ORELA Flight Path 1: Determinations of Its Effective Length vs Energy, Experimental Energies, and Energy Resolution Function and Their Uncertainties

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ORELA FLIGHT PATH 1: DETERMINATIONS OF ITS EFFECTIVE LENGTH vs ENERGY, EXPERIMENTAL ENERGIES, AND ENERGY RESOLUTION FUNCTION AND THEIR UNCERTAINTIES

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# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>1</td>
</tr>
<tr>
<td>1. Introduction and Definitions</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Method and Definitions</td>
<td>2</td>
</tr>
<tr>
<td>2. General Properties of Flight Path</td>
<td>6</td>
</tr>
<tr>
<td>3. Properties Associated With the Flight-Path Length</td>
<td>6</td>
</tr>
<tr>
<td>3.1 Distribution Function for $x_1$: The Contribution to the Flight-Path Length From the Target</td>
<td>9</td>
</tr>
<tr>
<td>3.1.1 Mean Value $t_1$ for Distribution of $x_1$</td>
<td>12</td>
</tr>
<tr>
<td>3.1.2 Uncertainty in $t_1$</td>
<td>12</td>
</tr>
<tr>
<td>3.1.3 Higher Moments of the Target Distribution Function</td>
<td>14</td>
</tr>
<tr>
<td>3.1.4 Width $\omega_1$ of Target Distribution Function</td>
<td>16</td>
</tr>
<tr>
<td>3.1.5 Uncertainty on $\omega_1$</td>
<td>17</td>
</tr>
<tr>
<td>3.2 Distribution Function for $x_2$: The Contribution to the Flight-Path Length From the Flight Tube</td>
<td>18</td>
</tr>
<tr>
<td>3.3 Distribution Function for $x_3$: The Contribution to the Flight-Path Length From the Detector</td>
<td>19</td>
</tr>
<tr>
<td>3.3.1 Mean Value $t_3$ for Distribution of $x_3$</td>
<td>20</td>
</tr>
<tr>
<td>3.3.2 Uncertainty in $t_3$</td>
<td>20</td>
</tr>
<tr>
<td>3.3.3 Higher Moments of the Detector Distribution Function</td>
<td>21</td>
</tr>
<tr>
<td>3.3.4 Widths $\omega_3$ of Detector Distribution Function</td>
<td>22</td>
</tr>
<tr>
<td>3.3.5 Uncertainty on $\omega_3$</td>
<td>22</td>
</tr>
<tr>
<td>3.4 Distribution Function for $x$: The Total Flight-Path Length</td>
<td>23</td>
</tr>
<tr>
<td>3.4.1 Mean Value $t$ for Distribution of $x$</td>
<td>23</td>
</tr>
<tr>
<td>3.4.2 Uncertainty in $t$</td>
<td>24</td>
</tr>
<tr>
<td>3.4.3 Higher Moments of the Flight-Path Length Distribution Function</td>
<td>25</td>
</tr>
<tr>
<td>3.4.4 Width $\omega$ of Flight-Path Length Distribution Function</td>
<td>25</td>
</tr>
<tr>
<td>3.4.5 Uncertainty on $\omega$</td>
<td>27</td>
</tr>
<tr>
<td>4. Properties Associated With the Flight Time</td>
<td>27</td>
</tr>
<tr>
<td>4.1 Distribution Function for $t_1$: The ORELA Pulse Width</td>
<td>28</td>
</tr>
<tr>
<td>4.1.1 Mean Value $t_1$ for Distribution of $t_1$</td>
<td>28</td>
</tr>
<tr>
<td>4.1.2 Uncertainty in $t_1$</td>
<td>29</td>
</tr>
<tr>
<td>4.1.3 Higher Moments of the Pulse Width Distribution Function</td>
<td>29</td>
</tr>
<tr>
<td>4.1.4 Width $\omega_1$ of Pulse Width Distribution Function</td>
<td>29</td>
</tr>
<tr>
<td>4.1.5 Uncertainty on $\omega_1$</td>
<td>29</td>
</tr>
<tr>
<td>4.2 Distribution Function for $t_2$: The Channel Width</td>
<td>30</td>
</tr>
<tr>
<td>4.2.1 Mean Value $t_2$ for Distribution of $t_2$</td>
<td>30</td>
</tr>
<tr>
<td>4.2.2 Uncertainty in $t_2$</td>
<td>30</td>
</tr>
<tr>
<td>4.2.3 Higher Moments of the Channel Width Distribution Function</td>
<td>30</td>
</tr>
<tr>
<td>4.2.4 Width $\omega_2$ of Channel Width Distribution Function</td>
<td>31</td>
</tr>
<tr>
<td>4.2.5 Uncertainty on $\omega_2$</td>
<td>31</td>
</tr>
</tbody>
</table>
ORELA FLIGHT PATH 1: DETERMINATIONS OF ITS EFFECTIVE LENGTH vs ENERGY, EXPERIMENTAL ENERGIES, AND ENERGY RESOLUTION FUNCTION AND THEIR UNCERTAINTIES

D. C. Larson, N. M. Larson, and J. A. Harvey

ABSTRACT

Flight path 1 at ORELA is nominally 200 m in length and has been extensively used for neutron transmission and scattering measurements. Due to moderation effects in the neutron-producing target and to the finite thickness of the neutron detector, the effective flight-path length is a function of neutron energy. In this report, we determine the effective length as a function of energy, its uncertainty, and time-of-flight energies and their uncertainties. Finally, we determine the resolution function and its uncertainty and compare the width of this resolution function with an experimental determination of this quantity.

1. INTRODUCTION AND DEFINITIONS

Flight path 1 at ORELA has been used for neutron transmission and scattering measurements on many materials and isotopes. It has a nominal length of 200 m, with intermediate flight-path stations at 80 and 18 m. With the recent development of an R-matrix code, SAMMY (LA80), based on Bayes' equations, more in-house data analyses are being performed. Such analyses demand precise information for the flight-path length, timing parameters, and the resolution function, and their uncertainties, in order to obtain reliable resonance parameters and realistic uncertainties. This report is a scoping study to develop this information using approximate analytic models for the neutron-producing target, the NE110 proton recoil detector, the neutron burst shape, and the timing channels. We evaluate energy dependent effects which contribute to the effective flight-path length and resolution function and give a first-order treatment of those which appear to be important. Finally, we note those areas which should be treated in more depth. This report is a companion to our report on the measurement of the total cross section of natural nickel (LA83), and the experimental parameters used in this report are consistent with the experimental configuration used for the nickel measurement.

In Sect. 2 we describe the general properties of flight path 1, including a description of a recent laser measurement of the drift tube length.

In Sect. 3 we develop distribution functions for components of the flight-path length from the target and the detector, and we use these to evaluate the first moment (mean value of the length) and second moment or width (variance) of the distributions. Uncertainties of the moments are also evaluated.

In Sect. 4 we concern ourselves with properties associated with the flight time. We develop distribution functions for the ORELA pulse width and the channel width of the data-acquisition program. Means and variances of these distributions are computed, along with their uncertainties.

In Sect. 5 the results from Sects. 3 and 4 are combined to develop a distribution function for the time-of-flight (t-o-f) energy. The first moment of this energy distribution function gives the mean t-o-f energy, and we identify the second moment with the width of the energy resolution function. Uncertainties of the mean energy and the resolution function parameters are also derived. Finally, in this section we compare parameters of our resolution function with those extracted from a recent analysis of experimental data.
Section 6 is a summary of what we have learned in this scoping study. Also noted are areas where more thorough treatments are called for.

1.1 METHOD AND DEFINITIONS

We approach this problem by developing analytic models for the distribution functions of the flight-path length and flight time. From these distribution functions we obtain the desired results for the energy scale and energy-resolution function and their associated uncertainties. Considering as an example the flight-path length, we identify the length with the mean value of the convolution of the various distribution functions associated with the effective flight-path length. The uncertainty in the length is identified as the uncertainty in the mean value. The contribution to the energy resolution function from the flight path is identified with the widths of the above distributions, and the uncertainties in the resolution functions result from the uncertainties associated with the distribution widths. Quantities associated with the flight time are similarly obtained from the distribution functions for the components of the flight time. Figures 1 and 2 are graphic representations of the various distributions, means and uncertainties.

In this work we will be dealing with distributions which describe physical properties of the target, detector, and beam from the accelerator. We now explicitly state our definitions and rules for calculating mean values and variances associated with these distributions and uncertainties on the means and variances. We feel that it is important to be very clear on this point. For example, looking ahead we will find that to first order, the expressions for the energy uncertainty $\Delta E$ and the resolution function width $\omega_E$ are identical in form; however, corresponding parameters in the two expressions have very different meanings. For this reason, we consistently apply the following definitions and methods of calculating the various means, variances, and uncertainties.

Each distribution, $p(y)$, where $y$ is an arbitrary parameter or set of parameters, will be expressed in an appropriate analytic form utilizing parameters which can be either calculated or estimated. We define the mean value of a quantity (assumed to be the experimentally observed value) as

\[
<y> = \int y p(y) dy ,
\tag{1.1.1}
\]

and the variance of $y$ (which is a measure of the width of the distribution) as

\[
\omega_y^2 = <(y - <y>)^2> ,
\tag{1.1.2}
\]

or equivalently

\[
\omega_y^2 = <y^2> - <y>^2 ,
\tag{1.1.3}
\]

where

\[
<y^2> = \int y^2 p(y) dy ,
\tag{1.1.4}
\]

and $<y>^2$ is obtained from Eq. (1.1.1).
Fig. 1. This figure schematically illustrates the various quantities and their uncertainties associated with the target and detector. $l_1$ is the mean distance from the effective point of neutron production to the front face of the Ta target. The distribution is a superposition of a Gaussian for the water and a parabola for the Ta for a neutron energy of 100 keV (see text). $l_2$ is the mean distance from the front face of the target to the front face of the detector, and $l_3$ is the mean distance from the front face of the detector to the first collision. Widths of the target and detector distributions are also shown. $\omega_t$ is the width of the neutron distribution associated with the target, and $\omega_d$ is the width of the distribution associated with the detector. Uncertainties (standard deviations) for the mean lengths $\Delta l_1$, $\Delta l_2$, and $\Delta l_3$, as well as for the widths $\Delta \omega_t$ and $\Delta \omega_d$, are also illustrated. The Ta contribution to the neutron production is offset from the center of the incident electron beam by $s$ (6 mm for 1-MeV neutrons), the correction for multiple scattering in the Ta (see text).
Fig. 2. This figure schematically serves to define the various times used in this report as well as the time distributions. The ORELA pulse is represented by a Gaussian of width $\omega_1 \pm \Delta \omega_1$ and mean value $t_1 \pm \Delta t_1$ ($t_1$ is taken as zero in the text). The flight time $t$ is the flight time of the neutron from the target to the detector. $t_2$ is the center of a time channel defined by the data-acquisition program, and $\omega_2$ is the width (standard deviation) of this channel. $t_n$ is the "observed flight time" of a neutron event in the data-acquisition system. Uncertainties in the various times and widths are also illustrated.
As noted earlier, in this work we will develop distribution functions for each component of the flight-path length and flight times. From the mean values of the flight-path length and flight-time distributions, we can calculate the observed mean energy $E$, and from the widths (and higher moments) of the distributions associated with the flight-path length and flight times we can calculate the width (square root of variance) $\omega_E$ of the energy resolution function. We can also calculate the uncertainties associated with the energy scale and resolution function, as well as the correlations among the parameters of the distributions, by calculating small increments of $E$ and $\omega_E$. In particular, for the energy uncertainty we will generate a small increment $\delta E$ via the chain rule

$$\delta E = \sum_i \frac{\partial E}{\partial p_i} \delta p_i,$$

where the $p_i$ are the values of the parameters in the expression for $E$, and $\delta p_i$ are small increments of those parameters. The variance $\Delta E^2$ (squared uncertainty) on $E$ is then given by forming the product

$$\langle \delta E \delta E \rangle = \Delta E^2.$$

The energy and its uncertainty is then given by $E \pm \Delta E$. Similarly, the uncertainty associated with the width of the resolution function is obtained from

$$\delta(\omega_E) = \sum_i \frac{\partial \omega_E}{\partial p_i} \delta p_i,$$

and the standard deviation $\Delta \omega_E$ is obtained from

$$\Delta \omega_E = \sqrt{\langle \omega_E \omega_E \rangle}.$$

Our evaluation of the quantities $\Delta E$, $\omega_E$, and $\Delta \omega_E$ (Sect. 5) involves second, third, and fourth moments of the length- and time-distribution functions as well as the uncertainties of those moments. The rest of this report is concerned with developing approximations for the various distribution functions $\rho(y)$ and evaluating the means, widths, uncertainties, and higher moments.

Before proceeding to the task of developing the distribution functions, we digress to point out that the term "distribution function" often has another meaning when used in an uncertainty analysis context. We are using the term to refer to a physical property of the experimental system. However, the parameters $p_i$ which occur in the analytic expressions for these distributions are also characterized by a mean value and an uncertainty. The parameters $p_i$ are distributed according to some probability density function (pdf). Nevertheless, when evaluating the parameters $p_i$ and their uncertainties $\Delta p_i$, we need only give the mean values and standard deviations and do not require knowledge of the explicit form of the pdf.
2. GENERAL PROPERTIES OF FLIGHT PATH 1

Flight path 1 is positioned at 90° with respect to the incident electron beam and is perpendicular to the face of the neutron-producing target (see Fig. 3). Three computer-controlled sample changers are available, located at 5 m, 9 m, and 10 m from the target. Two changers have five paddles and one has four positions available for filters or samples. Various collimators are inserted at ~8 m from the target to view neutrons from only the tantalum target, only the water moderator (above the tantalum), or a combination of the two sources. We treat the latter case in this report. The neutron spectrum from the tantalum has a higher average energy than that from the water moderator. A rectangular (5.4 by 4.8 cm) shadow bar which consists of 2.9 cm uranium, 2.5 cm thorium, and 2.5 cm tantalum is normally located at ~4 m from the source. The thickness and composition of the shadow bar have been chosen to reduce the gamma flash from the tantalum to an acceptable intensity, while still allowing neutrons from the tantalum to reach the detector.

The neutron time-of-flight spectrum from the bare tantalum target has been compared to that from the surrounding water moderator (HA75). The spectral comparisons are shown in Fig. 4. After renormalizing those results to collimator sizes, filters, shadow bars, and neutron production as a function of distance from the target center for the nickel measurement (LA83), we find that the neutron intensities from the water moderator and from the tantalum metal are equal at about 300 keV, with the tantalum target producing more neutrons at 1 MeV by a factor of about 10. The water-moderated neutrons have significantly greater intensity below ~100 keV, since the production cross section for neutrons in the tantalum is decreasing with decreasing energy, and multiple scattering is increasing since the tantalum target is >1 mean-free-path thick for neutrons less than ~100 keV.

For this report we need an accurate measurement of the length of the 200-m flight path. In March 1983 a Hewlett-Packard electronic distance meter (laser) was used to measure distances along the 200-m flight path. This measurement and analysis is detailed in (LA84). At that time we were not able to measure directly to the target center and had to satisfy ourselves with measurements to various benchmarks along the 200-m flight path and to the 20-m station, which is on line with, but on the other side of, the target from the 200-m flight path. Sufficient measurements were taken to determine the offset of the measuring device, and the statistical uncertainty associated with the measurements. Combining the laser results with results from engineering drawings (HA 69) which give distances from the center of the target room to existing benchmarks allowed us to determine a mean flight-path length from the center of the target room to the east benchmark at the 200-m station. In this analysis, provision was made for the possibility that the target is not located perfectly at the center of the target room. The mean value of this displacement parameter was taken as zero, with an uncertainty of ±2 mm. An uncertainty analysis using Bayes' equations and including correlations introduced by the measurement process provided the uncertainty. The preliminary result is 201858.6 ± 4.0 mm.

3. PROPERTIES ASSOCIATED WITH THE FLIGHT-PATH LENGTH

The total flight-path length \( l \) can be separated into three components (see Fig. 1): (1) \( l_1 \), the mean distance from the point where the neutron is "produced" to the front face of the neutron-producing target; (2) \( l_2 \), the distance from the face of the target to the face of the detector; and (3) \( l_3 \), the distance from the face of the detector to the point of the first collision. Each component will be discussed separately in Sects. 3.1, 3.2, and 3.3, and the combined flight-path length is discussed in Sect. 3.4.
Fig. 3. This figure is an illustration of the ORELA Ta target. The 200-m flight path is in the direction of the neutron arrows, perpendicular to the face of the target.
Fig. 4. This figure is an illustration of the measured flux as a function of neutron energy at 200 m from two different areas of the target (Fig. 3) and from a separate Be block target which is not treated in this report. The "Ta target" spectrum results from using a collimator which views only the Ta plates of the target, while the "H₂O moderator" spectrum results from using a collimator which views only neutrons emanating from the water moderator above the Ta plates. These spectral measurements were done with a different experimental configuration than that treated in the present work; however, the present analysis has been normalized for the different filters, collimator sizes, etc., used for the present work (see text). In particular, for the present analysis we have a larger collimator which views both the Ta and the water moderator, and we used a "thin" shadow bar to reduce the intensity of the gamma flash and neutrons emanating from electrons striking the Ta plates. This figure and analysis served to provide the "crossover energy" $E_m$, where the spectra from the Ta and H₂O have approximately equal intensity. For the present experimental configuration, this was found to be $\sim 300 \pm 90$ keV.
3.1 DISTRIBUTION FUNCTION FOR $x_1$: THE CONTRIBUTION TO THE FLIGHT-PATH LENGTH FROM THE TARGET

The first component $l_1$ of the total flight-path length $I$ is the most difficult to assess since in the experiments of immediate concern we view neutrons both from the tantalum metal target and from the cooling-water moderator surrounding the target. For the nickel measurement, the 7.6-cm-diam collimator at 8 m views 45.4 cm$^2$ of the target, of which 20.8 cm$^2$ is the tantalum and the remaining 24.6 cm$^2$ is water moderator. The cross-sectional area of the shadow bar is 25.9 cm$^2$ and thus shadows the tantalum target plus some of the water moderator. Neutrons which emanate from the tantalum target are assumed to be born near the centerline of the target (along the direction of the incident electron beam, see Figs. 1 and 3). For these neutrons, the length $W/2$ from the center of the tantalum target to the target face (from where $l_1$ is measured) is 18.3 mm and is assumed to be energy independent. However, scattering of the primary neutrons in the Ta target changes the distribution of neutrons leaving the exit surface of the Ta target. To estimate these effects, we consider what happens to 1-MeV neutrons produced in the Ta by the electron beam. We assume a value of $\sigma_{Ta} = 7.56$ b for the following approximate calculation. The unscattered neutrons produced at the center of the Ta target moving in the direction of the detector at 200 m escape with a 47% probability. These primary neutrons have a mean value length of $W/2 = 18.3$ mm. Numerical analytic calculations have been made to estimate the contributions to the effective flight-path length $q = W/2 + s$ of the scattered neutrons in the Ta target. The intensity of singly scattered neutrons in the direction of the detector is $\sim 60\%$ of the unscattered neutron intensity. Combining the unscattered and first scattered neutrons gives a value of $s = 3.3$ mm. Estimating the contribution of higher order scattering (to eighth order) approximately doubles this correction to 6 mm. In this work we use a value of $s = 6$ mm with $\Delta s = 30\%$, giving $q = W/2 + s = 24.3$ mm. We also note that the correction factor $s$ is expected to be energy dependent; however, that complication is not treated in this work. In addition, the possibility that the electron beam is not centered about $W/2$ must be considered. For instance, if the electron beam strikes the target between $W/2$ and the front face, the correction $s$ due to multiple scattering in the Ta may be reduced. To account for this possibility, we add another parameter $Z$, which we assume has a mean value $Z = 0$, but with the uncertainty $\Delta Z = \pm 2$ mm. Thus, finally $q = W/2 + s + Z$.

The neutrons which come from the water moderator are also difficult to characterize since the moderation process produces a distribution of neutron delay times. These delay time distributions are conventionally converted to distributions of equivalent moderation distances $d$, which are the product of delay times and escape velocities. The mean equivalent moderation distance $d$ for the ORELA target is described in (CO83). The quantity $d$ is energy-dependent and increases with energy according to

$$d(E) = 22.8 - 1.60 \times \ln E + 0.283 \times (\ln E)^2,$$  \hspace{1cm} (3.1.1)

where $d$ is in mm and $E$ in eV. This equation is quoted to be valid for 10-ev to 1-MeV neutrons which come from the water moderator (CO83). A calculation similar to this should be done for the Ta.

We now see that for the energy region from $\sim 1$ keV to 1 MeV (a region of interest for resonance parameter analysis) we have an energy-dependent flight-path length, since useful neutrons come from both the tantalum and the water sources. For the case of 300-keV neutrons emerging from the face of the target (recall that both sources contribute about equally at 300 keV), those neutrons coming from the tantalum target have a mean effective flight-path length of 24.3 mm, while those coming from the water moderator have a mean effective flight-path length of 47.6 mm. Thus, for the water-moderated neutrons, the effective flight path is $\sim 23$ mm longer than for the neutrons which come directly from the tantalum.
We now derive a distribution function for the production of neutrons from the target. If we let $\rho_i(x_i)$ describe the distribution of neutrons as a function of $x_i$, the distance from the front face of the target measured in the direction away from the detector, we can write

$$\rho_i(E,x_i) = f_w(E)\rho_w(E,x_i) + f_T(E)\rho_T(x_i) ,$$

where $f_w(E)$ and $f_T(E)$ represent the fraction of neutrons originating in the water moderator and in the tantalum target respectively for energy $E$. $\rho_w(E,x_i)$ and $\rho_T(x_i)$ represent the effective spatial distributions of neutrons originating in the water and tantalum, respectively. Note that for water the distribution is a function of energy and position, while the distribution from the tantalum is assumed to be independent of energy.

We somewhat arbitrarily specify the energy dependence of the fractions $f_w(E)$ and $f_T(E)$ as

$$f_w(E) = \frac{2}{E/E_m} ,$$

with $E_m = 300$ keV and

$$f_T(E) = 1 - f_w(E) .$$

Note that this choice gives

$$f_w(E=0) = 1 , \quad f_T(E=0) = 0 ,$$

$$f_w(E=E_m) = 0.5 , \quad f_T(E=E_m) = 0.5 ,$$

$$f_w(E=1\text{ MeV}) \approx 0.05 , \quad f_T(E=1\text{ MeV}) \approx 0.95;$$

i.e., this choice for the two fractions is consistent with the energy dependence of the observed spectra from the tantalum and water (Fig. 4).

For $\rho_w(E,x_i)$, we choose a Gaussian distribution centered at $d_i(E)$, with a standard deviation width $\omega_w(E)$. The distributions actually given in (CO83) are slightly asymmetric, and are described by the functional form $x^2e^{-x}$, but we ignore that complication in this work. Thus
11

\[ p_w(E,x_1) = \frac{1}{\omega_w \sqrt{2\pi}} e^{-\frac{(x_1-d)^2}{2\omega_w^2}}, \quad (3.1.5) \]

where the mean value is calculated from Eq. (1.1.1).

\[ \langle x_1 \rangle_w = d(E), \quad (3.1.6) \]

with \( d(E) \) given in Eq. (3.1.1). The variance of the distribution is calculated from Eq. (1.1.2):

\[ \langle (x_1 - \langle x_1 \rangle_w)^2 \rangle_w = \omega_w^2(E). \quad (3.1.7) \]

An expression for \( \omega_w(E) \) was obtained by fitting a curve to the square root of values given in the "Var d" column of Table 1 of (CO83):

\[ \omega_w(E) = 10.0 - 0.63 \times \ln E + 0.112 \times (\ln E)^2, \quad E \text{ in eV}. \quad (3.1.8) \]

We now develop an expression for \( \rho_T(x_1) \), the distribution function for neutrons from the tantalum. The electron beam strikes the tantalum target directly. Thus for \( \rho_T(x_1) \) we assume that a uniform electron beam of radius \( r = 9.2 \text{ mm} \) (LE70) is incident on the tantalum plates of the target, and we find

\[ \rho_T(x_1) = \begin{cases} \frac{2}{\pi r^2} \left[ r^2 - (q - x_1)^2 \right]^{1/2} & \text{for } |x_1 - q| \leq r \\ 0 & \text{otherwise} \end{cases} \quad (3.1.9) \]

where \( q = W/2 + s + Z \), \( W \) is the width of the tantalum target (= 36.6 mm), \( s \) (6 mm) is the correction for multiple scattering and attenuation in the tantalum, and \( Z \) is the possible offset of the electron beam from the center of the target (\( Z = 0 \)). From this distribution and Eq. (1.1.1) we find the mean value

\[ \langle x_1 \rangle_T = q, \quad (3.1.10) \]

and from Eq. (1.1.2) the variance
The distribution function $p_i(E,x_1)$ is then given by the weighted sum of the water plus tantalum distributions, as stated in Eq. (3.1.2), with Eqs. (3.1.5) and (3.1.9) for the individual distributions.

### 3.1.1 Mean Value $l_1$ for Distribution of $x_1$

From Eq. (1.1.1) we can write the expression for the mean value of the contribution to the flight path length from the neutron target:

$$l_1 = <x_1>_l = \int x_1 p_1(x_1) dx_1 = f_w <x_1>_w + f_T <x_1>_T = f_w d + f_T q .$$

Replacing $q$ by $W/2 + s + Z$ and $f_T$ by $1 - f_w$, this equation becomes

$$l_1 = (d - W/2 - s - Z)f_w + W/2 + s + Z .$$

This contribution to the flight-path length is given as a function of energy in Table 1.

### 3.1.2 Uncertainty in $l_1$

We now evaluate the uncertainty associated with $l_1$. Taking small increments and using the chain rule for partial derivatives give

$$\delta l_1 = \frac{\partial l_1}{\partial d} \delta d + \frac{\partial l_1}{\partial W} \delta W + \frac{\partial l_1}{\partial s} \delta s + \frac{\partial l_1}{\partial Z} \delta Z + \frac{\partial l_1}{\partial f_w} \delta f_w .$$

Using Eq. (3.1.3) for $f_w$ gives $\delta f_w$ in terms of $\delta E_m$, which is a parameter of interest,

$$\delta f_w = \frac{\partial f_w}{\partial E_m} \delta E_m = f_w^2 \frac{E \ln^3}{2E_m^2} e^{E/m} \delta E_m .$$
Table 1. The energy uncertainty $\Delta E$ and the total flight-path length $l$ and its uncertainty $\Delta l$ are given as a function of energy. Also given are the contributions from the target ($l_1$) and the detector ($l_3$) to $l$ and their uncertainties. All energies are in eV, lengths are in mm, and uncertainties are one standard deviation.

| $E$     | $\Delta E$ | $\Delta E/E$ | $l$ | $\Delta l$ | $l_1$ | $\Delta l_1$ | $l_3$ | $\Delta l_3$ | $l_1 + l_3$ | $\Delta (l_1 + l_3)$ |
|---------|------------|---------------|-----|------------|------|--------------|------|--------------|-------------|----------------|----------------|
| 10.000  | 0.001      | 6.699E-05     | 201466.604 | 5.413 | 20.616 | 2.062 | 5.986 | 0.220 | 26.602 | 2.073 |
| 1000.000| 0.069      | 6.853E-05     | 201471.238 | 5.604 | 25.250 | 2.521 | 5.989 | 0.220 | 31.239 | 2.530 |
| 2000.000| 0.138      | 6.917E-05     | 201472.984 | 5.681 | 26.979 | 2.689 | 6.005 | 0.220 | 32.984 | 2.698 |
| 5000.000| 0.351      | 7.021E-05     | 201475.703 | 5.806 | 29.653 | 2.943 | 6.051 | 0.222 | 35.704 | 2.952 |
| 10000.000| 0.711     | 7.114E-05     | 201478.055 | 5.913 | 31.928 | 3.149 | 6.127 | 0.224 | 38.055 | 3.157 |
| 20000.000| 1.443     | 7.215E-05     | 201480.592 | 6.022 | 34.330 | 3.349 | 6.263 | 0.230 | 40.592 | 3.357 |
| 50000.000| 3.670     | 7.341E-05     | 201483.910 | 6.136 | 37.311 | 3.548 | 6.600 | 0.244 | 43.911 | 3.557 |
| 100000.000| 7.430    | 7.430E-05     | 201485.699 | 6.180 | 38.705 | 3.623 | 6.995 | 0.266 | 45.700 | 3.633 |
| 200000.000| 15.273   | 7.636E-05     | 201485.490 | 6.305 | 38.023 | 3.830 | 7.466 | 0.296 | 45.489 | 3.841 |
| 300000.000| 23.788   | 7.929E-05     | 201483.803 | 6.541 | 35.966 | 4.205 | 7.835 | 0.323 | 43.802 | 4.218 |
| 400000.000| 32.764   | 8.191E-05     | 201481.605 | 6.744 | 33.668 | 4.513 | 7.937 | 0.331 | 41.604 | 4.525 |
| 700000.000| 59.068   | 8.438E-05     | 201476.617 | 6.698 | 28.338 | 4.442 | 8.279 | 0.360 | 36.618 | 4.457 |
| 1000000.000| 85.044  | 8.504E-05     | 201474.304 | 6.420 | 25.823 | 4.009 | 8.478 | 0.377 | 34.301 | 4.027 |
| 2000000.000| 186.289 | 9.314E-05     | 201473.156 | 6.318 | 24.346 | 3.841 | 8.811 | 0.408 | 33.157 | 3.862 |
| 5000000.000| 578.663 | 1.157E-04     | 201473.375 | 6.322 | 24.300 | 3.845 | 9.074 | 0.433 | 33.374 | 3.869 |
| 10000000.000| 1457.753| 1.458E-04     | 201473.486 | 6.323 | 24.300 | 3.845 | 9.186 | 0.444 | 33.486 | 3.870 |
| 20000000.000| 3845.079| 1.923E-04     | 201475.514 | 6.323 | 24.300 | 3.845 | 9.213 | 0.447 | 33.513 | 3.870 |

Note: Insignificant digits have been retained in all tables to facilitate comparisons with future calculations.

Evaluating the partial derivations in Eq. (3.1.14) explicitly from Eq. (3.1.13) gives

$$
\delta l_1 = f_W \delta d + \left( \frac{1 - f_W}{2} \right) \delta W + (1 - f_W)(\delta s + \delta Z)
$$

$$
+ \left( d - \frac{W}{2} - s - Z \right) \left[ f_W^2 \frac{E \ln 3}{2E_m^2} e^{-\frac{E}{E_m}} \right] \delta E_m .
$$

Forming the product $<\delta l_1 \delta l_1^* > = \Delta l_1^2$, and assuming all variables are uncorrelated, we find

$$
\Delta l_1 = \left\{ f_W \Delta d^2 + \left( \frac{1 - f_W}{2} \right)^2 \Delta W^2 + (1 - f_W)^2(\Delta s^2 + \Delta Z^2)
$$

$$
+ \left( d - \frac{W}{2} - s - Z \right)^2 \left[ f_W^2 \frac{E \ln 3}{2E_m^2} e^{-\frac{E}{E_m}} \right]^2 \Delta E_m^2 \right\}^{1/2}
$$
for the uncertainty on \( l_1 \).

We assume values of \( \Delta d/d = 10\% \), \( \Delta W/W = 2\% \), \( \Delta s/s = 30\% \), \( \Delta Z = 2 \text{ mm} \), \( \Delta r/r = 20\% \), and \( \Delta E_m/E_m = 30\% \). Using these values, we obtain results for \( \Delta l_1 \) as in Table 1.

This completes our evaluation of the contribution to the flight-path length from the target end and its uncertainty \((l_1 \pm \Delta l_1)\) as a function of neutron energy.

3.1.3 Higher Moments of the Target Distribution Function

Later on this report when we derive expressions for the mean energy and the energy resolution function, we will need higher moments of \( x_1 \); in particular, the second, third, and fourth moments. We shall evaluate these moments "about the mean" and define \( \omega^2_1 \) as the second moment about the mean (i.e., the variance),

\[
\omega^2_1 = <(x_1 - <x_1>_l)^2>_l , \tag{3.1.18}
\]

\( \nu^3_1 \) as the third moment,

\[
\nu^3_1 = <(x_1 - <x_1>_l)^3>_l , \tag{3.1.19}
\]

and \( \mu^4_1 \) as the fourth moment,

\[
\mu^4_1 = <(x_1 - <x_1>_l)^4>_l . \tag{3.1.20}
\]

We now evaluate these moments for future reference. The \( n^{th} \) moment about the mean is given by

\[
<x^*_n>_l = \int (x_1 - <x_1>_l)^n \rho_i(x_1) dx_1 , \tag{3.1.21}
\]

where \( \rho_i(x_1) \) is given by Eq. (3.1.2) in terms of the water and tantalum distributions. For the water contribution, \( \rho_w(x_1) \) is a Gaussian with moments given by

\[
<x_1>_w = d , \tag{3.1.22}
\]
\begin{align*}
\langle (x_1 - d)^2 \rangle_W &= \omega_W^2, \quad (3.1.23) \\
\nu_W^3 &= \langle (x_1 - d)^3 \rangle_W = 0, \quad (3.1.24) \\
\mu_W^4 &= \langle (x_1 - d)^4 \rangle_W = 3\omega_W^4. \quad (3.1.25)
\end{align*}

For the tantalum, \( \rho_T(x_i) \) is given by Eq. (3.1.9). The moments of this distribution are

\begin{align*}
\langle x_1 \rangle_T &= q, \quad (3.1.26) \\
\omega_T^2 &= \langle (x_1 - q)^2 \rangle_T = \frac{r^2}{4}, \quad (3.1.27) \\
\nu_T^4 &= \langle (x_1 - q)^4 \rangle_T = 0, \quad (3.1.28) \\
\mu_T^4 &= \langle (x_1 - q)^4 \rangle_T = \frac{r^4}{8}. \quad (3.1.29)
\end{align*}

Combining these results, we obtain

\begin{align*}
l_1 &= \langle x_1 \rangle_{l_1} = f_w d + (1 - f_w)q, \quad (3.1.30) \\
\omega_{l_1}^2 &= \langle (x_1 - l_1)^2 \rangle_{l_1} = f_w \omega_W^2 + (1 - f_w)\frac{r^2}{4} + f_w(1 - f_w)(d - q)^2, \quad (3.1.31) \\
\nu_{l_1}^3 &= \langle (x_1 - l_1)^3 \rangle_{l_1} \\
&= f_w(1 - f_w)(d - q)[3(\omega_W^2 - \frac{r^2}{4}) + (1 - 2f_w)(d - q)^2]. \quad (3.1.32)
\end{align*}

(Note that even though the third-order moments about their means are zero, the combined distribution is asymmetric about its mean, and hence the third-order moment is non-zero.)
\[ \mu_i^4 = \langle x_i - \mu_i \rangle^4 \]

\[ = f_w \mu_i^4 + (1 - f_w) \mu_i^4 + 6f_w(1 - f_w)(d - q)^2 [(1 - f_w)^2 \omega_i^2 + f_w^2 r^2/4] \]

\[ + f_w(1 - f_w)(1 - f_w)^3 + f_w^3](d - q)^2 \]  

(3.1.33)

3.1.4 Width \( \omega_i \) of Target Distribution Function

The distribution function \( \rho_i(x_i) \) is the contribution of the target end of the flight path to the energy resolution function. The width of this distribution function is given by the square root of Eq. (3.1.31). Tabulated values of \( \omega_i = \sqrt{\omega_i^2} \) are given as a function of energy in Table 2.

Table 2. The width of the energy resolution function \( \omega_E \) and its uncertainty \( \Delta \omega_E \) are given as a function of energy. In addition, the contribution of \( \omega_i \) to \( \omega_E \) is given, as well as the components \( \omega_{l_E} \) and \( \omega_{l_i} \) of \( \omega_i \).

All energy widths and uncertainties are in eV, and length widths and uncertainties are in mm. All uncertainties are one standard deviation. The contribution of \( \omega_i \) to \( \omega_E \) is energy independent and not given in the table. \( (\omega_i \pm \Delta \omega_i = 3.21 \pm 0.21 \text{ ns}) \) See Sect. 3.4.5 for a discussion of why \( \Delta \omega_i < \Delta \omega_i \).

<table>
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<th>( \omega_{l_E} )</th>
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3.1.5 Uncertainty on $w_i$

Eventually we wish to determine the uncertainty associated with the energy resolution function. For this it is necessary to determine the uncertainty on $w_i$, which is found by first evaluating the square of the uncertainty on $w_i^2$ or $(\Delta w_i^2)^2$. The uncertainty $\Delta w_i$ will then be expressed in terms of $(\Delta w_i^2)^2$.

Our procedure is the same as that used to evaluate $(\Delta l)^2$ in Sect. 3.1.2. A small increment on $w_i$ can be written as

$$
\delta w_i^2 = \frac{\partial w_i}{\partial f_w} \delta f_w + \frac{\partial w_i^2}{\partial w_w} \delta w_w + \frac{\partial w_i^2}{\partial d} \delta d
$$

$$+ \frac{\partial w_i^2}{\partial r} \delta r + \frac{\partial w_i^2}{\partial W} \delta W + \frac{\partial w_i^2}{\partial s} \delta s + \frac{\partial w_i^2}{\partial Z} \delta Z .
$$

(3.1.34)

If the partial derivatives in Eq. (3.1.34) are evaluated directly from Eq. (3.1.31) (remembering that $q = W/2 + s + Z$) and Eq. (3.1.15) is used for $\delta f_w$, then $\delta w_i^2$ becomes

$$
\delta w_i^2 = [w_i^2 + (1 - 2f_w)(d - q)^2 - r^2/4] \left[ f_w^2 \frac{E \ln 3}{2E_m} - \frac{E \ln 3}{e^2} \right] \delta E_m
$$

$$+ 2f_w w \delta w_w + 2f_w(1 - f_w)(d - q)(\delta d - \delta s - \delta Z - \delta W/2)
$$

$$+ (1 - f_w)r \delta r/2 .
$$

(3.1.35)

Forming the product $<\delta w_i^2 \delta w_i^2> = (\Delta w_i^2)^2$ and assuming all parameters are uncorrelated give

$$
(\Delta w_i^2)^2 = [w_i^2 + (1 - 2f_w)(d - q)^2 - r^2/4]^2 \left[ f_w^2 \frac{E \ln 3}{2E_m} - \frac{E \ln 3}{e^2} \right] \Delta E_m^2
$$

$$+ (2f_w w \Delta w_w)^2 + ((1 - f_w)r \Delta r/2)^2
$$

$$+ [2f_w(1 - f_w)(d - q)]^2 \left[ \Delta d^2 + \Delta s^2 + \Delta Z^2 + \Delta W^2/4 \right] .
$$

(3.1.36)
To convert from this expression to an uncertainty on \( \omega_i \), we note that

\[
\delta \omega_i^2 = 2 \omega_i \delta \omega_i
\]  

(3.1.37)

which gives \((\Delta \omega_i)^2\) as

\[
(\Delta \omega_i)^2 = \langle \delta \omega_i \delta \omega_i \rangle = \frac{1}{4 \omega_i^2} \langle \delta \omega_i^2 \delta \omega_i^2 \rangle
\]  

(3.1.38)

or

\[
\Delta \omega_i = \frac{1}{2 \omega_i} \Delta \omega_i^2
\]  

(3.1.39)

Thus, the uncertainty on \( \omega_i \) is evaluated using Eq. (3.1.39) with the square root of Eq. (3.1.36) for \( \Delta \omega_i^2 \) and the square root of Eq. (3.1.31) for \( \omega_i \). We assume \((\Delta \omega_w)/(\omega_w) = 10\%\). Values of \( \omega_i \) and \( \Delta \omega_i \) are tabulated in Table 2.

This completes derivation of the mean flight-path length from the target end [Eq. (3.1.13)], its uncertainty [Eq. (3.1.17)], the width of the distribution [Eq. (3.1.31)], and the uncertainty on the width [Eq. (3.1.39)]. We note that these quantities are all functions of neutron energy.

### 3.2 DISTRIBUTION FUNCTION FOR \( x_2 \): THE CONTRIBUTION TO THE FLIGHT-PATH LENGTH FROM THE FLIGHT TUBE

We now evaluate the second component of the flight-path length, \( l_2 \), measured from the face of the target to the face of the detector. The distance from the center of the neutron-producing target to the east benchmark in the 200-m station was measured to be (preliminary value) 201858.6 ± 4.0 mm (LA84). However, since the target flight-path length \( l_1 \) is measured from the front face of the target, we subtract one-half of the tantalum target thickness \( W_t \), 18.3 mm, which gives 201840.3 ± 4.0 mm. The distance from the benchmark to the front face of the NE110 detector was measured to be 400 ± 3 mm, giving a mean value of the flight-path length from the front face of the target to the face of the detector \( l_2 + \Delta l_2 = 201440 ± 5 \) mm.

No distribution is associated with this component of the flight-path length; i.e., \( \omega_i^2 = 0 \). Equivalently, we may say that the distribution function is a \( \delta \)-function:

\[
\rho_i(x_2) = \delta(x_2 - l_2)
\]  

(3.2.1)
(We note, however, that if the target vibrates back and forth, and we developed a physical model for this phenomenon described by a distribution function $p_1(x_2)$, $\omega l_i^2$ would not be zero. However, we neglect such phenomena in this study.) The mean of this distribution is

\[ <x_2>_{l_i} = l_2 \]

which has uncertainty $\Delta l_2$. Higher moments about the mean vanish, since

\[ <(x_2 - l_2)^m>_{l_i} = \int \delta(x_2 - l_2)(x_2 - l_2)^m \, dx_2 = 0 \]

Explicitly, the second, third, and fourth moments are

\[ \omega l_i^2 = 0 \]

\[ \nu l_i^3 = 0 \]

\[ \nu l_i^4 = 0 \]

3.3 DISTRIBUTION FUNCTION FOR $x_3$: THE CONTRIBUTION TO THE FLIGHT-PATH LENGTH FROM THE DETECTOR

The final contribution to the flight-path length is from the 19-mm-thick NE110 detector. Since the transmission of neutrons through the detector ranges from 9% at 10 keV (2.4 mfp thick) to 82% at 10 MeV (0.2 mfp thick), the effective flight-path length of the neutrons in the detector is a function of energy. The low-energy neutrons will interact in the first few millimeters of the detector, while the high-energy neutrons will interact, on the average, near the center. To estimate the flight-path length in the detector as a function of neutron energy, we calculate the mean-path length $l_3$ before the first scattering. This assumes that we detect light from both hydrogen and carbon first-collision recoils in the NE110. The neutron distribution in the detector should be calculated using Monte Carlo techniques to account for multiple scattering effects, but for this work we choose the simpler "distance to first collision" approximation.
We represent the neutron distribution in the detector by

\[ \rho_f(x_3) = \begin{cases} \Lambda e^{-\lambda x_3}, & \text{for } 0 \leq x_3 \leq L \\ 0 & \text{otherwise} \end{cases} \]  

(3.3.1)

where \( \lambda = 0.0047 \), the number of molecules per mm·b of NE110 (CH\(_{1.104}\)), \( \sigma(E) \) is the total cross section of CH\(_{1.104}\), with numerical values calculated from ENDF/B-V (EN79), and \( L \) is the average thickness of the NE110 scintillator, taken as 19 mm. The normalization \( A \) is found by setting

\[ \int_0^L \rho_f(x_3) dx_3 = 1 \]  

(3.3.2)

giving

\[ A = \frac{\lambda \sigma}{1 - e^{-\lambda L}} \]  

(3.3.3)

3.3.1 Mean Value \( \langle x_3 \rangle \) for Distribution of \( x_3 \)

We can write the mean distance to the first collision as

\[ \langle x_3 \rangle_l = A \int_0^L x_3 e^{-\lambda x_3} dx_3 \]  

(3.3.4)

which gives

\[ \langle x_3 \rangle_l = \frac{1}{\lambda \sigma} + \frac{L}{1 - e^{\lambda L}} \]  

(3.3.5)

This contribution to the flight-path length is given in Table 1.

3.3.2 Uncertainty in \( l_3 \)

We now calculate the uncertainty on \( l_3 \). Taking small increments gives

\[ \delta l_3 = \frac{\partial l_3}{\partial \lambda} \delta \lambda + \frac{\partial l_3}{\partial \sigma} \delta \sigma + \frac{\partial l_3}{\partial L} \delta L \]  

(3.3.6)
which, after evaluating derivatives, becomes

$$
\delta l_j = \left[ -\frac{1}{\lambda^2 \sigma^2} - \frac{e^{\lambda \sigma L}}{1 - e^{\lambda \sigma L}} \right] \delta \lambda + \left[ -\lambda \sigma^2 + \frac{\lambda L^2 e^{\lambda \sigma L}}{(1 - e^{\lambda \sigma L})^2} \right] \delta \sigma \\
+ \left[ \frac{1}{1 - e^{\lambda \sigma L}} + \frac{\lambda \sigma L e^{\lambda \sigma L}}{(1 - e^{\lambda \sigma L})^2} \right] \delta L .
$$

(3.3.7)

Forming the product $\langle \delta l_3 \delta l_3 \rangle = \Delta l_3^2$ and assuming $\langle \delta \lambda \delta \sigma \rangle = \langle \delta \sigma \delta L \rangle = \langle \delta \lambda \delta L \rangle = 0$, we obtain

$$
\Delta l_3 = \left\{ \left[ -\frac{1}{\lambda^2 \sigma^2 L^2} - \frac{e^{\lambda \sigma L}}{1 - e^{\lambda \sigma L}} \right] \left[ \left( \frac{\Delta \lambda}{\lambda} \right)^2 + \left( \frac{\Delta \sigma}{\sigma} \right)^2 \right] \right\} \\
+ \left[ \frac{1}{\lambda \sigma L (1 - e^{\lambda \sigma L})} + \frac{e^{\lambda \sigma L}}{(1 - e^{\lambda \sigma L})^2} \right] \left[ \frac{\Delta L}{L} \right]^2 \lambda \sigma L^2
$$

(3.3.8)

for the uncertainty on $l_3$.

We assume uncertainties of $\Delta \lambda / \lambda = 2\%$, $\Delta \sigma / \sigma = 5\%$, and $\Delta L / L = 5\%$. Results for $\Delta l_3$ are given in Table 1.

### 3.3.3 Higher Moments of the Detector Distribution Function

We will also need the higher moments of $x_3$, and we present those results here for future convenience. In general,

$$
\langle (x_3 - l_3)^m \rangle_{l_3} = \frac{\int_0^L (x_3 - l_3)^m e^{-\lambda x_3} dx_3}{\int_0^L e^{-\lambda x_3} dx_3} ,
$$

(3.3.9)

so that after considerable algebra we find

$$
\omega_3^1 = \langle (x_3 - l_3)^2 \rangle_{l_3} = \frac{1}{(\lambda \sigma)^2} - \frac{L^2 e^{\lambda \sigma L}}{(1 - e^{\lambda \sigma L})^2} ,
$$

(3.3.10)

$$
\nu_3^1 = \langle (x_3 - l_3)^3 \rangle_{l_3} = \frac{2}{(\lambda \sigma)^3} + \frac{L^3 e^{\lambda \sigma L} (1 - e^{\lambda \sigma L})}{(1 - e^{\lambda \sigma L})^3} ,
$$

(3.3.11)
and

$$\mu_4^i = \frac{9}{(\lambda \sigma)^2} - \frac{6L^2e^{\lambda L}}{(\lambda \sigma)^2(1 - e^{\lambda L})^2} - \frac{L^4e^{\lambda L}(1 + e^{\lambda L} + e^{2\lambda L})}{(1 - e^{\lambda L})^4}. \quad (3.3.12)$$

### 3.3.4 Width $\omega_i$ of Detector Distribution Function

The variance of the distribution function for $x_3$ is given by Eq. (3.3.10) and is identified with the contribution to the energy resolution function from the detector. Values of the width of the distribution $\omega_i = \sqrt{\omega_i^2}$ are given in Table 2 as a function of neutron energy.

### 3.3.5 Uncertainty on $\omega_i$

The uncertainty on $\omega_i^2$ is found from

$$\delta \omega_i^2 = \frac{\partial \omega_i^2}{\partial \lambda} \delta \lambda + \frac{\partial \omega_i^2}{\partial \sigma} \delta \sigma + \frac{\partial \omega_i^2}{\partial L} \delta L, \quad (3.3.13)$$

where the partial derivatives may be evaluated from Eq. (3.3.10),

$$\delta \omega_i^2 = L \left\{ - \frac{2}{\lambda \sigma L} + \frac{1 + e^{\lambda L}}{1 - e^{\lambda L}} \right\} \omega_i^2 \left[ \sigma \delta \lambda + \lambda \delta \sigma \right] + \lambda \sigma \left[ \frac{2}{\lambda \sigma L} + \frac{1 + e^{\lambda L}}{1 - e^{\lambda L}} \right] \omega_i^2 \delta L. \quad (3.3.14)$$

Thus the squared uncertainty on $\omega_i^2$ is

$$\langle \Delta \omega_i^2 \rangle^2 = \langle \delta \omega_i^2 \delta \omega_i^2 \rangle$$

$$= \left( \omega_i^2 \right)^2 \left\{ - \frac{2}{\lambda \sigma L} + \frac{1 + e^{\lambda L}}{1 - e^{\lambda L}} \right\}^2 \left[ L^2 \sigma^2(\Delta \lambda)^2 + L^2 \lambda^2(\Delta \sigma)^2 \right]$$

$$+ \left\{ \frac{2}{\lambda \sigma L} + \frac{1 + e^{\lambda L}}{1 - e^{\lambda L}} \right\}^2 \lambda^2 \sigma^2(\Delta L)^2. \quad (3.3.15)$$
To convert to the uncertainty on $\omega_{l_i}$, recall that

$$
\Delta \omega_{l_i} = \frac{1}{2 \omega_{l_i}} \Delta \omega_{l_i}^2 , \tag{3.3.16}
$$

in analogy with Eq. (3.1.39). Thus, the uncertainty on the width $\omega_{l_i}$ is given by

$$
\Delta \omega_{l_i} = \frac{\lambda \sigma L}{2} \left( \left\{ \frac{2}{\lambda \sigma L} + \frac{1 + e^{\lambda L}}{1 - e^{\lambda L}} \right\} \left\{ \left( \frac{\Delta \lambda}{\lambda} \right)^2 + \left( \frac{\Delta \sigma}{\sigma} \right)^2 \right\} + \left\{ \frac{2}{\lambda \sigma L} + \frac{1 + e^{\lambda L}}{1 - e^{\lambda L}} \right\} \left( \frac{\Delta L}{L} \right)^2 \right)^{1/2} . \tag{3.3.17}
$$

Values for $\Delta \omega_{l_i}$ are given in Table 2 as a function of neutron energy. This completes the derivation of the mean flight path length in the detector [Eq. (3.3.5)] and its uncertainty [Eq. (3.3.8)], and the width of the distribution [Eq. (3.3.10)] and its uncertainty [Eq. (3.3.17)]. We note that these quantities are all energy dependent.

### 3.4 DISTRIBUTION FUNCTION FOR $x$: THE TOTAL FLIGHT-PATH LENGTH

We now have the necessary results which allow us to calculate the distribution function for the total flight-path length $x$. Since $x$ is the sum of $x_1$, $x_2$, and $x_3$, the distribution function for $x$ is

$$
\rho_l(x) = \int \rho_l(x_1)dx_1 \int \rho_l(x_2)dx_2 \int \rho_l(x_3)dx_3 \delta((x_1 + x_2 + x_3) - x) , \tag{3.4.1}
$$

where $\delta(\cdot)$ is the Dirac delta function. By using Eqs. (3.1.2), (3.2.1), and (3.3.1), this expression could be explicitly evaluated to give $\rho_l(x)$ in terms of the parameters of the target ($d$, $W$, $s$, $Z$, and $f_W$), the flight tube ($l_2$), and the detector ($\lambda$, $\sigma$, and $L$). However, for this study we will not determine the combined distribution function directly, but will be satisfied with the moments of the distribution.

#### 3.4.1 Mean Value $L$ for Distribution of $x$

The mean value for the distribution function given in Eq. (3.4.1) is
The final step in Eq. (3.4.2) results from substituting Eq. (3.1.12), (3.2.2), and (3.3.5) for the three individual mean values. Note that the total flight-path length $l$ is energy dependent; i.e.,

$$l(E) = l_1(E) + l_2 + l_3(E)$$

(3.4.3)

or, substituting explicit results from Eqs. (3.1.13) and (3.3.5),

$$l = (d - \frac{W}{2} - s - Z)f_w + \frac{W}{2} + s + Z + l_2 + \frac{1}{\lambda \sigma} + \frac{L}{1 - e^{\lambda L}}$$

(3.4.4)

3.4.2 Uncertainty in $l$

A small increment in $l$ can be written in terms of small increments in $l_1$, $l_2$, and $l_3$ as

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

(3.4.5)

so that

$$(\Delta l)^2 = \langle \delta l \delta l \rangle = \langle (\delta l_1 + \delta l_2 + \delta l_3)^2 \rangle$$

(3.4.6)
Since $l_1$, $l_2$, and $l_3$ are independent, the cross terms (e.g., $<\delta l_1 \delta l_2>$) in Eq. (3.4.6) vanish, yielding for the uncertainty in $l$:

$$\Delta l = [\Delta l_1^2 + \Delta l_2^2 + \Delta l_3^2]^{1/2} \quad (3.4.7)$$

where $\Delta l_1$ and $\Delta l_3$ are evaluated from Eqs. (3.1.17) and (3.3.8) respectively, and $\Delta l_2$ is taken from the measurement process for $l_2$. The total flight-path length and its uncertainty $l \pm \Delta l$ are given in Table 1. We see that there is about a 20-mm variation in the effective flight-path length as a function of energy, and this is larger than the uncertainty at any energy. Figure 5 shows a plot of $l_1 + l_3$ as a function of neutron energy.

### 3.4.3 Higher Moments of the Flight-Path Length Distribution Function

In evaluating $<x>_1$ in the previous subsection, we explicitly displayed each step of the process, for clarity's sake. In evaluating the higher moments, those steps shall be implicitly understood. Details are given in Appendix A.

The second moment about the mean is given by

$$\omega_2^2 = <(x - l)^2>_1 = \omega_1^2 + \omega_2^2 + \omega_3^2 = \omega_1^2 + \omega_2^2 \quad \text{since} \quad \omega_3^2 = 0 \quad (3.4.8)$$

The third moment becomes

$$\nu_3^3 = \nu_1^3 + \nu_2^3 + \nu_3^3 = \nu_1^3 + \nu_3^3 \quad \text{since} \quad \nu_2^3 = 0 \quad (3.4.9)$$

Finally, the fourth moment is given by

$$\mu_4^4 = \mu_1^4 + \mu_2^4 + \mu_3^4 + 6(\omega_1^2 \omega_2^2 + \omega_1^2 \omega_3^2 + \omega_2^2 \omega_3^2)$$

$$= \mu_1^4 + \mu_3^4 + 6(\omega_1^2 \omega_3^2) \quad \text{since} \quad \mu_2^4 = \omega_2^2 = 0 \quad (3.4.10)$$

### 3.4.4 Width $\omega_l$ of Flight-Path Length Distribution Function

The width $\omega_l$ of the distribution function for the total flight-path length is given in Eq. (3.4.8), with values for $\omega_l$ and $\omega_l$ taken from Sects. 3.1 and 3.3 respectively. Values of $\omega_l$ as a function of energy are shown in Table 2.
Fig. 5. This figure illustrates the energy dependence of the mean flight-path length as a function of neutron energy. The energy-independent length $l_2$ was not included; only the sum $l_1 + l_3$ (and its uncertainty) is shown. We note there is approximately a 20-mm difference in the mean effective flight-path length as a function of neutron energy. Simply adding the measured distance from the center of the Ta target to its face, 18.3 mm, and (arbitrarily) one-third of the detector thickness, 6.3 mm, would give a length of 24.6 mm (shown as dashed line), significantly different from the results found in this report.
3.4.5 Uncertainty on $w_I$

The uncertainty on $w_I$ is found by using Eqs. (3.1.36) and (3.3.15) and Eq. (3.4.8), which gives

$$\langle (\Delta w_I)^2 \rangle = \langle \Delta w_I^2 \rangle + \langle \Delta w_{I'}^2 \rangle$$  \hspace{1cm} (3.4.11)

The uncertainty $\Delta w_I$ is then given by

$$\Delta w_I = \frac{1}{2\omega_I} \Delta w_I^2$$  \hspace{1cm} (3.4.12)

Values for this uncertainty are shown in Table 2. We note that although the uncertainties on $w_I^2$ and $w_{I'}^2$ combine quadratically to give $\Delta w_I^2$, this is not true for the uncertainties on $w_I$ and $w_{I'}$. $\Delta w_I$ must be obtained using results from Eq. (3.4.12). This is illustrated in Table 2, where $\Delta w_I$ is observed to be smaller than $\Delta w_{I'}$, a somewhat surprising result.

4. PROPERTIES ASSOCIATED WITH THE FLIGHT TIME

In order to calculate the neutron energy, we must have information about the flight time as well as the flight-path length. In this section we describe how the neutron flight times are determined. Figure 2 illustrates the various times defined below. For each burst, the time digitizer (clock) is started with a signal from a bare phototube viewing the gamma flash produced when the electron beam strikes the target. $t_0'$ is the time for the signal to travel from the phototube to the clock. The gamma flash reaches the detector at time $t_0''$, where $l$ is the length of the flight path and $c$ is the speed of light. The gamma-flash signal then leaves the detector and at time $t_0'$ later is stored in channel $C_\gamma$ (at time $t_\gamma$) in the data-acquisition computer. Thus

$$\frac{l}{c} + (t_0' - t_0'') = t_\gamma$$  \hspace{1cm} (4.1)

Properties of $t_0'$ and $t_0''$ need not be developed since we are only interested in the relative time $t_0' - t_0''$, so we define a time parameter $t_0$ as

$$t_0 = t_0' - t_0''$$

or

$$t_0 = t_\gamma - \frac{l}{c}$$  \hspace{1cm} (4.2)
Thus $t_0$ is related to the time to process a signal from the detector to the computer and is a constant for a given experiment. To get the flight time $t$ associated with a neutron-induced event, we subtract $t_0$ from the time $t_n$ the event is observed at the computer. That is,

$$ t = t_n - t_0 \quad (4.3) $$

This time $t_n$ can be identified with the difference in the means of two distributions: (1) $t_1$, the mean time at which the neutrons are born, and (2) $t_2$, the mean time at which the neutron is registered in the data-taking computer at channel $c_n$. That is,

$$ t_n = t_2 - t_1 \quad (4.4) $$

The mean flight time is then given by

$$ t = (t_2 - t_1) - t_0 \quad . \quad (4.5) $$

The first component, $t_1$, is associated with the pulse width of the ORELA beam and may be described approximately by a Gaussian distribution. The second component represents the finite channel width of the data-acquisition program and is approximated by a square distribution.

### 4.1 DISTRIBUTION FUNCTION FOR $t_1$: THE ORELA PULSE WIDTH

The ORELA pulse width may be approximated by a Gaussian shape (for pulse widths $<10$ ns) whose standard deviation is $a$. For the nickel measurement, the FWHM of the gamma flash pulse width was $7.5 \pm 0.5$ ns, corresponding to $a = 3.2 \pm 0.2$ ns. The distribution may be written as

$$ \rho_1(\tau_1) = \frac{1}{a\sqrt{2\pi}} e^{-\frac{\tau_1^2}{2a^2}} \quad . \quad (4.1.1) $$

#### 4.1.1 Mean Value $t_1$ for Distribution of $\tau_1$

The mean value for the distribution in Eq. (4.1.1) is

$$ t_1 = \langle \tau_1 \rangle_1 = \int \tau_1 \rho_1(\tau_1) d\tau_1 = 0 \quad . \quad (4.1.2) $$
4.1.2 Uncertainty in $t_1$

The uncertainty in $t_1$ is simply the accuracy with which $t_1$ can be determined experimentally, which we will call $\Delta t_1$. A more complete description is given in Sect. 4.3.2.

4.1.3 Higher Moments of the Pulse Width Distribution Function

The variance and other moments of the Gaussian distribution about the mean ($= 0$ for this case) in Eq. (4.1.1) are given by

$$\omega_i^2 = a^2,$$  \hspace{1cm} (4.1.3)

$$\mu_i^3 = 0,$$  \hspace{1cm} (4.1.4)

$$\mu_i^4 = 3a^4.$$  \hspace{1cm} (4.1.5)

4.1.4 Width $\omega_i$ of Pulse Width Distribution Function

The width, $\omega_i$, of the pulse is given by the square root of the variance, or

$$\omega_i = a.$$  \hspace{1cm} (4.1.6)

This width is identified with the contribution to the energy resolution function from the neutron burst width.

4.1.5 Uncertainty on $\omega_i$

The uncertainty on this width reflects the accuracy with which $a$ can be determined experimentally; that is

$$\Delta \omega_i = \Delta a = 0.2 \text{ ns}.$$  \hspace{1cm} (4.1.7)
4.2 DISTRIBUTION FUNCTION FOR \( t_2 \): THE CHANNEL WIDTH

Neutrons are accumulated in the data acquisition computer in channels of width \( b \) (typically 1 ns). Since the time distributions which enter in this work (e.g., \( \omega_i \)) are large compared with \( b \), this section will be important only if wide channel widths (>10 ns) are used. It is included here for completeness. This square distribution may be written as

\[
\rho_i(t_2) = \begin{cases} 
1/b & \text{for } -b/2 \leq t_2 - t_i \leq b/2 \\
0 & \text{otherwise.}
\end{cases}
\]  

4.2.1 Mean Value \( t_2 \) for Distribution of \( t_2 \)

The 1-ns-wide channels are probably not square distributions in fact, but trapezoidal or even triangular. However, for wide channels where this section is important, a square distribution is a good approximation. The mean of this distribution is

\[
\langle t_2 \rangle_{t_i} = \frac{1}{b} \int_{-b/2}^{b/2} t_2 dt_2 = t_i .
\]  

4.2.2 Uncertainty in \( t_2 \)

The uncertainty \( \Delta t_2 \) reflects the accuracy with which the value \( t_2 \) can be determined experimentally. See Sect. 4.3.2 for a more complete description.

4.2.3 Higher Moments of the Channel Width Distribution Function

The second moment about the mean (i.e., the variance) for the channel width distribution is given by

\[
\omega^2_{t_2} = \langle (t_2 - t_i)^2 \rangle = \frac{1}{b} \int_{-b/2}^{b/2} (t_2 - t_i)^2 dt_2 = \frac{b^2}{12} .
\]  

Similarly, the third moment is equal to

\[
\nu^3_{t_2} = 0 .
\]
and the fourth to

$$\mu_i^4 = b^4/80 \quad .$$  \hspace{1cm} (4.2.5)

### 4.2.4 Width $\omega_i$ of Channel Width Distribution Function

The width (square root of variance) $\omega_i$ is given by the full width $b$ of the square distribution divided by $\sqrt{12}$:

$$\omega_i = b/\sqrt{12} \quad .$$  \hspace{1cm} (4.2.6)

This width is the contribution to the energy resolution function from the data-acquisition channel width.

### 4.2.5 Uncertainty on $\omega_i$

The uncertainty on the width $\omega_i$ is simply

$$\Delta \omega_i = \Delta b/\sqrt{12} \quad ,$$  \hspace{1cm} (4.2.7)

where $\Delta b$ was estimated by accumulating ~100,000 cts/channel from a random source and looking for deviations from the statistical uncertainty. From this we estimate $\Delta b/b = 0.3\%$.

### 4.3 DISTRIBUTION FUNCTION FOR $\tau$: THE TOTAL FLIGHT TIME

The mean flight time $\tau = t_n - t_0$ (where $t_n = t_2 - t_1$) is given by the difference between the mean time $t_2$ at which the neutron is registered by the data-acquisition system and the mean time $t_1$ at which the neutron left the source, minus the time $t_0$ to transfer and process the signal. Thus, the t-o-f distribution is the convolution of the Gaussian and the square distributions. That is, the distribution function for $\tau$ is

$$\rho_i(\tau) = \int \rho_{i1}(\tau_1) d\tau_1 \int \rho_{i2}(\tau_2) d\tau_2 \delta((\tau_2 - \tau_1 - t_0) - \tau) \quad ,$$  \hspace{1cm} (4.3.1)

where $\rho_{i1}$ and $\rho_{i2}$ are given in Eqs. (4.1.1) and (4.2.1) respectively.
4.3.1 Mean Value $t$ for Distribution of $\tau$

The mean value $t$ for the distribution given in Eq. (4.3.1) is

$$ t = \langle \tau \rangle = \int_{-\infty}^{\infty} d\tau_1 \rho_1(\tau_1) = \int_{-b/2}^{1+b/2} d\tau_2 \rho_1(\tau_2)(\tau_2 - \tau_1 - t_0) \tag{4.3.2} $$

$$ = t_2 - t_1 - t_0 = t_0 - t_0 \tag{4.3.3} . $$

4.3.2 Uncertainty in $t$

The uncertainty in the flight time is given by

$$ \delta t = \delta < \tau_2 - \tau_1 - t_0 > = \delta t_n - \delta t_0 = \delta (t_n - t_0) \tag{4.3.4} , $$

so that

$$ (\Delta t)^2 = \Delta^2 (\tau_2 - \tau_1 - t_0) = \langle \delta (t_n - t_0)^2 \rangle \tag{4.3.5} . $$

To evaluate these terms, we consider the possibility of a scale error in both $t_n$ and $t_0$ and an absolute error in $t_0$. Thus

$$ t_n \rightarrow t_n g \quad \text{and} \quad t_0 \rightarrow t_0 g \tag{4.3.6} , $$

with $\langle g \rangle = 1$, so that

$$ \delta t_n = t_n \delta g \quad \text{and} \quad \delta t_0 = t_0 \delta g + g \delta t_0 \tag{4.3.7} , $$

or

$$ \delta (t_n - t_0) = (t_n - t_0) \delta g - g \delta t_0 \tag{4.3.8} . $$
Squaring and taking expectation values gives

\[(\Delta t)^2 = \Delta (t_n - t_0)^2 = (t_n - t_0)^2 \Delta g^2 + \Delta t_0^2 = t^2 \Delta g^2 + \Delta t_0^2 \quad (4.3.9)\]

The standard deviation of the uncertainty in \(t_0\), \(\Delta t_0\), is estimated to be 0.3 ns, based on comparison of the gamma flash centroids in the four bias spectra (LA83).

The scale uncertainty \(\Delta g\) may be identified with uncertainties in the timing clock oscillator. Such uncertainties have been measured for the ORELA clocks and are found to be less than 1 part in 50,000, i.e., \(\Delta g = 2 \times 10^{-5}\).

4.3.3 Higher Moments of the Total Flight-Time Distribution Function

We shall define \(\omega_i^2, \nu_i^3, \) and \(\mu_i^4\) as the second, third, and fourth moments about the mean, respectively, of the flight-time distribution function. These moments are given by

\[<(\tau - t)^m>_t = \int (\tau - t)^m \rho_i(\tau) d\tau\]

\[= \int \rho_i(\tau_1)d\tau_1 \int \rho_i(\tau_2)d\tau_2 ((\tau_2 - \tau_1 - t_0) - t)^m\quad (4.3.10)\]

in which we have used Eq. (4.3.1) to replace the integral over \(\tau\) by the double integrals over \(\tau_1\) and \(\tau_2\) and to replace \(\tau\) by \(\tau_2 - \tau_1 - t_0\). Since \(t\) is equal to \(t_2 - t_1 - t_0\) and \(t_1\) is zero, Eq. (4.3.10) can be written in the form

\[<(\tau - t)^m>_t = \int \rho_i(\tau_1)d\tau_1 \int \rho_i(\tau_2)d\tau_2 (\tau_2 - t_2 - \tau_1)^m\quad (4.3.11)\]

Thus the variance \(\omega_i^2\) becomes

\[\omega_i^2 = <(\tau - t)^2>_t = \omega_i^2 + \omega_i^2 = \frac{h^2}{12} + a^2\quad (4.3.12)\]

where we have used results from Eqs. (4.1.3) and (4.2.3) for values of \(\omega_i^2\) and \(\omega_i^2\).

Similarly, the third moment is

\[\nu_i^3 = <(\tau - t)^3>_t = \nu_i^3 - 3 \omega_i^2 <\tau_2>_t + 3 \omega_i^2 <t_2 - t>_t - \nu_i^3 = \nu_i^3 - \nu_i^3\quad (4.3.13)\]
Each of the two terms $\nu^3_i$ and $\nu^3_i$, are zero [see Eqs. (4.2.4) and (4.1.4)] by virtue of the symmetry of the square and Gaussian distributions. Thus, we have

$$\nu^3_i = 0$$

(4.3.14)

The fourth moment is

$$\mu^4_i = <(\tau - t)^4> - \mu^4_i - 4\nu^3_i <\tau_1>^2_i + 6\omega_i t^2 \omega^2_i - 4\nu^3_i <\tau_2 - t>^3_i + \mu^4_i$$

(4.3.15)

or, again using the value zero for $\nu^3_i$ and $\nu^3_i$,

$$\mu^4_i = \mu^4_i + 6\omega_i t^2 \omega^2_i + \mu^4_i$$

(4.3.16)

Substituting values found in Sect. 4.1 and 4.2, we find

$$\mu^4_i = b^4/80 + 6(b^2/12)(a^2) + 3a^4$$

(4.3.17)

In Sect. 5 we shall require expectation values of powers of $1/\tau$. These are evaluated in Appendix B in terms of the quantities $\omega_1$, $\nu^3_i$, and $\mu^4_i$ whose values we have just derived.

4.3.4 Width $\omega_i$ of Total Flight-Time Distribution Function

The width $\omega_i$ of the time-resolution function is the square root of the variance $\omega^2_i$ given in Eq. (4.3.12), or

$$\omega_i = \sqrt{b^2/12 + a^2}$$

(4.3.18)

4.3.5 Uncertainty on $\omega_i$

The uncertainty $\Delta \omega_i$ may be found directly from Eq. (4.3.18)

$$\Delta \omega_i = \left( b \Delta b/12 + a \Delta a \right) \frac{1}{\omega_i}$$

(4.3.19)
5. PROPERTIES OF THE ENERGY SCALE AND THE RESOLUTION FUNCTION

We now focus our attention on two objects of ultimate interest - the energy scale and the associated energy resolution function. Work done in the preceding two sections can be viewed as prologue, laying the framework for this section.

5.1 DISTRIBUTION FUNCTION OF $\epsilon$: THE ENERGY SCALE

The (non-relativistic) energy $\epsilon$ can be written in terms of the time-of-flight length $x$ and travel time $\tau$ as

$$\epsilon = \frac{m}{2} \left( \frac{x}{\tau} \right)^2,$$

where $m$ is the neutron mass. The distribution function for the energy can therefore be written

$$\rho(\epsilon) = \int \rho_{\tau}(\tau) d\tau \int \rho_{x}(x) dx \delta \left( \epsilon - \frac{m}{2} \left( \frac{x}{\tau} \right)^2 \right),$$

where the time and length distribution functions are given in Eqs. (4.3.1) and (3.4.1) respectively. We do not explicitly evaluate this expression for $\rho(\epsilon)$, but calculate the first and second moments of the distribution.

5.1.1 Mean Value $E$ for Distribution of $\epsilon$

The mean energy $E$ is found from Eq. (5.1.2) as

$$E = \langle \epsilon \rangle = \int \epsilon \rho(\epsilon) d\epsilon = \int \rho_{\tau}(\tau) d\tau \int \rho_{x}(x) dx \frac{m}{2} \left( \frac{x}{\tau} \right)^2.$$

This expression can be divided naturally into the product form

$$E = \frac{m}{2} <\tau^{-2}>_\tau <x^2>_x,$$

where
In Appendix A we show that the second moment about the origin is equal to the variance plus the square of the mean; thus Eq. (5.1.6) becomes

\[ \langle x^2 \rangle_i = \omega_i^2 + l^2 , \] (5.1.7)

where values of \( \omega_i^2 \) and \( l^2 \) are given in Sect. 3.

In Appendix B we show that the expression in Eq. (5.1.5) may be expanded about \( t \) to give the approximate value

\[ \langle \tau^{-2} \rangle_t = t^{-2} \left[ 1 + 3\omega_i^2 t^{-2} + 5\mu_i^4 t^{-4} \right] , \] (5.1.8)

where values for the second and fourth moments about the mean \( \omega_i^2 \) and \( \mu_i^4 \) are given in Sect. 4.

Substituting Eqs. (5.1.7) and (5.1.8) into (5.1.4) gives

\[ E \approx E_0 (1 + \omega_i^2 l^2/(1 + 3\omega_i^2 t^{-2} + 5\mu_i^4 t^{-4})) \] , (5.1.9)

where we have set

\[ \frac{\omega_i^2}{l^2} = \frac{m}{2} \frac{l^2}{t^2} . \] (5.1.10)

Thus, in addition to the usual term \( E_0 \) for the mean energy, we have correction factors which depend on the variances (and higher moments) of the distributions in length and time. We can estimate the magnitude of these correction terms from results previously determined. Values for \( l \) and \( \omega_i \) as a function of energy are given in Tables 1 and 2. At \( E_0 = 300 \text{ keV} \), we see that

\[ \frac{\omega_i^2}{l^2} \approx 1.0 \times 10^{-8} \] . (5.1.11)
Likewise, using $\omega_i^2 = a^2 + b^2/12$ gives

$$3\omega_i^2/t^2 = 4.1 \times 10^{-8}, \quad (5.1.12)$$

and a far smaller number for the $\mu_i$ term. Therefore, these corrections can be neglected, and the mean energy is well approximated by

$$E \approx E_0 = \frac{m}{2} \left( \frac{l}{I} \right)^2 \quad (5.1.13).$$

5.1.2 Uncertainty in $E$

We can now evaluate the uncertainty on the mean energy. Taking small increments gives

$$\delta E = \frac{\partial E}{\partial t} \delta l + \frac{\partial E}{\partial t} \delta t \quad (5.1.14).$$

Evaluation of the derivatives, using the approximation in Eq. (5.1.13), gives

$$\delta E = 2E_0 \left( \frac{\delta l}{l} - \frac{\delta t}{l} \right) \quad (5.1.15).$$

Forming the product $<\delta E \delta E> = \Delta E^2$, we find

$$\Delta E = 2E_0 \left[ \left( \frac{\Delta l}{l} \right)^2 + \left( \frac{\Delta t}{l} \right)^2 \right]^{1/2} \quad (5.1.16),$$

where $\Delta l^2$ is obtained from Table 1 as a function of energy by squaring the standard deviation, and $\Delta t^2$ was evaluated in Sect. 4.3.2. Table 1 contains the energy uncertainty as a function of energy.

5.1.3 Higher Moments of the Energy Distribution Function

We shall evaluate only the second moment of the energy distribution function, since higher moments are not needed. The second moment (the variance) is given by
\[ \omega_k^2 = \langle (e - E)^2 \rangle = \langle e^2 \rangle - E^2, \quad (5.1.17) \]

where \( \langle e^2 \rangle \), the second moment about the origin, is given by

\[ \langle e^2 \rangle = \int e^2 \rho(e) de = (m/2)^2 \langle r^{-4} \rangle \langle x^4 \rangle. \quad (5.1.18) \]

In Appendix A (Eq. A.9), the fourth moment about the origin \( \langle x^4 \rangle \) is shown to be given by

\[ \langle x^4 \rangle = \mu_4^4 + 4\nu_4 l + 6\omega_4^2 l^2 + t^4. \quad (5.1.19) \]

Values for \( \mu_4^4, \nu_4, \) and \( \omega_4^2 \) are given in Sect. 3.4 and in Table 3.

### Table 3. The second, third, and fourth moments of the flight-path length distribution function.

<table>
<thead>
<tr>
<th>( E ) (eV)</th>
<th>( \omega_4^2 ) (mm(^2))</th>
<th>( \nu_4^3 ) (mm(^3))</th>
<th>( \mu_4^4 ) (mm(^4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.</td>
<td>106.52</td>
<td>89.77</td>
<td>12944.99</td>
</tr>
<tr>
<td>1000.</td>
<td>143.59</td>
<td>90.29</td>
<td>18053.78</td>
</tr>
<tr>
<td>2000.</td>
<td>159.08</td>
<td>92.99</td>
<td>20232.67</td>
</tr>
<tr>
<td>5000.</td>
<td>184.94</td>
<td>108.80</td>
<td>23968.89</td>
</tr>
<tr>
<td>10000.</td>
<td>209.15</td>
<td>150.46</td>
<td>27696.27</td>
</tr>
<tr>
<td>20000.</td>
<td>238.05</td>
<td>266.91</td>
<td>32714.38</td>
</tr>
<tr>
<td>50000.</td>
<td>285.38</td>
<td>755.24</td>
<td>43905.53</td>
</tr>
<tr>
<td>100000.</td>
<td>329.54</td>
<td>1818.09</td>
<td>59994.09</td>
</tr>
<tr>
<td>200000.</td>
<td>371.08</td>
<td>4328.43</td>
<td>82187.19</td>
</tr>
<tr>
<td>300000.</td>
<td>372.50</td>
<td>6593.78</td>
<td>87614.68</td>
</tr>
<tr>
<td>400000.</td>
<td>345.74</td>
<td>7971.24</td>
<td>77657.39</td>
</tr>
<tr>
<td>500000.</td>
<td>213.06</td>
<td>6729.15</td>
<td>25648.56</td>
</tr>
<tr>
<td>700000.</td>
<td>119.29</td>
<td>3378.89</td>
<td>4965.63</td>
</tr>
<tr>
<td>1000000.</td>
<td>53.32</td>
<td>159.46</td>
<td>5741.62</td>
</tr>
<tr>
<td>2000000.</td>
<td>51.13</td>
<td>15.25</td>
<td>6307.69</td>
</tr>
<tr>
<td>5000000.</td>
<td>51.18</td>
<td>10.81</td>
<td>6142.88</td>
</tr>
<tr>
<td>10000000.</td>
<td>51.19</td>
<td>9.97</td>
<td>6187.93</td>
</tr>
</tbody>
</table>
In Appendix B we show that the inverse fourth moment of the time distribution is approximately

$$<r^{-4}>_t = t^{-4} \left( 1 + 10\omega_l^2 t^{-2} + 35\mu_l^4 t^{-4} \right), \quad (5.1.20)$$

where the various moments are evaluated explicitly in Sect. 4, and we have set $\nu_l^2 = 0$. Combining Eqs. (5.1.18) through (5.1.20) and keeping only terms of fourth order or less gives

$$<E^2> = E_0^2 \left( 1 + 6\omega_l^2 t^{-2} + 4\nu_l^4 t^{-3} + \mu_l^4 t^{-4} + 10\omega_l^2 t^{-2} + 35\mu_l^4 t^{-4} + 60\omega_l^2 t^{-2} \omega_l^2 t^{-2} \right). \quad (5.1.21)$$

Similarly, squaring Eq. (5.1.9) and keeping only terms of fourth order or less gives

$$E^2 = <E>^2 \left( 1 + 2\omega_l^2 t^{-2} + \omega_l^4 t^{-4} + 6\omega_l^2 t^{-2} + 10\mu_l^4 t^{-4} + 9\omega_l^2 t^{-4} + 12\omega_l^2 t^{-2} \omega_l^2 t^{-2} \right). \quad (5.1.22)$$

Finally, substituting Eqs. (5.1.21) and (5.1.22) into (5.1.17) yields

$$\omega_E^2 = E_0^2 \left\{ (2\omega_l t^{-1})^2 + (2\omega_l t^{-1})^2 + 3(2\omega_l t^{-1})^2 \omega_l t^{-1} \right\}$$

$$+ 4\nu_l^4 t^{-3} + (\mu_l^4 - \omega_l^4) t^{-4} + (2\mu_l^4 - 9\omega_l^4) t^{-4} \right\} \quad (5.1.23)$$

for the variance of our energy distribution function.

5.1.4 Width $\omega_E$ of Energy Distribution Function: Identification with the Energy Resolution Function

The energy distribution function given in Eq. (5.1.2) is precisely the energy resolution function by which experimental data are "broadened" and by which theoretical calculations must be "broadened" prior to comparison with experiments. The broadening width $\omega_E$ is given by the square root of the variance in Eq. (5.1.23). Note that the first two terms within the brackets correspond to the conventional expression for the width of the energy resolution function while the remaining terms are higher order corrections; i.e., to first order in powers of $\omega^2$ we can write

$$\omega_E = 2E_0 \left\{ \left( \frac{\omega_l}{t} \right)^2 + \left( \frac{\omega_l t}{t} \right)^2 \right\}^{1/2} \quad (5.1.24)$$
We note that this expression has the same form as Eq. (5.1.16) for the uncertainty in the mean energy. However, $\Delta l$ and $\Delta t$ in Eq. (5.1.16) are the uncertainties in the mean values of the length and time, while $\omega_l$ and $\omega_t$ in Eq. (5.1.24) are the standard deviations (widths) of the length and time distributions. It is clear that $\Delta l$ and $\omega_l$ are completely different quantities (as are $\Delta t$ and $\omega_t$) as seen from the equations from which they are calculated.

Values for $\omega_E$ are given in Table 2. These values were generated from Eq. (5.1.23) rather than from the more severe approximation of Eq. (5.1.24).

5.1.5 Uncertainty on $\omega_E$

To obtain an expression for the uncertainty on the width given by the square root of Eq. (5.1.23), we first note that a small increment of $\omega_E$ can be written in the form

$$\delta(\omega_E^2) = q_1 \delta(\omega_l^2) + q_2 \delta(\omega_t^2) + q_3 \delta l + q_4 \delta t$$

$$+ [4l^{-3} \delta (\nu_l^2) + l^{-4} \delta (\mu_t^2) + 25t^{-4} \delta (\mu_t^4)] E_o^2 .$$

We shall assume that the quantities within the square brackets in the preceding equation produce negligible effects; these terms will be dropped. Coefficients $q_1$, $q_2$, $q_3$, and $q_4$ in Eq. (5.1.25) can be evaluated directly by taking partial derivatives of Eq. (5.1.23):

$$q_1 = 4E_o^2 l^{-2}(1 + 12\omega_l^2 t^{-2} - \omega_l^2 l^{-2}/2) ,$$

$$q_2 = 4E_o^2 l^{-2}(1 + 12\omega_t^2 l^{-2} - 9\omega_t^2 t^{-2}/2) ,$$

$$q_3 = -2E_o^2 l^{-1}(4\omega_l^2 l^{-2} + 48\omega_l^2 t^{-2} l^{-2} + 6\nu_l^2 l^{-3} + 2(\mu_t^4 - \omega_t^4) t^{-4}) + 4\omega_E^2 l^{-1} ,$$

$$q_4 = -2E_o^2 l^{-1}(4\omega_t^2 l^{-2} + 48\omega_t^2 t^{-2} l^{-2} + 2(25\mu_t^4 - 9\omega_t^4) t^{-4}) - 4\omega_E^2 t^{-1} .$$

The square of the uncertainty in the variance is then given by squaring Eq. (5.1.25) and taking expectation values:

$$\langle (\Delta \omega_E^2) \rangle^2 = \langle (\delta(\omega_E^2))^2 \rangle$$

$$= q_1^2 (\Delta \omega_l^2)^2 + q_2^2 (\Delta \omega_t^2)^2 + q_3^2 (\Delta l)^2 + q_4^2 (\Delta t)^2 + 2q_1q_3 <\delta \omega_l \delta l> .$$
Only the final term in Eq. (5.1.30) requires explanation. Since $l$ and $\omega_l^2$ are determined from the same set of parameters, the two are not independent. In Sect. 3 of this report we determined the partial derivatives of both $\omega_l^2$ and $l$ with respect to the ten independent parameters ($f_w$, $d$, $W$, $s$, $Z$, $\omega_w$, $r$, $\lambda$, $\sigma$, $L$). Those partial derivatives may be used to evaluate

\[
<\delta \omega_l^2 \delta l> = \sum_{i=1}^{10} \frac{\partial \omega_l^2}{\partial p_i} \left( \Delta p_i \right)^2 \left( \frac{\partial l}{\partial p_i} \right), \tag{5.1.31}
\]

where $\{p\}$ represents those ten parameters. Note that the analogous term

\[
2q_2q_4 <\delta \omega_l^2 \delta t>, \tag{5.1.32}
\]

is zero here since $\omega_l^2$ depends only on parameters $a$ and $b$ and not on $t_0$ or $g$. Values of $\Delta \omega_E$ are given in Table 2.

5.2 COMPARISON OF THE RESOLUTION FUNCTION WITH EXPERIMENT

To facilitate comparison of our calculated results with an experimental determination of the resolution function, we use the relation

\[
E = (72.3 l/t)^2, \tag{5.2.1}
\]

with $l$ in mm, $t$ in ns, and $E$ in ev, to rewrite Eq. (5.1.23) as

\[
\omega_l^2 = E_0 \left\{ (2\omega_l l^{-1})^2 + (2\omega_l t^{-1})^2 + \ldots \right\} = E_0 \left\{ (2\omega_l l^{-1})^2 + (2\omega_l l^{-1} / 72.3)^2 E \right\}, \tag{5.2.2}
\]

or

\[
(\omega_E/E)^2 = b_1 + b_2 E, \tag{5.2.3}
\]

where $b_1 = (2\omega_l l)^2$ and $b_2 = (2\omega_l / 72.3 l)^2$.

Since $\omega_l$ and $l$ are energy dependent, $(\omega_E/E)^2$ is a non-linear function of energy. However, the energy dependence of $l$ is weak enough that $b_2$ is essentially independent of energy. Table 4 presents results for the parameters $b_1$ and $b_2$ and their uncertainties and $\omega_E^2$ as a function of energy.
Table 4. Values of $b_1$ and $b_2$ as a function of energy. Experimental values for the energy range 50 to 500 keV are $b_1 = (3.25 \pm 0.72) \times 10^{-8}$ and $b_2 = (1.75 \pm 0.18) \times 10^{-13}$, which are assumed to be independent of energy.

<table>
<thead>
<tr>
<th>$E$ (eV)</th>
<th>$\omega^2$ (eV$^3$)</th>
<th>$b_1$</th>
<th>$\Delta b_1$</th>
<th>$b_2$ (1/eV)</th>
<th>$\Delta b_2$ (1/eV)</th>
<th>$b_1 + b_2 E$</th>
<th>$\Delta(b_1 + b_2 E)$</th>
<th>$(\omega/E)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.050E-06</td>
<td>1.050E-08</td>
<td>1.657E-09</td>
<td>1.928E-13</td>
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<td>1.050E-08</td>
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</tr>
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</tr>
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<tr>
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</tr>
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</tr>
<tr>
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</tr>
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<td>3.861E-06</td>
<td>5.100E-07</td>
<td>3.861E-06</td>
</tr>
</tbody>
</table>

Johnson, Kernohan, and Winters (JO83) have analyzed isolated narrow resonances with a single-level R-function code, using a Gaussian resolution function, to obtain values of $b_1$ and $b_2$ (or equivalently, $\omega_1$ and $\omega_2$). The argon transmission measurement on which their analysis is based was performed immediately following our nickel measurement and utilized the same experimental setup. Based on the analysis of 13 resonances from 50 to 500 keV, they obtained values of $b_1 = (3.25 \pm 0.72) \times 10^{-13}$ and $b_2 = (1.75 \pm 0.18) \times 10^{-13}$. Averaging our values of $b_1$ and $b_2$ from Table 4 from 50 to 400 keV, we obtain $b_1 = (3.36 \pm 0.77) \times 10^{-8}$ and $b_2 = (1.93 \pm 0.26) \times 10^{-13}$, both values being consistent with their results.

5.3 COMPUTER PROGRAM FLIP1

The computer program FLIP1 was designed to evaluate the various expectation values, variances, higher moments, and uncertainties described in Sects. 3, 4, and 5 of this report. The code is organized in a manner similar to this report: quantities for the individual components of flight-path length are generated first, in subroutines TAH20X (for the tantalum water target, Sect. 3.1 of this report) and DETECT (for the detector end, Sect. 3.3), and the total mean flight-path length $l$ (with other moments and uncertainties) is produced from those individual results in subroutine COMBIN (Sect. 3.4). Quantities for the individual components of time are generated and combined in subroutine TIME (Sect. 4). The energy and its uncertainty and the energy resolution width and its uncertainty are generated from the length and time components in subroutine ENERGY (Sect. 5).

Additional subroutines are included to (1) produce plots of the distribution functions $\rho_i(x_i)$ (subroutine PLOT, Sect. 3.1) and $\rho_i(x_j)$ (subroutine PLTDET, Sect. 3.3), and (2) output both the input parameter values and values of the generated quantities. Tables 1 through 4 of this report were generated in subroutine DISPLAY.
No attempt has been made to cast this code into user-friendly format with convenient methods of changing parameter values, since the code is not expected to be used for a series of production runs. However, by isolating each component of length into a separate subroutine, we expect to have facilitated the substitution of more accurate distribution functions, for example.

Appendix C contains a listing of FLIP1; one of the output files produced by the code is given in Appendix D.

6. SUMMARY AND CONCLUSIONS

With the goal of developing expressions for the mean neutron energy and its uncertainty and the energy resolution function and its uncertainty, we have developed approximate analytic forms for the necessary distribution functions from which the desired quantities can be determined. The distribution functions for the neutrons which come from the Ta target and surrounding water moderator and the distribution function for neutrons which interact in the NE110 detector were found to be dependent on the neutron energy. From these results we obtained expressions for the mean energy-dependent flight-path length and its uncertainty and found as much as a 20-mm difference in the mean effective flight-path length, due to the energy dependence. This difference is much larger than the uncertainty in the flight-path length at any energy. From the distribution functions we also calculated the contribution to the width of the energy resolution function from the components of the flight-path length and the uncertainty on the width as functions of energy.

We then developed distribution functions for the t-o-f, i.e., expressions for the neutron burst width and the data-acquisition channel width. From these we evaluated the mean t-o-f and its associated uncertainty. We also evaluated the contribution to the width of the energy resolution function from the time distribution functions and the associated uncertainty.

Having developed the above information for the length and time distributions, we then used those results to develop an expression for the energy distribution function \( p(t) \). From this expression, we found the mean neutron energy and its uncertainty. In addition to the conventional energy term [Eq. (5.1.10)], our expression for the mean energy contains small correction terms associated with the widths of the length and time distributions. Neglecting the small corrections to the conventional expression for the neutron energy, the expression for the energy uncertainty is the conventional expression [Eq. (5.1.16)], with the usual dependence on the uncertainties \( \Delta t \) and \( \Delta \).

We identified the second moment of the energy distribution function about the mean as the width of the energy resolution function, but did not evaluate the expression [Eq. (5.1.2)] for the energy distribution function itself. (Evaluation of that expression would provide the shape of the resolution function.) Comparing our result for the width of the resolution function with that determined from measurement, we find agreement well within the uncertainty.

A number of aspects of this report could be studied in more detail. It is clear to us that we have used too simple a treatment of the neutrons emerging from the Ta target. Multiple scattering and attenuation within the Ta produce an asymmetric tail to \( p_T(x) \), and in addition the process is energy dependent. The Ta target neutron emission should properly be ascertained by Monte Carlo calculations, similar to the treatment in CO83 for neutrons from the water moderator. Ideally, Monte Carlo calculations should be done for the complete (Ta + H\(_2\)O) target at one time. Proper treatment of the Ta would increase the value of \( \omega_T \), the contribution to the width of the energy resolution function from
the source end, and thus increase the width \( \omega_E \) of the energy resolution function. However, it would probably not have a significant effect on the mean path length \( l_t \). A correct treatment of the Ta would simply modify \( \rho_T(x_t) \) to \( \rho_T(x_t,E) \) and change its shape, but the formalism we have developed to propagate \( \rho_T \) would be unaffected.

When a proper treatment of the Ta becomes available, it would be worthwhile attempting to evaluate the integral for the energy distribution (i.e., resolution) function [Eq. (5.1.2)]. The resulting shape could then be parameterized and used as the resolution function in the computer code SAMMY (1.480) for analysis of resonance parameters. Again, the framework for the t-o-f energy and energy resolution function, and their uncertainties, which we have developed would still be valid, although requiring some modification to SAMMY.

One small point should be noted regarding the treatment of the NE110 detector. The face of the detector which matches onto the phototube is curved to match the phototube face. Thus the thickness \( l \) of the detector is not a constant 19 mm, but has about a 15% variation in thickness across its face which should be accounted for in the description of the distribution function for the detector [Eq. (3.3.1)]. Also, a Monte Carlo treatment of neutrons in the NE110 would be more correct than our "distance to first collision" approximation; however, these effects are not expected to change the results significantly.

In discussion of resolution functions, references are often made to a correction for broadening due to "electronics." We have not included such a term because our method of identifying \( \omega_t \) (Sect. 4.1) with the observed width of the gamma flash automatically includes effects of the incident electron burst width, broadening due to electronics, time dependence of the electron energy during a pulse due to depletion of stored energy in the accelerator cavity, and any contribution to the resolution due to the bremsstrahlung process.

ACKNOWLEDGEMENTS

We wish to thank Francis Perey for useful discussions regarding the uncertainty analysis component of this work, Cleland Johnson for providing the results of the resolution function analysis of their argon data, and Sue Damewood for a careful typing of this report. This work is an extension of work described in report ORNL/TM-8203, and was partially sponsored by the Division of Reactor Research and Technology of the Department of Energy.

REFERENCES


HA69 Engineering Drawing No. EA-023-D, "Flight Tube as-built, Data Sheet No. 2," September 1969, J. A. Harvey files. This drawing contains measurements from the center of the target room (which is also assumed to be the target center) to various benchmarks in the flight stations.


APPENDIX A. MOMENTS OF DISTRIBUTION FUNCTIONS

In Sects. 3 and 4 of this report, we evaluated the first, second, third, and fourth moments about the mean of the length and time distribution functions, respectively. In this appendix we (1) relate the moments about the origin to the moments about the mean; and (2) derive the moments of a combined distribution function. It is these moments about the origin of the combined distribution which enter into our determination of the resolution broadening function.

We begin by developing some notations. Let \( \rho_\alpha(y) \) be an arbitrary distribution function for some variable \( y \). This distribution function has a mean which we shall call \( Y_\alpha \) and which is given by

\[
Y_\alpha = \langle y \rangle_\alpha = \int y \rho_\alpha(y) dy.
\]  

The variance \( \omega_\alpha^2 \) for this distribution function is the "second moment about the mean," or

\[
\omega_\alpha^2 = \langle (y - Y_\alpha)^2 \rangle_\alpha = \langle y^2 \rangle_\alpha - Y_\alpha^2,
\]  

where \( \langle y^2 \rangle_\alpha \) is the "second moment about the origin," or

\[
\langle y^2 \rangle_\alpha = \int y^2 \rho_\alpha(y) dy.
\]  

In like manner the "third and fourth moments about the mean" are defined:

\[
\nu_\alpha^3 = \langle (y - Y_\alpha)^3 \rangle_\alpha = \int (y - Y_\alpha)^3 \rho_\alpha(y) dy
\]  

and

\[
\mu_\alpha^4 = \langle (y - Y_\alpha)^4 \rangle_\alpha = \int (y - Y_\alpha)^4 \rho_\alpha(y) dy
\]  

Expanding the integrand of Eq. (A.4) gives

\[
\nu_\alpha^3 = \langle y^3 \rangle_\alpha - 3\langle y^2 \rangle_\alpha Y_\alpha + 2Y_\alpha^3
\]  

or
\[
\langle y^3 \rangle_\alpha = \nu_\alpha^3 + 3\omega_\alpha^2 Y_\alpha + Y_\alpha^3 \quad .
\] (A.7)

Similarly, Eq. (A.5) becomes
\[
\mu_\alpha^4 = \langle y^4 \rangle_\alpha - 4\langle y^3 \rangle_\alpha Y_\alpha + 6\langle y^2 \rangle_\alpha Y_\alpha^2 - 3Y_\alpha^4
\] (A.8)
or
\[
\langle y^4 \rangle_\alpha = \mu_\alpha^4 + 4\nu_\alpha^3 Y_\alpha + 6\omega_\alpha^2 Y_\alpha^2 + Y_\alpha^4 \quad .
\] (A.9)

We now wish to convolute two arbitrary distributions to obtain moments of the combined distribution. For example, in Sect. 3.4 we set \( x = x_1 + x_2 + x_3 \); in Sect. 4.3 we set \( t_n = t_2 - t_1 \). Let us assume that we have \( s = y \pm z \), where \( \rho_\beta(z) \) is the distribution for variable \( z \). The distribution function for \( s \) is then
\[
\rho(s) = \int \rho_\alpha(y)dy \int \rho_\beta(z)dz \delta(s - (y \pm z)) \quad .
\] (A.10)
The mean of this distribution is
\[
S = \langle s \rangle = \int sp(s)ds = \langle y \rangle_\alpha \pm \langle z \rangle_\beta = Y \pm Z \quad .
\] (A.11)
The second moment about the mean is
\[
\omega_i^2 = \langle (s - S)^2 \rangle = \int \rho_\alpha(y)dy \int \rho_\beta(z)dz ((y \pm z) - (Y \pm Z))^2)
\[
= \langle (y - Y)^2 \rangle_\alpha \pm 2\langle y - Y \rangle_\alpha \langle z - Z \rangle_\beta + \langle (z - Z)^2 \rangle_\beta
\[
= \omega_\alpha^2 + \omega_\beta^2 \quad .
\] (A.12)
Similarly the third moment is

\[ \nu_3 = \nu_\alpha^3 \pm \nu_\beta^3 \]

and the fourth is

\[ \mu_4 = \mu_\alpha^4 + 6\omega_\alpha \omega_\beta^2 + \mu_\beta^2 \]
APPENDIX B. EXPECTATION VALUES OF NEGATIVE POWERS OF $\tau$

In Sect. 4.3 of this report we described the distribution function for the time of flight and evaluated the mean and variance of that distribution. In this appendix we derive approximate expressions for the expectation values of negative powers of $\tau$. These approximations are needed to evaluate our expression for $E$ as given in Sect. 5, which involves inverse powers of $\tau$.

The fraction $1/\tau$ may be written in the form

$$\frac{1}{\tau} = \frac{1}{t} \frac{1}{1 + y},$$

where $y$ is given by

$$y = \frac{\tau - t}{t}.$$

Thus the expectation value of $\tau^{-m}$ may be written as

$$\langle \tau^{-m} \rangle_t = \int \tau^{-m} \rho(\tau) d\tau$$

$$= t^{-m} \int \rho_t(\tau_1)d\tau_1 \int \rho_t(\tau_2)d\tau_2 \left\{1 + \frac{\tau_2 - \tau_1 - t_0 - t}{t}\right\}^{-m},$$

or, substituting $t = t_2 - t_1 - t_0$ into the numerator of the fraction, with $t_1 = 0$,

$$\langle \tau^{-m} \rangle_t = t^{-m} \int \rho_t(\tau_1)d\tau_1 \int \rho_t(\tau_2)d\tau_2 \left\{1 + \frac{(\tau_2 - t_2) - (\tau_1)}{t}\right\}^{-m}. \tag{B.4}$$

Since $\rho_t(\tau_1)$ is large only for small $\tau_1$, and $\rho_t(\tau_2)$ is non-zero only for $\tau_2$ near $t_2$, it is sufficient to replace $1/(1 + y)$ by a value which is correct only for small $y$. Note that by "$y$" we now mean

$$y = \frac{\tau_2 - t_2 - \tau_1}{t}. \tag{B.5}$$
so that the fraction $1/(1 + y)$ becomes

$$\frac{1}{1 + y} = 1 - y + y^2 - y^3 + y^4,$$  \hspace{1cm} (B.6)

where terms higher than fourth order in $y$ have been dropped. Squaring Eq. (B-6) gives

$$(1 + y)^{-2} = 1 - 2y + 3y^2 - 4y^3 + 5y^4,$$  \hspace{1cm} (B.7)

and squaring Eq. (B.7) gives

$$(1 + y)^{-4} = 1 - 4y + 10y^2 - 20y^3 + 35y^4.$$  \hspace{1cm} (B.8)

The expectation value of $\tau^{-k}$, where $k$ is 2 or 4, can then be found directly once the expectation values of the powers of $y$ are known. From the definition of $y$, Eq. (B.2), we see that

$$<y^n> = <(\tau - t)^n> t^{-n},$$  \hspace{1cm} (B.9)

so that

$$<\tau^{-2}> = \tau^{-2}(1 - 0 + 3\omega_l^2 t^{-2} - 4\nu_l^2 t^{-3} + 5\mu_l^4 t^{-4}) \hspace{1cm} (B.10)$$

where the $\nu_l^2$ term is retained for the sake of completeness; in this specific case $\nu_l^2$ is zero. Values of $\omega_l^2$ and $\mu_l^4$ are given in Sect. 4. Similarly, we have

$$<\tau^{-4}> = \tau^{-4}(1 - 0 + 10\omega_l^2 t^{-2} - 20\nu_l^2 t^{-3} + 35\mu_l^4 t^{-4}) \hspace{1cm} (B.11)$$

Finally, we note that

$$<\tau^{-2}>^2 = \tau^{-4}(1 + 6\omega_l^2 t^{-2} - 8\nu_l^2 t^{-3} + 10\mu_l^4 t^{-4} + 9\omega_l^4 t^{-4}) \hspace{1cm} (B.12)$$

to fourth order in $t^{-1}$. 
APPENDIX C. FORTRAN LISTING OF FLIP1
PURPOSE -- GENERATE MOMENTS OF ENERGY DISTRIBUTION
BY COMBINING THE VARIOUS COMPONENTS OF
LENGTH AND TIME

JANUARY, 1984

*** INPUT INTERNAL TO PROGRAM, NOT FROM EXTERNAL FILES
*** OUTPUT TAH20.ODF (FOR PLOTS OF ENERGY VS ENERGY-DEPENDENT
QUANTITIES FOR WATER-TANTULUM TARGET)
*** XXXXXX.ODF (FOR PLOT OF DISTRIBUTION IN TARGET, AT E=XXXXXX)
*** XXXXXX.DET (FOR PLOT OF DISTRIBUTION IN DETECTOR)
*** FOR25.DAT (COMPLETE OUTPUT)
*** FOR26.DAT (DEBUG OUTPUT)
*** FOR27.DAT (SUMMARY OUTPUT, AS INCLUDED IN TM REPORT)

*** NOTATION USED IN VARIABLE NAMES

EL, ELL1, ELL2, ELL2
FLIGHT PATH LENGTH
THREE COMPONENTS OF FPL

1 (ONE)
TARGET (TANTALUM + WATER)
WATER, WATR, WA REFER TO
WATER COMPONENT OF TARGET
TA REFERS TO TANTULUM PART
WITH PARAMETERS UUUUUU,
WWWWW, SSSSSS, RRRRRR

2 (TWO)
FLIGHT TUBE

3 (THREE)
DETECTOR END, WITH PARAMETERS
ALMBDA, THICKN, CROS

TIME OR TIM OR TI
"TOTAL" TIME

T1
PULSE COMPONENT (BRST)

T2
CHANNEL COMPONENT (CHNL)

TZERO
T-SUB-ZERO

ENERGY, EN
ENERGY

PREFIXES WITH SPECIAL MEANING

D, .
UNCERTAINTY

D2, .
SQUARED UNCERTAINTY

W, .
WIDTH

W2, .
SQUARED WIDTH (VARIANCE)

V3, .
THIRD MOMENT ABOUT MEAN

U4, .
FOURTH MOMENT ABOUT MEAN
SQUARED UNCERTAINTY ON VARIANCE
UNCERTAINTY ON VARIANCE
SQUARED UNCERTAINTY ON WIDTH
UNCERTAINTY ON WIDTH
PERCENT UNCERTAINTY
DELTA ... (ABSOLUTE UNCERTAINTY)
COVARIANCE TWEEN .. & ..

**SUFFIXES**
******Z,******Z,******Z,******Z

**GENERAL NOTES**

**ALL DISTANCES ARE IN UNITS OF MILLIMETERS (MM)**
**ALL TIMES ARE IN NANOSECONDS (NS)**
**ALL ENERGIES ARE IN ELECTRON-VOLTS (EV)**

**COMMON /TOTAL/ ENZZZZ(17), FUDGEZ(17), DENVZZZ(17),
  * D2ENZZZ(17), WENZZZ(17), W2ENZZZ(17), D2ENNZZ(17),
  * DW2ENZZ(17), D2W2ENZZ(17), WVELZZZ(17), WWTIM2ZZZ(17),
  * WVEL3ZZZ(17), WWTIM3ZZZ(17), WVEL14ZZZ(17),
  * WWTIM4ZZZ(17), PW2W2LZZZ(17), PW2W2TZZZ(17), PWZLZZZ(17),
  * PW2TIMZZZ(17)

**COMMON /ELXXX/ ELZZZZZZZ(17), DLZZZZZZZ(17), D2LZZZZZZZ(17),
  * WLZZZZZZZ(17), DWLZZZZZZZ(17), W2LZZZZZZZ(17), D2LZZZZZZZ(17),
  * D2W2LZZZZZZZ(17), V3LZZZZZZZ(17), U4LZZZZZZZ(17), CVLW2ZZZZZZZ(17)

**COMMON /ELL1XX/ DWAZZZZZZZ(17), WWAZZZZZZZ(17), FWAZZZZZZZ(17),
  * FTNZZZZZZ(17), ELLLZZZ(17), DL1ZZZZZZZ(17), D2L1ZZZZZZZ(17),
  * WL1ZZZZZZZ(17), DWL1ZZZZZZZ(17), W2L1ZZZZZZZ(17), D2L1ZZZZZZZ(17),
  * D2W2L1ZZZZZZZ(17), V3L1ZZZZZZZ(17), U4L1ZZZZZZZ(17), DFWAZZZZZZZ(17),
  * CVL1WZZZZZZZ(17)

**COMMON /TAWATV/ PERDWA, PERWVA, WATER1, WATER2, WATER3,
  * WWATR1, WWATR2, WWATR3, UUUUUU, WWWWWW, SSSSSS,
  * DELUUU, DELWWW, DELSSS, RRRRRR, DELRRR, EMMMMM,
  * PEREMM, ABSUNC
COMMON /FL13XX/ CROSSTM(17), DL3ZZZ(17),
* D2L3ZZ(17), W2L3ZZ(17), W2L3ZZM(17),
* D2W2L3(17), W3L3ZZ(17), U4L3ZZ(17),
* CVL3W2(17)
COMMON /DETECT/ ALMBA, THICKN, PERLMB, PERTHI, PECRO

COMMON /TIMER/ TZERO, DTZED, AFWMZ, AFWNM, BCHNLZ,
* BCHNL, TSCALE
COMMON /T1XXX/ T1ZZZZ, DT1ZZZ, D2T1ZZZ, WT1ZZZ, DWT1ZZZ,
* W2T1ZZZ, D2W2T1Z, D2W2T12, D3T1ZZZ, DWT2ZZZ, D3W2T1Z, W2T2ZZZ, D2W2T2Z, D2W2T22, D3W2T22, V3TZZZ, W4TZZZ, DWT2ZZZ, D3W2T22, V3TZZZ, U4TZZZ, CVT2W2
COMMON /TIMEXX/ TIMEZZ(17), DTIMEZ(17), D2TIME(17),
* WTZZZ, DWZZZ, WTZZZ, DWZZZ, D2TIME, V3TIME,
* U4TIME, CVT2W2

COMMON /AL2ZZZ/ EEEEEE(17), ELL2ZZ, DL2ZZZ, SHORTZ,
* IEGRID, IPILOT

LOGICAL SHORTZ
DATA SHORTZ /.TRUE./

DATA EEEEEE /10.0,1000.,2000.,5000.,10000.,20000.,50000.,
* 100000.,200000.,300000.,400000.,700000.,1000000.,
* 2000000.,5000000.,10000000.,20000000./

DATA IEGRID /17/
DATA IPILOT /3/

CALL TAH20
CALL QQTAH
CALL DETECT
CALL QQDQET
CALL COMBOM
CALL QQCOM
CALL OUTCMP
CALL PLOTT13
CALL TIME
CALL ENERGY
CALL QQTIM
CALL QQENE
CALL DSPLAY

STOP
END

SUBROUTINE TAH20

*** PURPOSE -- ORGANIZE TREATMENT OF TANTALUM-WATER TARGET
CALL OUTTAH
CALL TAH20X
CALL PLOT
RETURN
END
SUBROUTINE OUTTAH
C C *** PURPOSE -- OUTPUT THE INPUT FOR TA-H2O CALCULATION C
C COMMON /TAWATV/ PERDWA, PERWWA, WATER1, WATER2, WATER3,
* WWATR1, WWATR2, WWATR3, UUUUU, WWWW, SSSSSS,
* DELUUU, DELWWW, DELSSS, RRRRRR, DELRRR, EMMMMM,
* PEREMM, ABSUNC
C COMMON /ALLZZZ/ EEEEE(17), ELLZZZ, DL2ZZZ, SHORTZ,
* IEGRID, IPLOT
LOGICAL SHORTZ
C
W2PLUS = 0.5*WWWWWW + SSSSSS
UUUUU = UUUUUU + W2PLUS
RDELWW = DELWWW/WWWWWW*100.0
RDELRR = DELRRR/RRRRRR*100.0
RDELSS = DELSSS/SSSSSS*100.0
C
WRITE (25,99999)
WRITE (25,99998) UUUUUU, DELUUU
WRITE (25,99997) WWWW
WRITE (25,99996) DELWWW, RDELWW
WRITE (25,99995) SSSSSS
WRITE (25,99994) DELSSS, RDELSS
WRITE (25,99993) UUWWWW
WRITE (25,99992) RRRRRR
WRITE (25,99991) DELRRR, RDELRR
WRITE (25,99990) WATER1, WATER2, WATER3
WRITE (25,99989) WWATR1, WWATR2, WWATR3
WRITE (25,99988) PERDWA
WRITE (25,99987) PERWWA
WRITE (25,99986) ABSUNC
WRITE (25,99985) EMMMMM
WRITE (25,99984) PEREMM
IF (SHORTZ) GO TO 10
C
WRITE (5,99998) UUUUUU, DELUUU
WRITE (5,99997) WWWW
WRITE (5,99996) DELWWW, RDELWW
WRITE (5,99995) SSSSSS
WRITE (5,99994) DELSSS, RDELSS
WRITE (5,99993) UUWWWW
WRITE (5,99992) RRRRRR
WRITE (5,99991) DELRRR, RDELRR
WRITE (5,99990) WATER1, WATER2, WATER3
WRITE (5,99989) WWATR1, WWATR2, WWATR3
WRITE (5,99988) PERDWA
WRITE (5,99987) PERWWA
WRITE (5,99986) ABSUNC
WRITE (5,99985) EMMMMM
WRITE (5,99984) PEREMM
10 CONTINUE
C
WRITE (25,99983)
IF (.NOT.SHORTZ) WRITE (5,99983)
C
RETURN
99999 FORMAT (/25H ***** TARGET END *****)
99998 FORMAT (46H UUU = DISTANCE TO SURFACE OF TA TARGET ,
* 15H =, F7.3, 4H +/-, F7.3)
99997 FORMAT (46H WWW = WIDTH OF TA TARGET ,
* 15H =, F7.3)
99996 FORMAT (46H DELWWW = ERROR ON WWW ,
* 15H =, F/.3, 2H =, F5.1, 8H PERCENT)
99995 FORMAT (46H SSS = ESTIMATE OF MULTIPLE SCATTERING IN T ,
* 15H TARGET =, F7.3)
99994 FORMAT (46H DELSSS = ERROR ON SSS ,
* 15H =, F7.3, 2H =, F5.1, 8H PERCENT)
99993 FORMAT (46H UW2 = UUU + WWW/2 + SSS = DISTANCE TO MEAN ,
* 15H OF TA TARGET =, F7.3)
99992 FORMAT (46H RRR = RADIUS OF CIRCULAR DISTRIBUTION IN T ,
* 15H TARGET =, F/.3)
99991 FORMAT (46H DELRRR = ERROR ON RRRR ,
* 15H =, F7.3, 2H =, F5.1, 8H PERCENT)
99990 FORMAT (/44H DWA(E) = MEAN OF DISTRIBUTION IN WATER = ,
* F5.1, F6.2, 8H*LN(E) +, F6.3, 9H*LN(E)**2)
99989 FORMAT (44H WWA(E) = STD DEV OF DISTRIBUTION IN WATER = ,
* F5.1, F6.2, 8H*LN(E) +, F6.3, 9H*LN(E)**2)
99988 FORMAT (44H PERDWA = UNCERTAINTY ON MEAN DWA (E) = ,
* F5.1, 8H PERCENT)
99987 FORMAT (44H PERWWA = UNCERTAINTY ON STD DEV WWA (E) = ,
* F5.1, 8H PERCENT)
99986 FORMAT (/45H ABSUNC = ABSOLUTE UNCERTAINTY ON FRACTION FR ,
* 20HOM TA AND FROM H2O =, F6.2)
99985 FORMAT (46H EMM = ENERGY AT WHICH THE TWO FRACTIONS ARE ,
* 19H EQUAL =, F9.0)
99984 FORMAT (46H PEREMM = RELATIVE UNCERTAINTY IN EMM = DEL(EM, 
* 19HM)/EMM =, F6.1, 8H PERCENT/)
99983 FORMAT (46H FORMAT /44H ERROR DI ,
* 44HSTANCE WIDTH WIDTH/
* 49H ON TO ME ,
* 49HAN OF MEAN OF UNCERT. OF UNCE ,
* 3HRT./43H ENERGY FRACTN FRACTN FRACTN ,
* 49H OF WATER WATER TA+WATER ON TA+WATER ,
* 6H ON/40H (EV) OF TA OF WATER OF WATE ,
* 49HR DISTN DISTN DISTN MEAN DIST ,
* 11HN WIDTH, //31H EN FTA FWA ,
* 49H DFWA ELWATR WWATR ELL1 DLI ,
* 20H WL1 DWL1)
END
SUBROUTINE TAH20X

*** PURPOSE -- GENERATE MOMENTS OF WATER-TANTALUM DISTRIBUTION

COMMON /ELL1XX/ DWAZZZ(17), WWAZZZ(17), FWAZZZ(17),
* FTNZZZ(17), ELLLZZ(17), DL1ZZZ(17), D2L1ZZZ(17),
* WLLZZZ(17), DWLLZZZ(17), W2L1ZZZ(17), DW2L1ZZZ(17),
* D2W2L1(17), V3L1ZZZ(17), U4L1ZZZ(17), DFWAZZ2(17),
* CVLW2(17)

COMMON /TAWATV/ PERDWA, PERWWA, WATER1, WATER2, WATER3,
* WWATR1, WWATR2, WWATR3, UUUUUU, WWWWWW, SSSSSS,
* DELUUU, DELWWW, DELSSS, RRERRR, DELRRR, EMMMMM,
* PEREMM, ABSUNC

COMMON /ALLZZZ/ EEEEEE(17), ELLLZZ, DL2ZZZ, SHORTZ,
* IEGRID, IPLOT

LOGICAL SHORTZ

DIMENSION IPP(17)

DATA IPP /i,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17/

DATA WATER1 /22.8/, WATER2 /-1.60/, WATER3 /0.283/,
* WWATR1 /10.0/, WWATR2 /-0.63/, WWATR3 /0.112/,
* UUUUUU /0.0/, WWWWWW /36.6/, SSSSSS /6.0/, RRERRR
* /9.20/, DELUUU /2.0/, DELWWW /5.492/, DELSSS /1.8/,
* DELRRR /1.840/, EMMMMM /300000./, PEREMM /30.0/,
* PERDWA /10.0/, PERWWA /10.0/

W2PLUS = 0.5*WWWWWW + SSSSSS
UWZZZZ = UUUUUU + W2PLUS
PERDWA = PERDWA*0.01
PERWWA = PERWWA*0.01
PEREMM = PEREMM*0.01

IPL = 1
DO 10 IE=1,IEGRID

*** GENERATE FRACTION FROM TA AND FRACTION FROM H2O

QQQQQQ = EEEEEE(IE)*ALOG(3.0)/EMMMMM
EQQQQQ = EXP(QQQQQ)
FWAZZZ(IE) = 2.0/(1.0+EQQQQQ)
FTNZZZ(IE) = 1.0 - FWAZZZ(IE)
DFWAZZ(IE) = 0.5*FWAZZZ(IE)**2*EQQQQQ*QQQQQQ*PEREMM +
* ABSUNC
C *** DETERMINE PARAMETERS OF WATER DISTRIBUTION

\[ \text{ELOG} = \text{ALOG}(\text{EEEEEE}(1E)) \]
\[ \text{DWAZZZ}(1E) = \text{WATER1} + \text{WATER2} \times \text{ELOG} + \text{WATER3} \times \text{ELOG}^2 \]
\[ \text{WAZZZZ}(1E) = \text{WATR1} + \text{WATR2} \times \text{ELOG} + \text{WATR3} \times \text{ELOG}^2 \]
\[ \text{ELWATR} = \text{DWAZZZ}(1E) \]
\[ \text{DLWATR} = \text{ELWATR} \times \text{PERW} \]
\[ \text{WZWATR} = \text{WAZZZZ}(1E) \times \text{ELOG}^2 \]
\[ \text{DW2WAT} = 2.0 \times \text{W2WATR} \times \text{PERW} \]
\[ \text{V3WATR} = 0.0 \]
\[ \text{U4WATR} = 3.0 \times \text{WAZZZZ}(1E) \times \text{ELOG}^4 \]
\[ \text{FFWATR} = \text{FWAZZZ}(1E) \]
\[ \text{DFWATR} = \text{DFWAZZZ}(1E) \]

C *** DETERMINE PARAMETERS OF TANTALUM DISTRIBUTION

\[ \text{ELTNTL} = \text{UUUDDU} + \text{WWWWWW} / 2.0 \times \text{SSSSSS} \]
\[ \text{D2LINT} = \text{DELUUU}^2 + \text{DELWWW}^2 / 4.0 + \text{DELSSS}^2 \]
\[ \text{DLTNTL} = \text{SQRT}(D2LINT) \]
\[ \text{W2TNTL} = \text{RRRRRR}^2 / 4.0 \]
\[ \text{DW2TNT} = \text{RRRRRR} \times \text{DELRRR} / 2.0 \]
\[ \text{V3TNTL} = 0.0 \]
\[ \text{U4TNTL} = \text{RRRRRR}^4 / 8.0 \]
\[ \text{FFTNTL} = \text{FTNZZZ}(1E) \]

C *** COMBINATION OF WATER AND TANTALUM

\[ \text{FFWFFT} = \text{FWATR} \times \text{FFTNTL} \]
\[ \text{FTMNFW} = \text{FFTNTL} - \text{FWATR} \]
\[ \text{ELWATN} = \text{ELWATR} - \text{ELTNTL} \]

C *** GENERATE CONTRIBUTION TO ELL1, W2L1, V3L1, AND U4L1 FROM WATER DISTRIBUTION

\[ \text{ELL1ZZ}(1E) = \text{FFWATR} \times \text{ELWATR} \]
\[ \text{W2L1ZZ}(1E) = \text{FFWATR} \times \text{W2WATR} \]
\[ \text{V3L1ZZ}(1E) = \text{FFWATR} \times \text{V3WATR} \]
\[ \text{U4L1ZZ}(1E) = \text{FFWATR} \times \text{V4WATR} \]

C *** GENERATE CONTRIBUTION FROM TANTALUM DISTRIBUTION

\[ \text{ELL1ZZ}(1E) = \text{ELL1ZZ}(1E) + \text{FFTNTL} \times \text{ELTNTL} \]
\[ \text{W2L1ZZ}(1E) = \text{W2L1ZZ}(1E) + \text{FFTNTL} \times \text{W2TNTL} \]
\[ \text{V3L1ZZ}(1E) = \text{V3L1ZZ}(1E) + \text{FFTNTL} \times \text{V3TNTL} \]
\[ \text{U4L1ZZ}(1E) = \text{U4L1ZZ}(1E) + \text{FFTNTL} \times \text{U4TNTL} \]

C *** GENERATE CONTRIBUTION FROM COMBINATION

\[ \text{W2L1ZZ}(1E) = \text{W2L1ZZ}(1E) + \text{FFWFFT} \times \text{ELWATN}^2 \]
\[ \text{V3L1ZZ}(1E) = \text{V3L1ZZ}(1E) + \text{FFWFFT} \times \text{ELWATN}^2 \]
\[ \text{U4L1ZZ}(1E) = \text{U4L1ZZ}(1E) + \text{FFWFFT} \times \text{ELWATN}^4 \]

C
C *** NOW THE ASSOCIATED UNCERTAINTIES

PLFWAT = ELWATN
PLLWAT = FFWATR
PLLTNT = FFNTNL
PW2FWA = W2WATR - W2TNTL + FTMNFW*ELWATN**2
PW2W2W = FFWATR
PW2LWT = FFWFT*2.0*ELWATN
PW2W2T = FFTNTL
D2LIZZ(IE) = (PLFWAT*DFWATR)**2 + (PLLWAT*DLWATR)**2
* + (PLLTNT*DLTNTL)**2
D2W2L1(IE) = (PW2FWA*DFWATR)**2 + (PW2W2W*DFWATR)**2
* + (PW2W2T*DFW2T)**2 +
* PW2LWT**2*(DLWATR**2+D2LNTT)
CVL1W2(IE) = PLFWAT*PW2FWA*DFWATR**2 +
* PLLWAT*PW2LWT*DLWATR**2 - PLLTNT*PW2W2T*D2LNTT

C

C *** OBTAIN WIDTH (AND UNCERTAINTIES) FROM VARIANCE

WL1ZZZ(IE) = SQRT(WL1ZZZ(IE))
DL1ZZZ(IE) = SQRT(DL1ZZZ(IE))
DW2L1Z(IE) = SQRT(DW2L1Z(IE))
DWL1ZZ(IE) = 0.5*DW2L1Z(IE)/WL1ZZZ(IE)

C

C *** WRITE RESULTS

WRITE (25,99999) EEEEE(E), FTNZZZ(E), FWAZZZ(E),
* DFWAZZ(E), DWAZZZ(E), WWAZZZ(E), ELL1ZZZ(E),
* DL1ZZZ(E), WL1ZZZ(E), DWL1ZZZ(E)

C

C *** PLOT SOME OF THE DISTRIBUTIONS

IF (IPP(IPL).NE.IE) GO TO 10
CALL PLOT(EEEEE(IE), FTNZZZ(E), FWAZZZ(E),
* DWAZZZ(E), WWAZZZ(E), ELL1ZZZ(E), WL1ZZZ(E),
* IPL, UUUU, WWWWWW, WWZZZZ, RRRRRR)
IPL = IPL + 1
10 CONTINUE

C

WRITE (25,99998)
DO 20 IE=1,IEGKID
CRL1W2 = 0.0
IF (DL1ZZZ(IE)*DW2L1Z(IE).NE.0.0) CRL1W2 =
* CVL1W2(E)/(DL1ZZZ(IE)*DW2L1Z(IE))
WRITE (25,99997) EEEEE(E), D2L1Z(IE),
* D2W2L1Z(IE), CVL1W2(E), CRL1W2
20 CONTINUE

C

RETURN
99999 FORMAT (F10.0,3F10.4,6F10.3)
99998 FORMAT (/45H EN D2L1 D2W2L1 CVL1W2 C,
* 5HRIW2)
99997 FORMAT (F10.0,6F10.3)
END
SUBROUTINE DETECT
C
C *** PURPOSE -- GENERATE CONTRIBUTION TO LENGTH FROM DETECTOR
C *** END OF FLIGHT PATH
C
COMMON /ELL3XX/ CROSSS(17), ELL3ZZ(17), DL3ZZZ(17),
* DZL3ZZ(17), WL3ZZZ(17), DWL3ZZZ(17), W2L3ZZ(17),
* L2W2L3(17), V3L3ZZ(17), U4L3ZZ(17),
* CVL3W2(17)
COMMON /DETECV/ ALMBDA, THICKN, PERLMB, PERTHI, PERCRO
C
COMMON /ALLZZZ/ EEEEEE(17), ELL2ZZ, DL2ZZZ, SHORTZ,
* IEGRID, IPILOT
C
LOGICAL SHORTZ
C
DATA ALMBDA /0.0047/, THICKN /19.0/, PERLMB /0.02/,
* PERTHI /0.05/, PERCRO /0.05/
C
DATA CROSSS /27.200,27.17,27.02,26.60,25.91,24.69,21.71/
* 18.51,14.80,11.999,11.24,8.72,7.28,4.89,3.02,2.22,
* 2.03/
C
CCCcccCCCcccccccccccccccccccccccccccccccccccccccccccccccccccc
CCC 1INPUT CCCccccccccccccccccccccccccccccccccccccccccccccCCC
CCC
CCC CROSSS = CROSS SECTION FOR NE110 AT ENERGY E
CCC NE110 = CH1.104
CCC SS1GMA = CROSS SECTION ALSO
CCC DS1GMA = UNCERTAINTY ON CROSS SECTION FOR NE110
CCC PERCRO = PERCENT UNCERTAINTY ON CROSS SECTION
CCC
CCC THICKN = THICKNESS OF NE110 IN CM
CCC DTHICK = UNCERTAINTY ON THICKN
CCC PERTHI = PERCENT UNCERTAINTY ON THICKN
CCC
CCC ALMBDA = DENSITY OF NE110 * .6023 / 12+1.104
CCC (DENSITY OF NE110 IS 1.032G/CM3)
CCC DLMBDA = UNCERTAINTY ON ALMBDA
CCC PERLMB = PERCENT UNCERTAINTY ON ALMBDA
CCC
CCC OUTPUT CCCccccccccccccccccccccccccccccccccccccccccccccccc
CCC
CCC ELL3ZZ = MEAN DISTANCE TO FIRST SCATTER FROM FRONT
CCC OF DETECTOR
CCC DL3ZZZ = UNCERTAINTY ON ELL3ZZ
CCC WL3ZZZ = STANDARD DEVIATION OF DISTRIBUTION
CCC CENTERED ABOUT ELL3ZZ
CCC DWL3ZZ = UNCERTAINTY (STD DEV) ON WL3ZZZ
CCC
CCC OUTPUT IS IN MILLIMETERS
CCCcccccccccccccccccccccccccccccccccccccccccccccccccc
WRITE (25,99999)

C

DLMBDA = PERLMB*ALMBDA
DTHICK = PERTHI*THICKN
PPPPP = 100.0*PERCRO
WRITE (25,99998) ALMBDA, DLMBDA
WRITE (25,99997) THICKN, DTHICK
WRITE (25,99994) PPPPP
WRITE (25,99996)

C DO 10 IE=1,iegird

SSIGMA = CROSSS(IE)
RSTZZZ = ALMBDA*SSIGMA*THICKN
RSTZZZ = RSTZZZ**2
EXPRST = EXP(RSTZZZ)
DENOMN = 1.0 - EXPRST
ELL3ZZ(IE) = (1.0/RSTZZZ + 1.0/DENOMN)*THICKN

C

XXXXXX = 1.0/RSTZZZ - EXPRST/DENOMN**2
W2L3ZZ(IE) = XXXXXX*THICKN**2
WL3ZZZ(IE) = SQRT(XXXXXX)*THICKN

C

PLRSTX = -XXXXX*RSTZZZ*THICKN
PLTHXX = ELL3ZZ(IE) + PLRSTX
D2L3ZZ(IE) = PLRSTX**2*(PERLMB**2+PERCRO**2) +
(PLTHXX*PERTHI)**2
DL3ZZZ(IE) = SQRT(D2L3ZZ(IE))

C

XXXXXX = 2.0/(RSTZZZ*RSTZZZ) +
EXPRST*(1.0+EXPRST)/DENOMN**3
PW2RST = -XXXXX*RSTZZZ*THICKN**2
PW2THI = 2.0*W2L3ZZ(IE)+PW2RST
D2W2L3(IE) = PW2RST**2*(PERLMB**2+PERCRO**2) +
(PW2THI*PERTHI)**2
DW2L3Z(IE) = SQRT(D2W2L3(IE))
DWL3ZZ(IE) = DW2L3Z(IE)/(2.0*WL3ZZZ(IE))

C

CVL3W2(IE) = PLRSTX*PW2RST*(PERLMB**2+PERCRO**2) +
PLTHXX*PW2THI*PERTHI**2

C

V3L3ZZ(IE) = XXXXXX*THICKN**3
XXXXXX = 9.0/RSTZZZ**2 - 6.0*EXPRST/(RSTZZZ*DENOMN)**2
-U4L3ZZ(IE) = XXXXXX*THICKN**4
CRL3W2 = CVL3W2(IE)/(DL3ZZZ(IE)*DW2L3Z(IE))
C
WRITE (25,99995) EEEEEE(IE), SS1GMA, ELL3ZZ(IE),
  * DL3ZZZ(IE), WL3ZZZ(IE), DL3ZZZ(IE),
  * V3L3ZZZ(IE), U4L3ZZZ(IE), D2L3ZZZ(IE),
  * D2W2L3(IE), CVL3W2(IE), CRL3W2

C
IEE=IE
CALL PLTDET(EEEEEE(IE), ELL3ZZ(IE), WL3ZZZ(IE),
  * CROSSS(IE), ALMBDA, THICKN, IEE)

C
10 CONTINUE
RETURN

C
99999 FORMAT (//27H ***** DETECTOR END *****)
99998 FORMAT (/10H RHO = , F8.6, 5H +/- , F8.6)
99997 FORMAT (10H THICKN = , F8.4, 5H +/- , F8.4)
99994 FORMAT (/10H PERCHO = , F8.4,
  * 37H PERCENT UNCERTAINTY ON CROSS SECTION/)
99996 FORMAT (/45H ENERGY CRSSCTN ELL3 DL3 ,
  * 49H WL3 DLW3 V3L3 U4L3 D2L3 ,
  * 35H D2W2L3 CVL3W2 CRL3W2)
99995 FORMAT (F10.0, 6F10.3, 3X, 6G12.5)
END
SUBROUTINE COMBIN
C
C *** PURPOSE -- COMBINE THE VARIOUS COMPONENTS OF FLIGHT-PATH-
C *** LENGTH AND WRITE THEM ON UNIT 25
C
C COMMON /ELXXXX/ ELZZZZ(17), DLZZZZ(17), D2LZZZ(17),
C * WLZZZZ(17), DWLZZZ(17), W21ZZZ(17), DW1ZZZ(17),
C * DZLZZZ(17), V3LZZZ(17), U4LZZZ(17), CVLWWZ(17)
C COMMON /ELLXXX/ DWAZZZ(17), WWAZZZ(17), FWAZZZ(17),
C * FTNZZZ(17), ELLlZZ(17), DLlZZZ(17), D2LLZZ(17),
C * WL1ZZZ(17), DW1ZZZ(17), W21ZZZ(17), DW21ZZ(17),
C * DZW1ZZ(17), V3LlZZ(17), U4L1ZZ(17), DFWAZZ(17),
C * CVL1WZ(17)
C COMMON /TAWATV/ PERDWA, PERWWA, WATERR, WATER2, WATER3,
C * WAT1R1, WAT2R2, WAT3R3, UUUUU, WWWWW, SSSSSS,
C * DELUUN, DELWWW, DELSSSS, RRRRRR, EMMNMM,
C * PERENN, ABSUNC
C COMMON /ELLXXX/ CROSST(17), ELL3ZZ(17), DL3ZZZ(17),
C * D2L3ZZZ(17), WL3ZZZ(17), DWL3ZZ(17), W21L3ZZ(17),
C * DWL3ZZ(17), V3L3ZZ(17), U4L3ZZ(17),
C * CVL3WZ(17)
C COMMON /ALLZZZ/ EEEEE(17), ELLZZZ, DL2ZZZ, SHORTZ,
C * IEGRID, IPLOT
C LOGICAL SHORTZ
C DATA ELL2ZZ /201440.0/, DL1ZZZ /5.0/
C
C WRITE (25,99999)
C
C DO 10 IE=1,IEGRID
C E1ZZZ(IE) = ELL1ZZZ(IE) + ELL2ZZ + ELL3ZZZ(IE)
C D21ZZZ(IE) = D2LLZZZ(IE) + DL1ZZZ**2 + D2L3ZZZ(IE)
C DL2ZZZ(IE) = SQRT(D2L2ZZZ(IE))
C W21ZZZ(IE) = W21L2ZZZ(IE) + W21L3ZZZ(IE)
C WLZZZZ(IE) = SQRT(W2LZZZ(IE))
C DZW2LZ(IE) = DZW2L1(IE) + DZW2L3(IE)
C DW2LZZZ(IE) = SQRT(DZW2LZ(IE))
C DWL2ZZZ(IE) = 0.5*DW21ZZZ(IE)/WLZZZZ(IE)
C CVLW2Z(IE) = CVL1WZ(IE) + CVL3W2Z(IE)
C V3LZZZ(IE) = V3L3ZZZ(IE) + V3L1ZZZ(IE)
C U4LZZZ(IE) = U4L3ZZZ(IE) + U4L1ZZZ(IE) +
C * 6.0*W2L3ZZZ(IE)*W2L1ZZZ(IE)
C
C WRITE (25,99998) EEEEE(IE), E1ZZZ(IE), DL2ZZZ(IE),
C * WLZZZZ(IE, DWLZZZ(IE),
C IF (.NOT.SHORTZ) WRITE (5,99998) EEEEE(IE), E1ZZZ(IE),
C * DL2ZZZ(IE), WLZZZZ(IE, DWLZZZ(IE)
C 10 CONTINUE
C RETURN
C
C 99999 FORMAT (/^
C * 51H ENERGY EL DL WL DWL
C 99998 FORMAT (F10.0, F12.3, 5F10.3)
C END
SUBROUTINE OUTCIP

C *** PURPOSE -- WRITE ON UNIT 25 COMPONENTS OF WIDTHS ETC FOR FPL
C
COMMON /ELXXX/ ELZZZZ(17), D1ZZZZ(17),
* WLZZZZ(17), DWZZZZ(17), W2LZZZ(17), DW2LZZ(17),
* D2W2LZ(17), V3LZZZ(17), U4LZZZ(17), CVLW2Z(17)
COMMON /ELL1XX/ DWAZZZ(17), WWAZZZ(17), FWAZZZ(17),
* FWNZZZ(17), ELL1ZZ(17), DL1ZZZ(17), D2L1ZZ(17),
* W1ZZZZ(17), DW1ZZZ(17), W2L1ZZ(17), DW2L1Z(17),
* D2W2L1(17), V3L1ZZ(17), U4L1ZZ(17), DFWAZZ(17),
* CVL1W2Z(17)
COMMON /ELL3XX/ CROSSS(17), ELL3ZZ(17), DL3ZZZ(17),
* D2L3ZZ(17), WL3ZZZ(17), DWL3ZZ(17), W2L3ZZ(17),
* D2W2L3(17), D2W1L3(17), V3L3ZZ(17), U4L3ZZ(17),
* CVL3W2(17)
COMMON /A1LZZZ/ EEEEEE(17), ELL2ZZ, DL2ZZZ, SHORTZ,
* IEGRID, IPLOT
LOGICAL SHORTZ
C
WRITE (25,99999)
WRITE (25,99998)
DO 10 IE=I,IEGRID
   WRITE (25,99997) EEEEEE(IE), ELZZZZ(IE), ELL2ZZ,
** ELL2ZZ(IE), ELL3ZZ(IE)
10 CONTINUE
C
WRITE (25,99996)
WRITE (25,99995)
DO 20 IE=I,IEGRID
   WRITE (25,99994) EEEEEE(IE), ELZZZZ(IE), DLZZZZ(IE),
** DL1ZZZ(IE), DL2ZZZ, DL3ZZZ(IE)
20 CONTINUE
C
WRITE (25,99993)
WRITE (25,99992)
DO 30 IE=I,IEGRID
   D2LZZZ = DLZZZZ**2
   WRITE (25,99994) EEEEEE(IE), ELZZZZ(IE), D2LZZZ(IE),
** D2L1ZZZ(IE), D2L2ZZ, D2L3ZZ(IE)
30 CONTINUE
C
WRITE (25,99991)
WRITE (25,99990)
DO 40 IE=I,IEGRID
   WRITE (25,99989) EEEEEE(IE), ELZZZZ(IE), WLZZZZ(IE),
** WL1ZZZ(IE), WL3ZZZ(IE)
40 CONTINUE
C
WRITE (25,99988)
WRITE (25,99987)
DO 50 IE=I,IEGRID
   WRITE (25,99986) EEEEEE(IE), ELZZZZ(IE), W2LZZZ(IE),
** W2L1ZZ(IE), W2L3ZZ(IE)
50 CONTINUE
WRITE (25, 99985)
WRITE (25, 99984)
DO 60 IE=1, IEGRID
   WRITE (25, 99983) EEEEE(EI), ELLLL(EI), WLLL(EI), 
   DWLZZZ(EI)
   CONTINUE
60
WRITE (25, 99982)
WRITE (25, 99981)
DO 70 IE=1, IEGRID
   WRITE (25, 99980) EEEEE(EI), DWLZZZ(EI), DWLZZZ(EI), 
   DWLZZZ(EI)
   CONTINUE
70
WRITE (25, 99979)
WRITE (25, 99978)
DO 80 IE=1, IEGRID
   WRITE (25, 99976) EEEE(EI), CVLW2(EI), CVLW2(EI), 
   CVLW2(EI)
   CONTINUE
80
RETURN

99999 FORMAT (/4H *** COMPONENTS OF FLIGHT PATH LENGTH EL ***)
99998 FORMAT (/4H ENERGY EL ELL2 ELL11 , 
   * 10H ELL3 )
99997 FORMAT (1X, F10.0, F12.3, F13.3, 2F8.3)
99996 FORMAT (/4H *** COMPONENTS OF UNCERTAINTY ON FLIGHT PATH, 
   * 12H LENGTH ***)
99995 FORMAT (/45H ENERGY EL DL DL1 , 
   * 19H DL2 DL3 )
99994 FORMAT (1X, F10.0, F12.3, 8F10.3)
99993 FORMAT (/4H *** COMPONENTS (SQUARED) OF UNCERTAINTY ON , 
   * 2HFLIGHT PATH LENGTH ***)
99992 FORMAT (/45H ENERGY EL D2L D2L1 , 
   * 19H D2L2 D2L3)
99991 FORMAT (/4H *** COMPONENTS OF WIDTH "WL" OF FLIGHT PATH, 
   * 24H LENGTH DISTRIBUTION ***)
99990 FORMAT (/45H ENERGY EL WL WL1 , 
   * 7H WL3)
99989 FORMAT (1X, F10.0, F12.3, 3F10.3)
99988 FORMAT (/23H *** DITTO, SQUARED ***)
99987 FORMAT (/45H ENERGY EL W2L W2L1 , 
   * 8H W2L3 )
99986 FORMAT (1X, F10.0, F12.3, 3F10.3)
99985 FORMAT (/4H *** UNCERTAINTY ON WIDTH OF FPL DISTRIBUTIO, 
   * 5HN ***)
99984 FORMAT (/4H ENERGY EL WL DWL )
99983 FORMAT (1X, F10.0, F12.3, 8F10.3)
99982 FORMAT (/4H *** UNCERTAINTY ON VARIANCE OF FPL DISTRIBUT, 
   * 8HTION ***)
99981 FORMAT (/4H ENERGY DW2L DW2L1 DW2L3)
99980 FORMAT (1X, F10.0, 8F12.3)
99979 FORMAT (/4H *** COVARIANCE ON FLIGHT PATH LENGTH DISTRIBUT, 
   * 6HUTION)
SUBROUTINE TIME
C
C *** PURPOSE -- GENERATE MOMENTS OF TIME DISTRIBUTIONS
C
COMMON /THER/ TZEROZ, DTZERO, AFWHMZ, DAFWHM, BCHNLZ,
* DBCHNL, TSCALE
COMMON /T1XXX/  T1ZZZ, DT1ZZZ, D2T1ZZ, WT1ZZZ, DWT1ZZ,
* WT1ZZ, DW2T1, D2W2T1, V3T1ZZ, U4T1ZZ, CVT1W2
COMMON /T2XXX/  T2ZZZZ, DT2ZZZ, D2T2ZZ, WT2ZZZ, DWT2ZZ,
* WT2ZZ, DW2T2, D2W2T2, V3T2ZZ, U4T2ZZ, CVT2W2
COMMON /TIMEXX/ TIMEZZ(17), DTIMEZ(17), DTIME(17),
* WTIMEZ, DWTIME, W2TIME, DW2TIM, D2W2T, V3TIME,
* U4TIME, CVT2W2
C
COMMON /ALLZZZ/ EEEEE(17), ElliszZ, DL2ZZZ, SHORTZ,
* IEGRID, IPLOT
C
LOGICAL SHORTZ
C
DATA AFWHMZ /7.5/, DAFWHM /0.5/, BCHNLZ /1.0/, DBCHNL
* /0.0010/, DTZERO /0.288675/, TSCALE /0.00002/
C
DTZERO IS 1/SQRT(12.0)
C
C
C *** T-SUB-1
D2T1ZZ = 0.0
WT1ZZZ = AFWHMZ/(2.0*SQRT(2.0*ALOG(2.0)))
DWT1ZZ = AFWHMZ/(2.0*SQRT(2.0*ALOG(2.0)))
WT1ZZ = WT1ZZZ*WT1ZZZ
DW2T1 = 2.0*WT1ZZZ*DW2T1
D2W2T1 = DW2T1**2
V3T1ZZ = 0.0
U4T1ZZ = 3.0*WT1ZZZ
C
C *** PRINTOUT FOR T-SUB-1 AND T-SUB-2
WRITE (25,99999)
WRITE (25,99998) WT1ZZZ, DWT1ZZ, AFWHMZ, DAFWHM,
* BCHNLZ, DBCHNL, DTZERO, DTZERO, TSCALE, TSCALE
C
C *** T-SUB-2
D2T2ZZ = 0.0
WT2ZZZ = BCHNLZ/SQRT(12.0)
WT2ZZZ = BCHNLZ*BCHNLZ/12.0
DWT2ZZ = DBCHNL/SQRT(12.0)
DW2T2 = (BCHNLZ/6.0)*DBCHNL
D2W2T2 = DW2T2**2
V3T2ZZ = 0.0
U4T2ZZ = (BCHNLZ/2.0)**4/5.0
C
C *** COMBINE THE TWO
W2TIME = W2T2ZZ + W2T1ZZ
V3TIMEZ = SQRT(W2TIME)
V3TIME = 0.0
U4TIME = U4T2ZZ + U4T1ZZ + 6.0*W2T2ZZ*W2T1ZZ
D2W2TZ = D2W2T2 + D2W2T1
D2WTIM = SQRT(D2W2TZ)
DWTIME = D2WTIM/(2.0*W2TIMEZ)
CVTW2Z = CVT1W2 + CVT2W2

WRITE (25,9999/)
WRITE (25,99996) WT1ZZZ, DW1TZZ, W2T1ZZ, DW2T1Z, D2W2T1,
*     V3T1ZZ, U4T1ZZ
WRITE (25,99995) WT2ZZZ, DW2T2Z, W2T2ZZ, DW2T2Z, D2W2T2,
*     V3T2ZZ, U4T2ZZ
WRITE (25,99994) WT2TIMEZ, DW2TIME, W2TIME, DW2TIM, D2W2TZ,
*     V3TIME, U4TIME
RETURN

C
99999 FORMAT (///30H ***** TIME RESOLUTION *****)
99998 FORMAT (/37H ASTD = WIDTH (STD.DEV.) OF BURST =, F8.5,
*     5H +/-, F8.5/35H AFWHM = WIDTH (FWHM) OF BURST
*     2H =, F8.5, 5H +/-, F8.5/23H CHNL = WIDTH OF DETE,
*     14HCTOR-CHANNEL =, F8.5, 5H +/-, F8.5/10H DTZERO =,
*     37H TSCALE = RELATIVE TIME-UNCERTAINTY =, UPF8.5,
*     2H =, 1PE10.3)
99997 FORMAT (/45H ?? WT? DWT? W2T? ,
99996 FORMAT (12H ONE , 7G12.4)
99995 FORMAT (12H TWO , 7G12.4)
99994 FORMAT (12H TOTAL , 7G12.4)
END
SUBROUTINE ENERGY

*** PURPOSE -- GENERATE ENERGY UNCERTAINTY AND MOMENTS OF THE
    ENERGY-RESOLUTION FUNCTION

COMMON /TOTAL/ ENZZZZ(17), FUDGEZ(17), DENZZZ(17),
    * DZENZZ(17), WENZZZ(17), W2ENZZZ(17), DWENZZZ(17),
    * DW2ENZ(17), D2W2ENZ(17), W2ELZZZ(17), WWTIM2Z(17),
    * W2EL3Z(17), W2TIM3(17), W2ELTI(17), W2EL4Z(17),
    * W2TIM4(17), PW2W2L(17), PW2W2T(17), PW2LZ(17),
    * PW2TIM(17)

COMMON /ELLLLL/ ELZZZZ(17), DLZZZZ(17), DDLZZZ(17),
    * WLZZZZ(17), DWLZZZ(17), W2LZZZ(17), DW2LZ(17),
    * D2W2LZ(17), WLZZZ(17), ULZZZZ(17), CVW2ZZ (17)

COMMON /TINTER/ TZEROZ, DTZEROZ, AFWHMZ, DAFWHRM, BCHNLZ,
    * DBCHNL, TSACLE
COMMON /TIMEX/ TIMEZZ(17), DTIMEZ(17), D2TIMEZ(17),
    * WTIMEZ, DTIMEZ, W2TIMEZ, D2W2TIMEZ, V3TIMEZ,
    * U4TIMEZ, CVW2ZZ

COMMON /ALLLLL/ EEEEEE(17), EL1ZZZ, DLZZZZ, SHORTZ,
    * IEGRID, IPILOT

WRITE (25,99999)
WRITE (25,99998)
DO 30 IEG = 1, IEGRID
    TMTZER = 72.29*ELZZZZ(IE)/SQRT(EEEEEE(IE))
    TIMEZZ(IE) = TMTZER
    D2TIMEZ(IE) = DTZERO**2 + (TMTZER*TSCALE)**2
    DTIMEZ(IE) = SQRT(D2TIMEZ(IE))
    FUDGE1 = W2LZZZ(IE)/ELZZZZ(IE)**2
    FUDGE2 = 3.0*W2TIMEZ/TMTZER**2
    FUDGE3 = 5.0*U4TIMEZ/TMTZER**4
    FUDGE4 = 3.0* (W2LZZZ(IE)/ELZZZZ(IE)**2)**2 *
        (W2TIMEZ/TMTZER**2)
    FUDGEZ(IE) = FUDGE1 + FUDGEZ + FUDGE3 + FUDGE4
    ENZZZZ(IE) = EEEEEE(IE)*(1.0+FUDGEZ(IE))
    TMTZZZ = 72.29/ELZZZZ(IE)/SQRT(ENZZZZ(IE))

D2ENZZZ(IE) = EEEEEE(IE)**2*4.0 *
    ( D2LZZZZ(IE)/ELZZZZ(IE)**2 + D2TIMEZ(IE)/TMTZER**2 )
    D2ENZZZ(IE) = SQRT(D2ENZZZ(IE))

AAELZZZ = 2.0*WLZZZZ(IE)/ELZZZZZ(IE)
WWEI2Z(IE) = AAELZ**2
AAATIM2 = 2.0*W2TIMEZ/TMTZER
WWTIM2Z(IE) = AAATIM2**2
W2EL3Z(IE) = 4.0*W2LZZZ(IE)/ELZZZZ(IE)**3
AAEL3Z = SQRT(W2EL3Z(IE))
WWTIM3(IE) = 0.0
AATIM3 = 0.0
WELTIE(IE) = 3.0*WHELZ(IE)*WWTIM2(IE)
AAELTI = SQRT(WELTI(IE))
WHEL4Z(IE) = (U4LZZ(IE)-W2LZZZ(IE)**2)/ELZZZ(IE)**4
AAEL4Z = SQRT(WHEL4Z(IE))
WWTIM4(IE) = (25.0*U4TIME-9.0*W2TIME**2)/TMTZER**4
*AATIM4 = SQRT(WWTIM4(IE))
W2ENZZ(IE) = EEEEE(IE)**2*(WHELZ(IE)+WWTIMZ(IE))
   +WHELZ(IE)+WTELZ(IE)+WWTIMZ(IE)+WHEL4Z(IE)
   +WWTIM4(IE))
WENZZZ(IE) = SQRT(W2ENZZ(IE))
C
XAWLZ = (1.0+3.0*WWTIMZ(IE)-WHELZ(IE)/8.0)
   *4.0/ELZZZZZ(IE)**2
PW2W2(IE) = XAWLZ*EEEE(IE)**2
XAWST = (1.0+3.0*WHELZ(IE)-9.0*WWTIM2(IE)/8.0)
   *4.0/TMTZER**2
PW2WT(IE) = XAWST*EEEE(IE)**2
XAEZZ = -2.0*(WHELZ(IE)**(1.0+3.0*WWTIM2(IE))
   +WHELZ(IE)**1.5 + WHEL4Z(IE)**2.0) / ELZZZ(IE)
PW2LZZ(IE) = XAEZZ*EEEE(IE)**2 + 4.0*W2ENZZ(IE)/ELZZZ(IE)
XATIME = -2.0*(WWTIM2(IE)**(1.0+3.0*WHELZ(IE))
   +2.0*WWTIM4(IE)) / TMTZER
PW2TIM(IE) = XATIME*EEEE(IE)**2 - 4.0*W2ENZZ(IE)/TMTZER
C
D2W2N(IE) = PW2W2L(IE)**2*DW2LZ(IE) + PW2W2(IE)**2*DW2TIM**2 + PW2LZZ(IE)**2*DW2LZ**2 + PW2TIM(IE)**2*DW2TIME**2 + 2.0*PW2W2L(IE)*PW2LZZ(IE)**CWLW2Z(IE) + 2.0*PW2W2T(IE)*PW2TIM(IE)**CVTW2Z
D2W2(IE) = SQRT(D2W2N(IE))
D2WZEN(IE) = D2W2N(IE)/(2.0*W2ENZZ(IE))
C
WRITE (25,99997) TMTZER, TMTZZZ, FUDGE1, FUDGE2, FUDGE3,
   * FUDGE4, AAELZZ, AATIM2, AAEL3Z, AATIM3, AAELTI,
   * AAEL4Z, AATIM4
30 CONTINUE
C
WRITE (25,99994)
DO 40 IE=1,IEGRID
   WRITE (25,99993) EEEEE(IE), W2ENZZ(IE), WHELZ(IE),
      * WWTIM2(IE), WHELZ(IE), WTELZ(IE), WWTIM3(IE),
      * WHEL4Z(IE), WWTIM4(IE)
40 CONTINUE
C
WRITE (25,99996)
DO 50 IE=1,LEGGRID
   XX = DENZZZ(IE)/ENZZZZ(IE)
   YY = WENZZZ(IE)/ENZZZZ(IE)
   ZZ = DWENZZ(IE)/WENZZZ(IE)
   WRITE (25,99995) EEEEEE(IE), ENZZZZ(IE), FUDGEZ(IE),
      * DENZZZ(IE), XX, WENZZZ(IE), YY, DWENZZ(IE), ZZ
50 CONTINUE
RETURN
C
99999 FORMAT (///28H ***** FINAL RESULTS *****)
99998 FORMAT (///43H T-TO T-TOX FUDGE1 FUDGE2 FUD, *
      * 49HGE3 FUDGE4 AAEI2 AATIM3 AAEI4 AATIM4)
99997 FORMAT (1X, 2F9.0, 1P11G9.2)
99994 FORMAT (/// ENERGY W2EN WWEL2 WWTIM2 WW *
      *EL3 WWELT1 WWTIM3 WWEL4 WWTIM4)
99993 FORMAT (F10.0, 1P8G12.4)
99996 FORMAT (///44H ENERGY ADJ. ENE. FUDGE DEN, *
      * 49H DEN/EN WEN WEN/EN DWEN , *
      *12H DWEN/WEN)
99995 FORMAT (1X, F10.0, F11.0, 1P7G12.3)
END
SUBROUTINE DISPLAY

*** PURPOSE -- WRITE TABLES FOR IM REPORT

COMMON /TOTAL/ ENNZZZ(17), FUDGEZ(17), DENVZZZ(17),
*       D2ENZZ(17), WENZZZ(17), W2ENZZ(17), DWENZZ(17),
*       D2W2NZ(17), D2W2EN(17), W2ENZZ(17), WW1T1M2(17),
*       WW2L3Z(17), WW2L1M3(17), WW2L1T1(17), WW2L1Z(17),
*       WW2L1M4(17), PW2W2L(17), PW2W2T1(17), PW2L2ZZ(17),
*       PW2T1M(17)

COMMON /ELXXX/ ELZZZZ(17), DLZZZZ(17), D2LZZZ(17),
*       DLWZZZ(17), DWLZZZ(17), W2LZZZ(17), DW2LZZZ(17),
*       D2W21Z(17), V3LZZZ(17), U4LZZZ(17), CVLW2Z(17)

COMMON /ELLXXX/ DWAZZZ(17), WWAZZZ(17), FWAZZZ(17),
*       FTNZZZ(17), ELLZZZ(17), DL1ZZZ(17), D2LZZZ(17),
*       W1LZZZ(17), DW1LZZZ(17), W2L1ZZZ(17), DW2L1ZZZ(17),
*       D2W2L1(17), V3L1ZZZ(17), U4L1ZZZ(17), DF1A1ZZZ(17),
*       CVL1W2(17)

COMMON /ELLXXX/ CROSSS(17), ELL3ZZ(17), DL3ZZZ(17),
*       D2L3ZZ(17), W1L3ZZ(17), W1L3ZZ(17), W2L3ZZ(17),
*       D2W31ZZ(17), D2W3L3(17), DL3ZZZ(17), U4L3ZZZ(17),
*       CVL3W2(17)

COMMON /TIMEXX/ TIMEXZ(17), DT1MEZ(17), D2TIM(17),
*       WTIMEZ, DTME, W2IME, D2W2IME, D2W2T, V3TIME, U4TIME,
*       CTVW2Z

COMMON /ALL2ZZ/ EEEEEE(17), ELL2ZZ, DL2ZZZ, SHORTZ,
*       IEGRD, IPILOT

WRITE (2/,99999)
WRITE (2/,99998)
DO 10 IE=1,IEGRD
    ELLP13 = ELL1ZZZ(IE) + ELL2ZZ(IE)
    DL1P13 = SQRT(DL2LZZZ(IE) + DL2L3ZZ(IE))
    DEOE = DENVZZZ(IE)/EEEEEE(IE)
    WRITE (2/,99997) EEEEEE(IE), DENVZZZ(IE), DEOE, ELZZZZ(IE),
        DLZZZZ(IE), ELL1ZZZ(IE), DL1ZZZ(IE), ELL3ZZZ(IE),
        DL3ZZZ(IE), EL1P13, DL1P13
10 CONTINUE

WRITE (2/,99996)
WRITE (2/,99995)
DO 20 IE=1,IEGRD
    WOE = WENZZZ(IE)/EEEEEE(IE)
    WRITE (2/,99994) EEEEEE(IE), WENZZZ(IE), WOE, DENVZZZ(IE),
        W1ZZZZ(IE), DW1ZZZ(IE), W1L1ZZZ(IE), DWL1ZZZ(IE),
        W1L3ZZZ(IE), DW1L3ZZZ(IE)
20 CONTINUE
C
WRITE (2,99993)
WRITE (2,99992)
DO 30 IE=1,IEGRID
   WRITE (27,99991) EEEEEE(IE), W2IZZ(IE), V3LZZZ(IE),
   * U4IZZZ(IE)
30 CONTINUE
C
WRITE (27,99990)
WRITE (27,99989)
DO 40 IE=1,IEGRID
   B1 = 4.0*W2IZZ(IE)/ELZZZ(IE)**2
   D2B1 = B1**2 * (D2W2IZ(IE)/W2IZZ(IE)**2
   * -4.0*CVLWZZ(IE)/(W2IZZ(IE)*ELZZZ(IE))
   * 4.0*D2IZZZ(IE)/ELZZZ(IE)**2)
   DB1 = SQRT(D2B1)
   B2 = 4.0*W2TIME/(72.29*ELZZZ(IE)**2
   D2B2 = B2**2 * (D2W2T2/2TIME**2 +
   * 4.0*D2IZZZ(IE)/ELZZZ(IE)**2)
   DB2 = SQRT(D2B2)
   B1P2E = B1 + B2*EEEEE(IE)
   D2BBEZ = D2B1 + 16.0/TIMEZZ(IE)**4 *
   (D2W2TZ+4.0*D2TIME(IE)/TIMEZZ(IE)**2)
   DBBEZZ = SQRT(D2BBEZ)
   W2ENZ = W2ENZZ(IE)/EEEEEE(IE)**2
WRITE (27,99988) EEEEEE(IE), W2ENZZ(IE), B1, DB1, B2,
   * DB2, B1P2E, DBBEZZ, W2ENZ
40 CONTINUE
RETURN
C
99999 FORMAT (/// 8H TABLE 1)
99998 FORMAT (/31H ENERGY DELTA E DE/ENERGY L,
   * 48H DELTA L L1 DELTA L1 L3 DELTA L3,
   * 25H L1+L3 DELTA (L1+L3))
99997 FORMAT (F13.3, F11.3, P12.3, OPF12.3, 7F10.3)
99996 FORMAT (/// 8H TABLE 2)
99995 FORMAT (///45H ENERGY WEN WEN/ENERGY DWE,
   * 49HN WL DWL WL1 DWL1 W,
   * 12HL3 DWL3)
99994 FORMAT (F13.3, F11.3, P12.3, UP/F10.3)
99993 FORMAT (/// 8H TABLE 3)
99992 FORMAT (/41H ENERGY W2L V3L U4L)
99991 FORMAT (1X, F10.0, 3F10.2)
99990 FORMAT (/// 8H TABLE 4)
99989 FORMAT (/41H ENERGY W2EN B1,
   * 48H DB1 B2 DB2 B1+B2*E,
   * 30H D(B1+B2*E) W2EN/ENERGY**2)
99988 FORMAT (1X, F10.0, 1P8E13.3)
END
SUBROUTINE PLT

** PURPOSE -- MAKE PLOT FILES FOR ENERGY-DEPENDENT
PARAMETERS OF TANTALUM-WATER DISTRIBUTION

COMMON /ELL1XX/ DWAZZZ(17), WWAZZZ(17), FWAZZZ(17),
* FTNZZZ(17), ELL1ZZ(17), DL1ZZZ(17), D2L1ZZ(17),
* WL1ZZZ(17), DW1ZZZ(17), W2L1ZZ(17), DW2L1ZZ(17),
* D2W1L1(17), V311ZZ(17), U411ZZ(17), DFWAZZ(17),
* CVL1W2(17)

COMMON /ALLZZZ/ EEEEEE(17), ELL2ZZ, DL2ZZZ, SHORTZ,
* IEGRID, IPLOT
LOGICAL SHORTZ

IU = 21
INS = 9
IFB = 3
MODE = 3
NDSTRT = 0
NEW = 1
CALL ODFIO(IU, 'TAH20.0DF', IFB, NEW, INS, IEGRID, MODE,
* NDSTRT, -1, 0)
CALL OUTODF(IU, IFB, INS, 1, MODE, NDSTRT, 1, IEGRID,
* EEEEEE, 1)
CALL OUTODF(IU, IFB, INS, 2, MODE, NDSTRT, 1, IEGRID,
* FTNZZZ, 1)
CALL OUTODF(IU, IFB, INS, 3, MODE, NDSTRT, 1, IEGRID,
* FWAZZZ, 1)
CALL OUTODF(IU, IFB, INS, 4, MODE, NDSTRT, 1, IEGRID,
* DWAZZZ, 1)
CALL OUTODF(IU, IFB, INS, 5, MODE, NDSTRT, 1, IEGRID,
* WWAZZZ, 1)
CALL OUTODF(IU, IFB, INS, 6, MODE, NDSTRT, 1, IEGRID,
* ELL1ZZZ, 1)
CALL OUTODF(IU, IFB, INS, 7, MODE, NDSTRT, 1, IEGRID,
* DL1ZZZ, 1)
CALL OUTODF(IU, IFB, INS, 8, MODE, NDSTRT, 1, IEGRID,
* WL1ZZZ, 1)
CALL OUTODF(IU, IFB, INS, 9, MODE, NDSTRT, 1, IEGRID,
* DW1ZZZ, 1)
CLOSE (UNIT=21)
RETURN
END
SUBROUTINE PLOT13

*** PURPOSE -- MAKE PLOT FILES FOR ENERGY-DEPENDENT L1+L3 AND UNCERTAINTY THEREON

COMMON /ELL3XX/ CROSSS(17), WWAZZZ(17), FWAZZZ(17),
* FTWZZZ(17), ELL1ZZ(17), DL1ZZZ(17), D2L1ZZ(17),
* WL1ZZZ(17), DWL1ZZ(17), W2L1ZZ(17), DW2L1ZZ(17),
* D2W2L1(17), V3L1ZZ(17), U4L1ZZ(17), DFWAZZZ(17),
* CVL1WZ(17)

COMMON /ELL3XX/ CROSSLZ(17), ELL3ZZ(17), DL3ZZZ(17),
* D2L3ZZ(17), WL3ZZZ(17), DWL3ZZ(17), W2L3ZZ(17),
* DW2L3(17), D2W2L3(17), V3L3ZZ(17), U4L3ZZ(17),
* CVL3W2(17)

COMMON /ALIZZZ/ EEEEEEE(17), ELL2ZZ, DL2ZZZ, SHORTZ,
* IEGRID, IPILOT
LOGICAL SHORTZ

DIMENSION DDUMMY(17)

IU = 21
INS = 3
IFB = 3
MODE = 3
NDSTRT = 0
NEW = 1
CALL ODFIO(IU, 'L1PL3.ODF', IFB, NEW, INS, IEGRID, MODE,
* NDSTRT, -1, 0)

DO 10 IE=1,IEGRID
   DDUMMY(IE) = ELL1ZZ(IE) + ELL3ZZ(IE)
10 CONTINUE

CALL OUTODF(IU, IFB, INS, 1, MODE, NDSTRT, 1, IEGRID,
* EEEE, 1)
CALL OUTODF(IU, IFB, INS, 2, MODE, NDSTRT, 1, IEGRID,
* DDUMMY, 1)

DO 20 IE=1,IEGRID
   DDUMMY(IE) = SQRT(D2L1ZZ(IE)+D2L3ZZ(IE))
20 CONTINUE

CALL OUTODF(IU, IFB, INS, 3, MODE, NDSTRT, 1, IEGRID,
* DDUMMY, 1)
CLOSE UNIT=21
RETURN
END
SUBROUTINE PLOT(E, FTNZZZ, FWAZZZ, D, WTA, ELL1ZZZ,
* WL1ZZZ, IPL, U, W, UW2, R)
C
C *** PURPOSE -- GENERATE ODF FILE FOR TA-WATER DISTRIBUTION
C
DIMENSION Q(501), RHO(501)
DOUBLE PRECISION FILE(l7)
DATA FILE /10H0 .ODF, 10H1 .ODF, 10H2 .ODF,
* 10H5 .ODF, 10H10 .ODF, 10H20 .ODF,
* 10H50 .ODF, 10H100 .ODF, 10H200 .ODF,
* 10H300 .ODF, 10H400 .ODF, 10H500 .ODF,
* 10H1000 .ODF, 10H2000 .ODF, 10H5000 .ODF,
* 10H10000 .ODF, 10H20000 .ODF/
DATA IQQ /501/
C
QMIN = AMIN1(-U+W),-ELL1ZZZ-3.*WL1ZZZ)
QMAX = AMAX1(0.,-ELL1ZZZ+3.*WL1ZZZ)
TYPE 99, QMIN, QMAX
DELIQ = (QMAX-QMIN)/500.
DO 10 IQ=1,IQQ
    Q(IQ) = -(QMIN+DELIQ*(IQ-1))
10 CONTINUE
C
AH20 = FWAZZZ/(SQRT(2.*3.141592654)*WTA)
ATA = FTNZZZ*2./(3.141592654*R*R)
DEN = 2.*WTA**2
DO 20 IQ=1,IQQ
    RHO(IQ) = EXP(-(Q(IQ)-D)**2/DEN)*AH20
    IF (Q(IQ),GE.UW2-R .AND. Q(IQ),LE.UW2+R) RHO(IQ) =
    * RHO(IQ) + ATA*SQRT((R-UW2+Q(IQ))*(R+UW2-Q(IQ)))
20 CONTINUE
C
DO 30 IQ=1,IQQ
    Q(IQ) = -Q(IQ)
30 CONTINUE
C
IU = 22
INS = 2
IFB = 3
MODE = 3
NDSTRT = 0
NEW = 1
CALL ODFIO(IU, FILE(IPL), IFB, NEW, INS, IQQ, MODE,
* NDSTRT, -1, 0)
CALL OUTODF(IU, IFB, INS, 1, MODE, NDSTRT, 1, IQQ, Q, 1)
CALL OUTODF(IU, IFB, INS, 2, MODE, NDSTRT, 1, IQQ, RHO, 1)
CLOSE(UNIT=1U)
RETURN
END
SUBROUTINE PLTDET(E, EL, W, S, R, T, IPL)
C
C *** PURPOSE -- GENERATE ODF FILES FOR DETECTOR-END DISTRIBUTION (L3)
C
DIMENSION Q(501), RHO(501)
DOUBLE PRECISION FILE(17)
DATA FILE /10H0 .DET, 10H1 .DET, 10H2 .DET, 10H5 .DET,
* 10H10 .DET, 10H15 .DET, 10H20 .DET, 10H30 .DET,
* 10H40 .DET, 10H50 .DET, 10H60 .DET, 10H70 .DET,
* 10H80 .DET, 1OH100 .DET, 1OH200 .DET, 1OH300 .DET,
* 1OH400 .DET, 1OH500 .DET, 1OH600 .DET, 1OH700 .DET,
* 1OH800 .DET, 1OH900 .DET, 1OH1000 .DET, 1OH1100 .DET/
DATA IQQ /501/
C
QMIN = EL-3.0*W
QMAX = EL+3.0*W
C TYPE 99, QMIN, QMAX
DELQ = (QMAX-QMIN)/500.
DO 10 IQ=1,IQQ
   Q(IQ) = (QMIN+DELQ*(IQ-1))
10 CONTINUE
C
RS = R*S
RST = RS*T
A = RS/(1.0-EXP(-RST))
DO 20 IQ=1,IQQ
   IF (Q(IQ).GE.0. .AND. Q(IQ).LE.T)
      RHO(IQ) = A*EXP(-RS*Q(IQ))
   20 CONTINUE
C
IU = 22
INS = 3
IFB = 3
MODE = 3
NDSTRT = 0
NEW = 1
CALL ODF1O(IU, FILE(IPL), IFB, NEW, INS, IQQ, MODE,
* NDSTRT, -1, 0)
CALL OUTODF(IU, IFB, INS, 1, MODE, NDSTRT, 1, IQQ, Q, 1)
CALL OUTODF(IU, IFB, INS, 2, MODE, NDSTRT, 1, IQQ, RHO, 1)
CLOSE(UNIT=IU)
RETURN
END
SUBROUTINE QQQTAH

C WHITE DEBUG-PRINT FOR TANTALUM-WATER ARRAYS.
COMMON /ELL1XX/ DWAZZZ(17), WWAZZZ(17), FWAZZZ(17),
* FTNZZZ(17), ELL1ZZ(17), DL1ZZZ(17), DL1ZZZ(17),
* W2L1ZZZ(17), W2L1ZZZ(17), W2L1ZZZ(17), DW2L1ZZ(17),
* D2W2L1(17), V3L1ZZZ(17), U4L1ZZZ(17), DF2WAZ(17),
* CV1LW2(17),
COMMON /TAWATV/ PERDWA, PERWVA, WATER1, WATER2, WATER3,
* WWATR1, WWATR2, WWATR3, UUUUUV, WWWWWW, SSSSSS,
* DELUUU, DELWWW, DELSSS, RRERRR, DELRRR, EMMMNN,
* PEREMM, ABSUNC
DIMENSION AAAAAA(19)
EQUIVALENCE (AAAAAA(1), PERDWA)
CALL QQQZZZ(6HDWA , 6HWWA , 6HFVA , 6HFTN , DWAZZZ,
* WWAZZZ, FWAZZZ, FTNZZZ)
CALL QQQZZZ(6HEL1L , 6HD1L , 6HD2L1 , 6HW1L , ELL1ZZ,
* DL1ZZZ, DL1ZZZ, W2L1ZZ)
CALL QQQZZZ(6HDWL1 , 6H2L1 , 6HD2L1 , 6HD2L1 , 6L1ZZ,
* W2L1ZZ, W2L1ZZ, W2L1ZZ)
CALL QQQZZZ(6HV3L1 , 6HU4L1 , 6HDFVA , 6HCV1LW2, V3L1ZZ,
* U4L1ZZZ, DF2WAZZ, CV1LW2)
CALL QQQXXX(AAAAAA, 19)
RETURN
END

SUBROUTINE QQQDET

C *** PURPOSE -- DEBUG-PRINT FOR DETECTOR ARRAYS (L3)
COMMON /ELL3XX/ CROSSS(17), ELL3ZZ(17), DL3ZZZ(17),
* DL3ZZZ(17), W3L3ZZ(17), DWL3ZZ(17), W2L3ZZ(17),
* DWL3ZZ(17), D2W2L3(17), V3L3ZZZ(17), U4L3ZZZ(17),
* CV1L3ZZ(17)
COMMON /DETECV/ ALMBDA, THICKN, PERLMB, PERTHN, PERCRO
DIMENSION AAAAAA(5)
EQUIVALENCE (AAAAAA(1), ALMBDA)
CALL QQQZZZ(6HCROSSS, 6HEL1L , 6HD3L , 6HD2L3 , CROSSS,
* ELL3ZZ, DL3ZZZ, DL3ZZZ)
CALL QQQZZZ(6HW3L , 6H4L3 , 6HDFVA , 6HCV1LW2, V3L1ZZ,
* U4L1ZZZ, DF2WAZZ, CV1LW2)
CALL QQQXXX(AAAAAA, 5)
RETURN
END
SUBROUTINE QQQCOM

C *** PURPOSE -- DEBUG-PRINT LENGTH ARRAYS
COMMON /ELXXX/ ELZZZZ(17), DLZZZZ(17), D1LZZZ(17),
* WLZZZZ(17), DW1ZZZ(17), W21ZZZ(17), DW21ZZZ(17),
* D2W1LZZ(17), V3LZZZ(17), U4LZZZ(17), CV1W2Z(17)
COMMON /ALLZZZ/ EEEEE(17), EL2ZZZ, DL2ZZZ, SHORTZ,
* IEGRID, IPLOT
DIMENSION AAAAAA(2)
EQUIVALENCE (AAAAAAA(1), ELL2ZZZ)
CALL QQZZZZ(ELHEL , HDL , HD2L , HDW , ELZZZZ,
* DLZZZZ , D2LZZZ, WLZZZZ)
CALL QQZZZZ(HW2L , HDWL , HDW2L , HDW2L , W21ZZZ,
* DW1ZZZ, DW21ZZZ, D2W1LZ)
CALL QQXXXX(AAAAAA, 2)
RETURN
END

C

C

SUBROUTINE QQQTIM

C *** PURPOSE -- DEBUG-PRINT TIME ARRAYS
COMMON /TIER/ TZ00Z, DT00Z, AFH0M, DAFH0M, BCHNLZ,
* DBCHNL , TSCALE
COMMON /T1XXX/ T1ZZZZ , DT1ZZZ , DT1ZZZ , W1ZZZZ , DW1ZZZ,
* W21ZZZ , DW21ZZZ , D2W1ZZZ , V3ZZZZ , U4T1ZZZ , CV1W2Z
COMMON /T2XXX/ T2ZZZZ , DT2ZZZ , DT2ZZZ , W21ZZZ , DW21ZZZ,
* W2T2ZZZ , DW2T2ZZZ , D2W2T2ZZZ , V3TZZZZ , U4T2ZZZ , CV2WZ
COMMON /TIMEXX/ TIMEZZZ(17), DTIMEXX(17), DTMEZZ(17),
* WTIME, DWTIME, W2TIME, DW2TIME, D2TIME, V3TIME,
* U4TIME, CV2W2Z
DIMENSION AAAAAAA(6), BBBB(11), CCCCC(11), DDDDDD(8)
EQUIVALENCE (AAAAAAA(1), T00Z), (BBBBBBB(1), T1ZZZZ),
* (CCCCCCC(1), T2ZZZZ), (DDDDDD(1), WTIME)
CALL QQXXXX(AAAAAA, 6)
CALL QQXXXX(BBBB, 11)
CALL QQXXXX(CCCCC, 11)
CALL QQXXXX(6HTIME , HDTIME , HD2TIME, 6H , TIMEZZZ,
* DTIME, D2TIME, X)
CALL QQXXXX(DDDDDD, 8)
RETURN
END
SUBROUTINE QQQENE
C *** PURPOSE -- DEBUG-PRINT ENERGY ARRAYS
COMMON /TOTAL/ ENZZZZ(17), FUDGEZ(17), DENZZZ(17),
* D2ENZZ(17), WENZZZ(17), W2ENZZ(17), DWNZZ(17),
* DW2ENZ(17), D2W2EN(17), WWL2Z(17), WWTM2(17),
* WT3Z(17), WWTM3(17), WELT(17), WEL4Z(17),
* WWTM4(17), PW2W2L(17), PW2W2T(17), PW2LZZZ(17),
* PW2TIM(17)
COMMON /ALLZZZ/ EEEEEE(17), ELLZZZ, DL2ZZZ, SHORTZ,
* IEGRID, IPILOT
CALL QQQZZZ(6HEN, 6HFUDGE, 6HDEN, 6HD2EN, ENZZZ,
* FUDGEZ, DENZZZ, D2ENZZ)
CALL QQQZZZ(6HEN, 6HEWEN, 6HDWEN, 6HD2WEN, WENZZZ,
* W2ENZZ, DWNENZZ, DW2ENZ)
CALL QQQZZZ(6HDW2EN, 6HWWEHL, 6HWWL2, 6HWWL3, 6D2WEN,
* WEL2Z, WWTM2, WEL3Z)
CALL QQQZZZ(6HWWL3, 6HWWTM, 6HWWL4, 6HWWL5, 6WWL3,
* WELT, WEL4Z, WWTM4)
CALL QQQZZZ(6HPW2W2L, 6HPW2W2T, 6HPW2L, 6HPW2TIM, PW2W2L,
* PW2W2T, PW2LZZ, PW2TIM)
RETURN
END

C
C
C

SUBROUTINE QQQZZZ(A, B, C, D, E, F, G, H)
C *** PURPOSE -- DEBUG-PRINT ARRAYS
DOUBLE PRECISION A, B, C, D, BLANK
COMMON /ALLZZZ/ EEEEEE(17), ELLZZZ, DL2ZZZ, SHORTZ,
* IEGRID, IPILOT
DIMENSION A(I), B(I), C(I), D(I)
DATA BLANK /'.'/
WRITE (26,99999) A, B, C, D
DO 10 I=1,IEGRID
   IF (D.NE.BLANK) WRITE (26,99998) E(I), F(I), G(I), H(I)
   IF (D.EQ.BLANK) WRITE (26,99997) E(I), F(I), G(I)
10 CONTINUE
RETURN

99999 FORMAT (/1X, 4(10X, A10))
99998 FORMAT (1X, 4F20.10, 2X, 1P4G10.2)
99997 FORMAT (1X, 3F20.10, 22X, 1P3G10.2)
END

C
C
C

SUBROUTINE QQQXXX(A, N)
C *** PURPOSE -- DEBUG-PRINT VARIABLES
DIMENSION A(N)
WRITE (26,99999) A
RETURN

99999 FORMAT (/5F20.10/, (5F20.10))
END

C
C
C
### APPENDIX D. OUTPUT FROM FLIP1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>UUU</td>
<td>Distance to surface of TA target</td>
<td>0.000 ± 2.000</td>
</tr>
<tr>
<td>WWW</td>
<td>Width of TA target</td>
<td>36.600</td>
</tr>
<tr>
<td>DELWWW</td>
<td>Error on WWW</td>
<td>5.492 ± 15.0%</td>
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<tr>
<td>SSS</td>
<td>Estimate of multiple scattering in TA target</td>
<td>6.000</td>
</tr>
<tr>
<td>DELSSS</td>
<td>Error on SSS</td>
<td>1.800 ± 30.0%</td>
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<tr>
<td>UW2</td>
<td>UUU + WWW/2 + SSS = Distance to mean of TA target</td>
<td>24.300</td>
</tr>
<tr>
<td>RRR</td>
<td>Radius of circular distribution in TA target</td>
<td>9.200</td>
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<tr>
<td>DELRRR</td>
<td>Error on RRRRRR</td>
<td>1.840 ± 20.0%</td>
</tr>
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</table>

**DWA(E)** = Mean of distribution in water = \(22.8 - 1.60 \times \ln(E) + 0.283 \times \ln(E)^2\)

**WWA(E)** = Std dev of distribution in water = \(10.0 - 0.63 \times \ln(E) + 0.112 \times \ln(E)^2\)

**PERDWA** = Uncertainty on mean DWA (E) = 10.0 ± 20.0%

**PERWWA** = Uncertainty on std dev WWA (E) = 10.0 ± 20.0%

**ABSUNC** = Absolute uncertainty on fraction from TA and from H2O = 0.00

**EMM** = Energy at which the two fractions are equal = 300000.0

**PEREMM** = Relative uncertainty in EMM = \(\frac{DEL(EMM)}{EMM}\) = 30.0 ± 20.0%
<table>
<thead>
<tr>
<th>EN</th>
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<th>D2W2L1</th>
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<th>CR1LW2</th>
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RHO = .004700 +/- .000094
THICKN = 19.0000 +/- 0.9500
PERCRO = 5.0000 % UNCERTAINTY ON CROSS SECTION

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<th>DWL3</th>
<th>V3L3</th>
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<th>CRL3W2</th>
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**** Combine the Three Lengths ****

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<th>DWL</th>
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*** Components of Flight Path Length EL ***

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### COMPONENTS OF UNCERTAINTY ON FLIGHT PATH LENGTH

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***** TIME RESOLUTION *****

ASTD = WIDTH (STD.DEV.) OF BURST = 3.18496 +/- 0.21233
AFWHM = WIDTH (FWHM) OF BURST = 7.50000 +/- 0.50000
CHNL = WIDTH OF DETECTOR-CHANNEL = 1.00000 +/- 0.00100
DTZERO = UNCERTAINTY IN TZERO = 0.28867 = 2.887E-01
TSCALE = RELATIVE TIME-UNCERTAINTY = 0.00002 = 2.000E-05

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