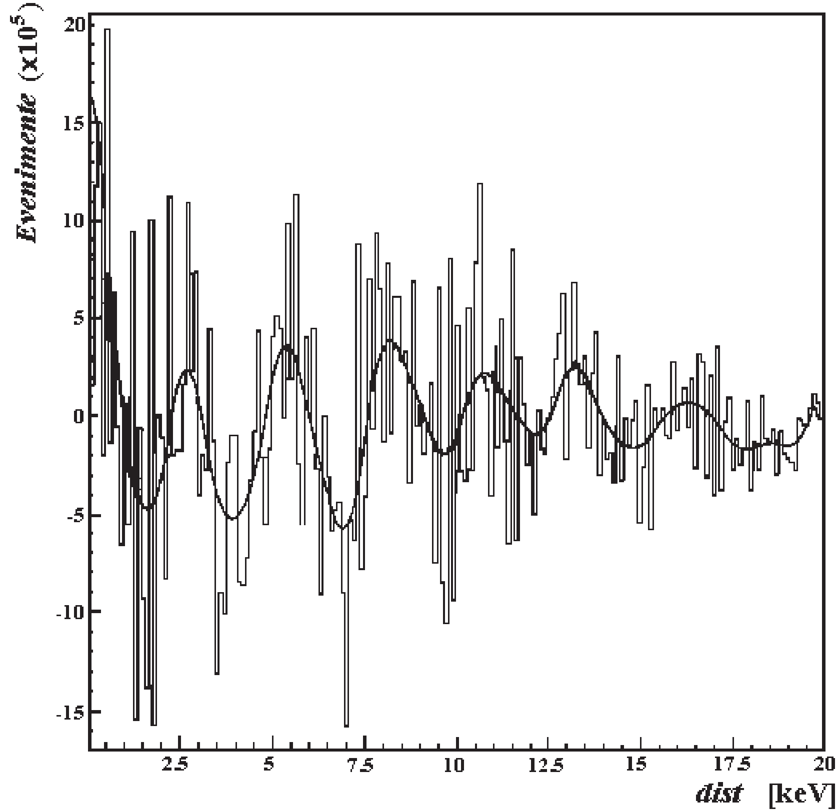


Detail of same ΔE_{γ} distribution (0.1 kev/ch – default value)

Repeatability in ^{163}Tm (all-bands reference) – cont.

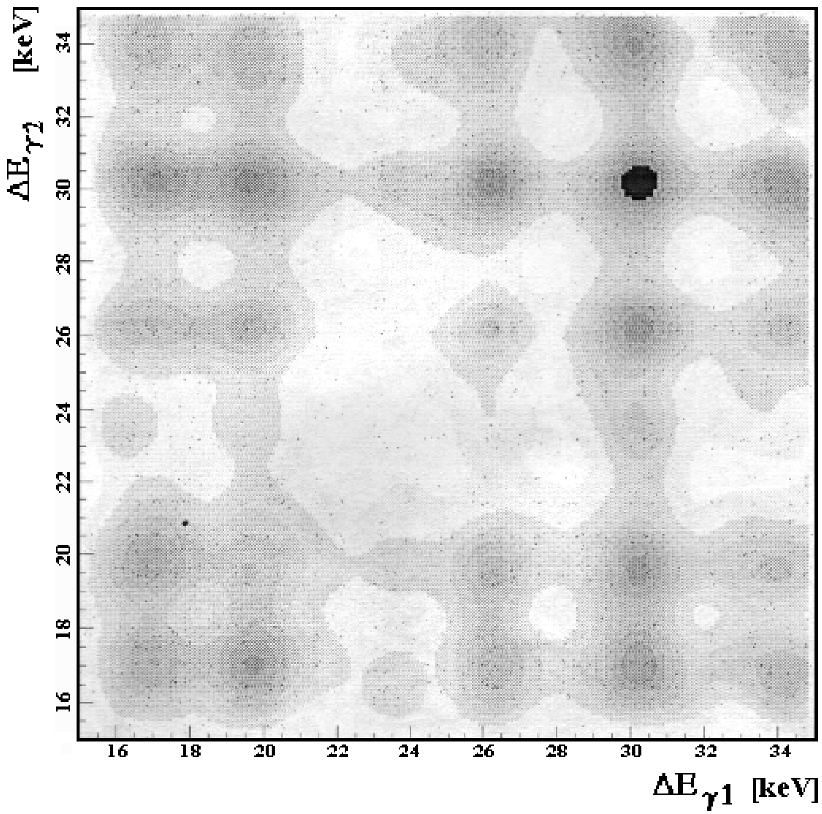


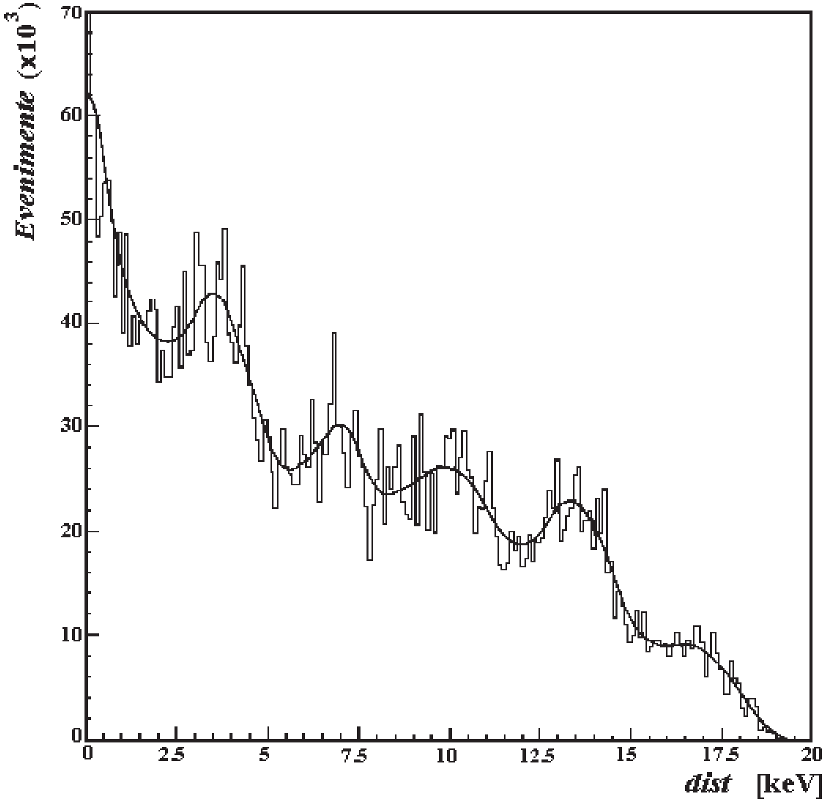
D_R(dist) for the detail of ΔE_γ (previous figure), revealing oscillations around plateau



Repeatability pattern with $d_{grid} \approx 2.65$ keV

Repeatability in ^{162}Tm (all-bands reference)

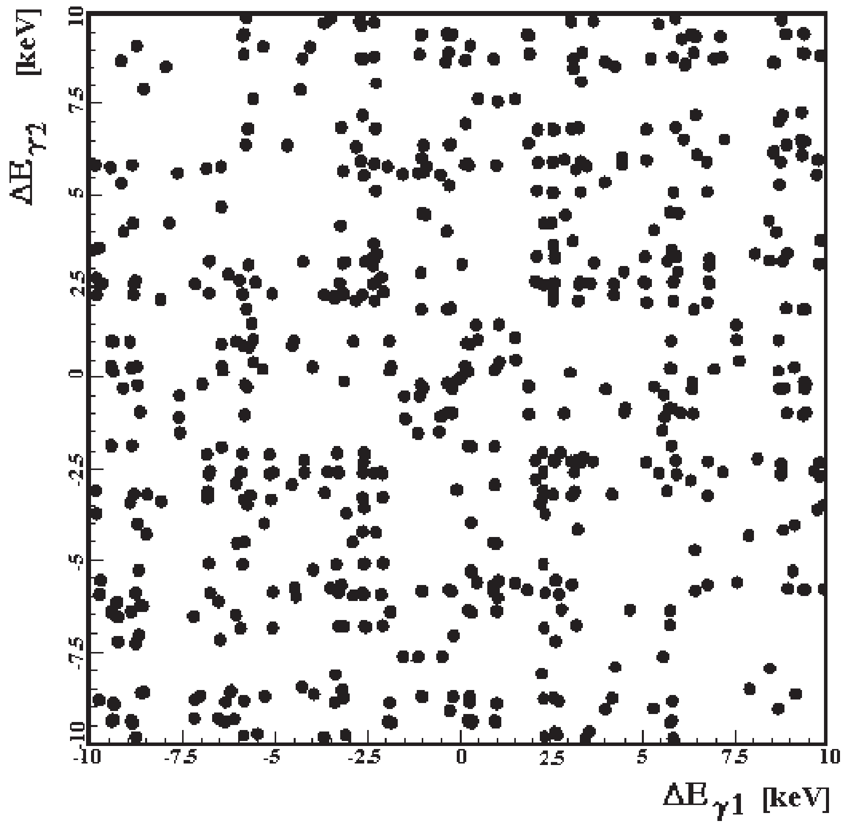




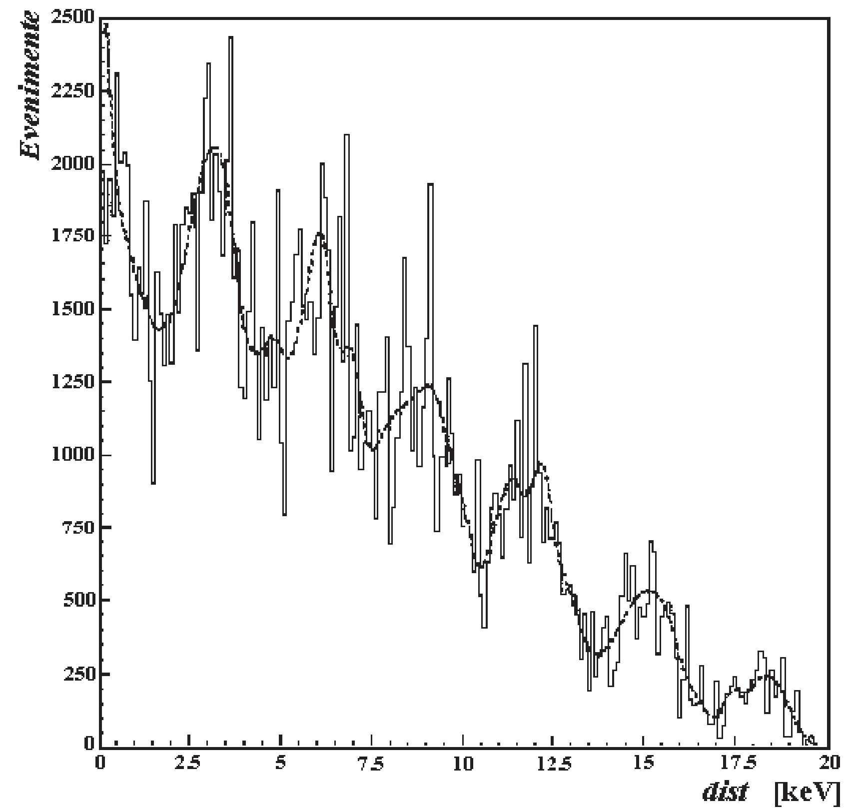
ΔE$_\gamma$ distribution of type "*R*" (black points, superposed with their fit with 2D spline functions)

***D_R(dist)* revealing repeatability pattern with $d_{grid} \approx 3.4$ keV**

Repeatability in ^{168}Yb (all-bands reference)



ΔE$_{\gamma}$ distribution of type "*S*"



D$_{S}$(dist) revealing repeatability pattern with $d_{grid} \approx 3.0$ keV

Repeatability findings

Nucleus		D_{grid} *(keV)* / Ref. type		Obs.
^{163}Tm *odd*	2.65 (total)	4.5 ([411]1/2^{+})	0.8 ([541]1/2^{-})	2.65 = (4.5+0.8)/2
^{162}Tm *odd-odd*	3.4 (total)			
^{168}Yb *even-even*	3.0 (total)			

Repeatability:

- regular symmetrical grid of repeated satellite peaks
- everywhere in the coincidence matrix including central valley
- fractal-like structure of hierarchized maxina

Consequences on levels schemes