How important are Response-Parameter Correlations?

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Outline

- Motivation
- Review of notation and formalism
- One response one parameter formalism
- One response two parameters formalism
- Numerical example: one-group bare critical sphere
- Summary and conclusions

Motivation(1)

- “The principle deficiency we wanted to remove
- was an under prediction of reactivity –
- for instance, the calculated keff for Godiva,
- a fast critical assembly based upon HEU
- in a spherical configuration,
- was 0.9966, compared to experiment of 1.0000.”

Motivation(2)

- “Prompt $\bar{V}$ is based on covariance analysis of experimental data,
- with consideration of
- consistency with fast critical benchmark experiments.”

Deviation

$$d = \bar{r}(p) - r$$

$$C_d = \left\langle \delta(\bar{r} - r)\,\delta(\bar{r} - r)^{\dagger} \right\rangle =$$

$$S C_p S^{\dagger} - S C_{pr} - C_{rp} S^{\dagger} + C_r$$

Adjustment formalism

$$r' = r + \left(C_r - C_{rp}S^\dagger\right)C_d^{-1}d$$

$$p' = p + \left(C_{pr} - C_pS^\dagger\right)C_d^{-1}d$$

$$C_{r'} = C_r - \left(C_r - C_{rp}S^\dagger\right)C_d^{-1}\left(C_r - SC_{pr}\right)$$

$$C_{p'} = C_p - \left(C_{pr} - C_pS^\dagger\right)C_d^{-1}\left(C_{rp} - SC_p\right)$$

$$C_{p'r'} = C_{pr} - \left(C_{pr} - C_pS^\dagger\right)C_d^{-1}\left(C_r - SC_{pr}\right)$$

One Parameter - One Response

$$C_p = \sigma_p^2 \qquad C_r = \sigma_r^2$$

$$C_{pr} = C_{rp} = \rho \sigma_p \sigma_r$$

$$S^\dagger = S$$

$$C_d = S^2 \sigma_p^2 - 2S\rho\sigma_p\sigma_r + \sigma_r^2$$

$$r' = r + \left(\sigma_r - \rho\sigma_p S\right)\sigma_r C_d^{-1} d$$

$$p' = p + \left(\rho\sigma_r - \sigma_p S\right)\sigma_p C_d^{-1} d$$

$$\sigma_{r'}^2 = S^2\sigma_{p'}^2 = S^2\left(1-\rho^2\right)\sigma_p^2\sigma_r^2 C_d^{-1}$$

$$\sigma_{p'}^2 = \left(1-\rho^2\right)\sigma_p^2\sigma_r^2 C_d^{-1}$$

$$\rho' = 1$$

Yehuda Yeivin
1924 - 2007

One Response – Two Parameters

$$C_r = \sigma_r^2 \qquad C_{rp} = C_{pr}^{\dagger} = \left(\rho\sigma_r\sigma_{p_1}, 0\right) \quad C_{p_{11}} = \sigma_{p_1}^2 \quad C_{p_{22}} = \sigma_{p_2}^2$$

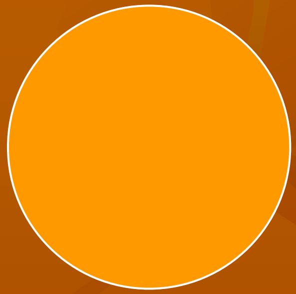
$$r' = r + \left(\sigma_r - \rho S_1 \sigma_{p_1}\right)\sigma_r \frac{d}{C_d}$$

$$\sigma_{r'}^2 = \sigma_r^2 \frac{\left[\left(1-\rho^2\right)S_1^2\sigma_{p_1}^2 + S_2^2\sigma_{p_2}^2\right]}{C_d}$$

$$p_1' = p_1 + \left(\rho\sigma_{p_1}\sigma_r - S_1\sigma_{p_1}^2\right)\frac{d}{C_d}$$

$$p_2' = p_2 - S_2\sigma_{p_2}^2 \frac{d}{C_d} \qquad \sigma_{p_1'}^2 = \sigma_{p_1}^2 \frac{\left[\left(1-\rho^2\right)\sigma_r^2 + S_2^2\sigma_{p_2}^2\right]}{C_d}$$

Homogeneous multiplying sphere



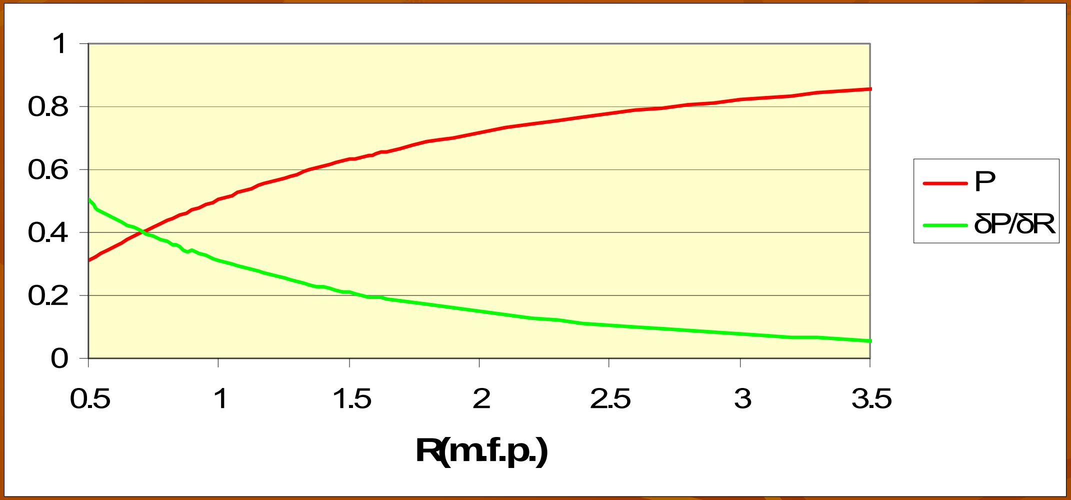
$$R(m.f.p.) = \Sigma R$$

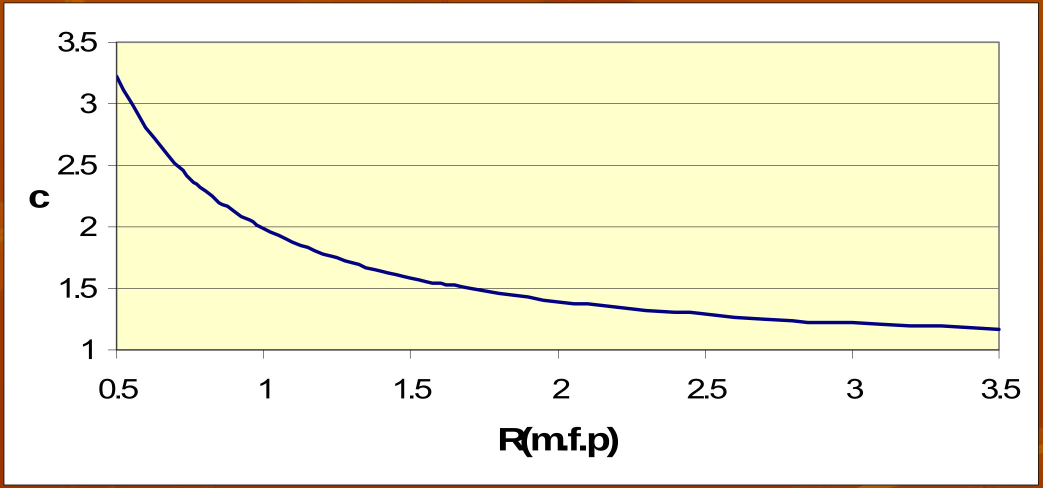
$$c = \frac{\bar{\nu}\sigma_f + 2\sigma_{n,2n} + 3\sigma_{n,3n} + \ldots + \sigma_s}{\sigma_{tot}}$$

$$\Omega \cdot \nabla \psi + \Sigma \psi = \frac{1}{\gamma}\left\{S_s[\psi] + S_f[\psi]\right\}$$

$$\gamma = c\,P(\Sigma R)$$

Universal Reactivity Curve





Numerical example

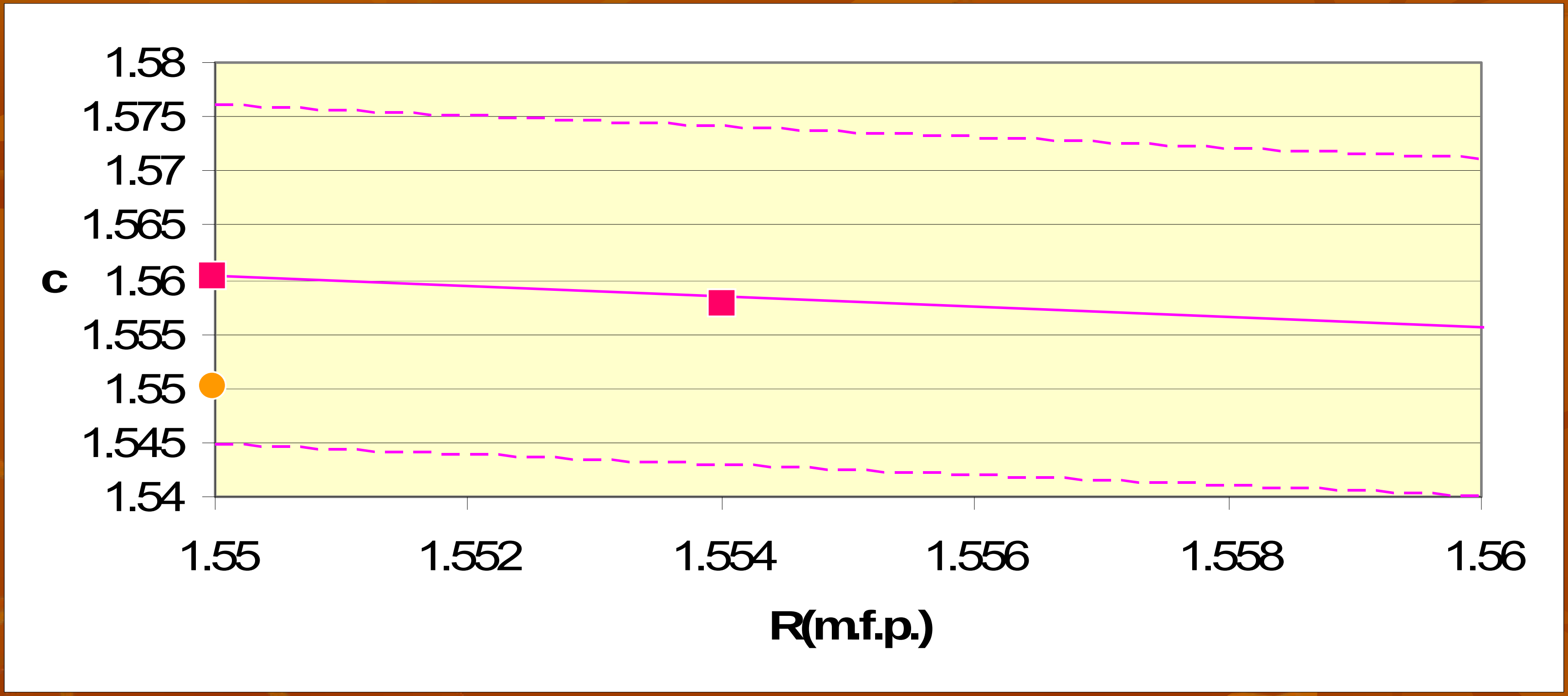
$$\gamma = 1 \pm 0.001\left(0.1\%\right)$$

$$c = 1.55 \pm 0.0155\left(1\%\right)$$

$$R\left(m.f.p.\right) = 1.55 \pm 0.0155\left(1\%\right)$$

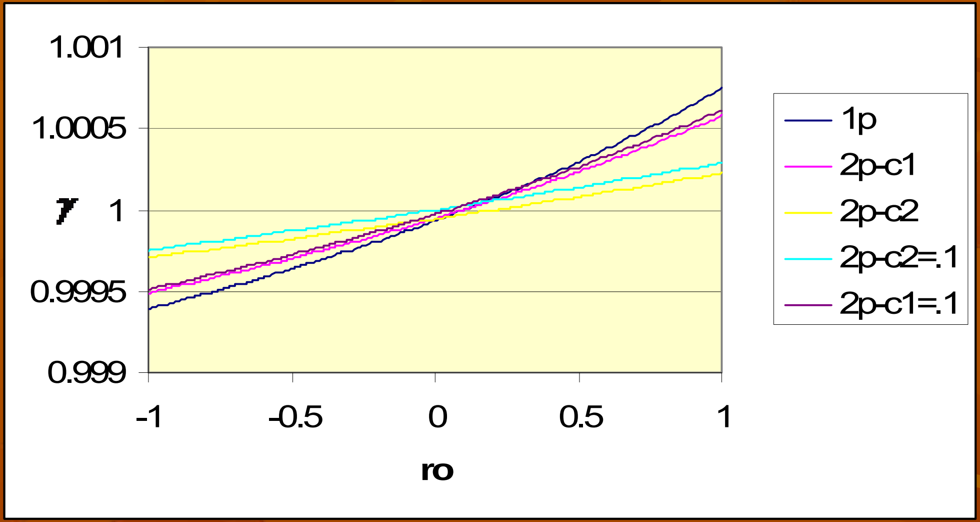
$$\overline{\gamma} = cP = c \cdot \frac{1}{c(1.55)} = 1.55 \cdot 0.64086 = 0.99333$$

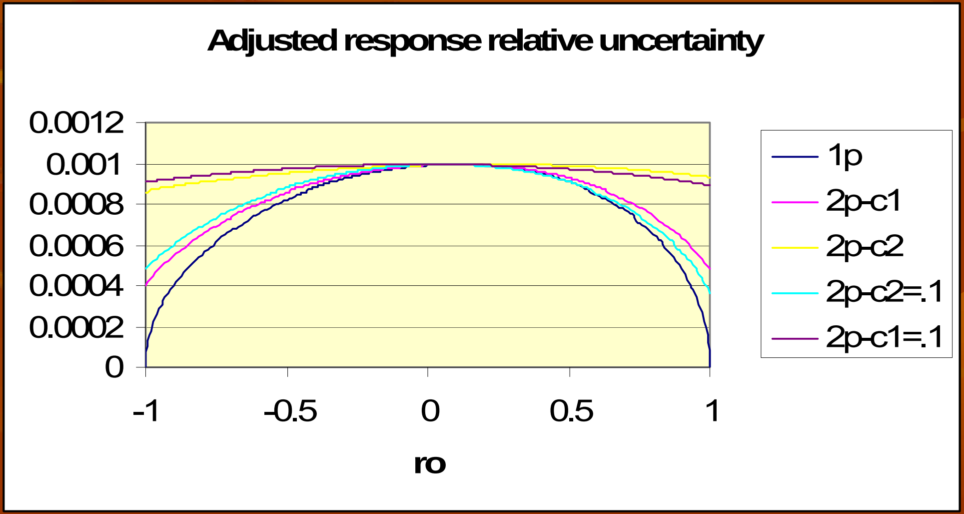
Adjustment



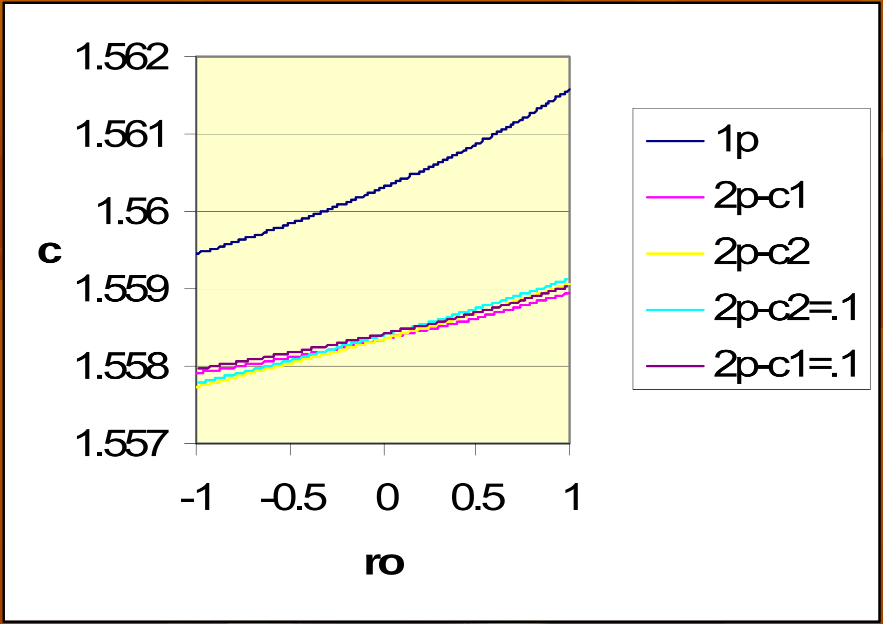
1.58
1.575
1.57
1.565
c
1.56
1.555
1.55
1.545
1.54
1.55
1.552
1.554
1.556
1.558
1.56
R(m.f.p.)

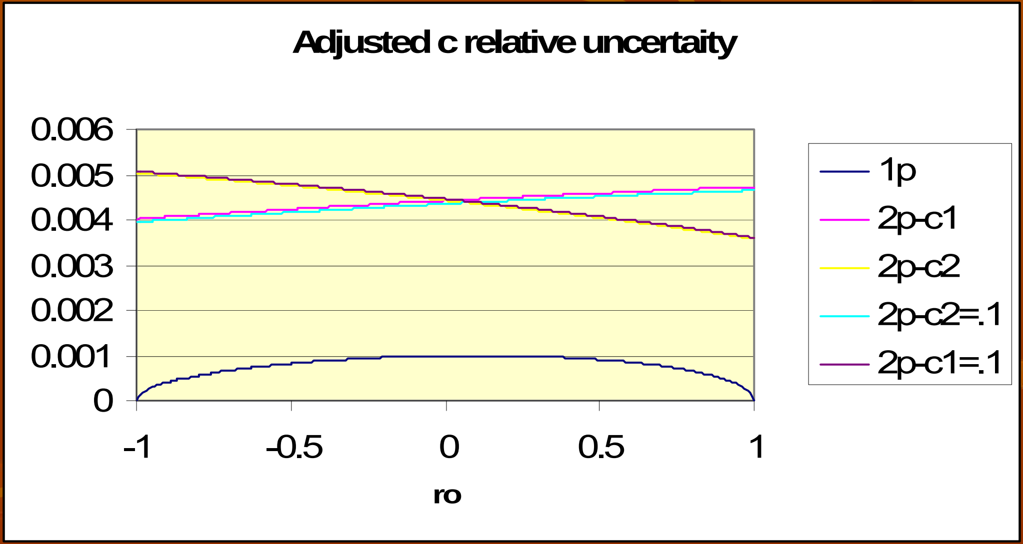
Adjusted response



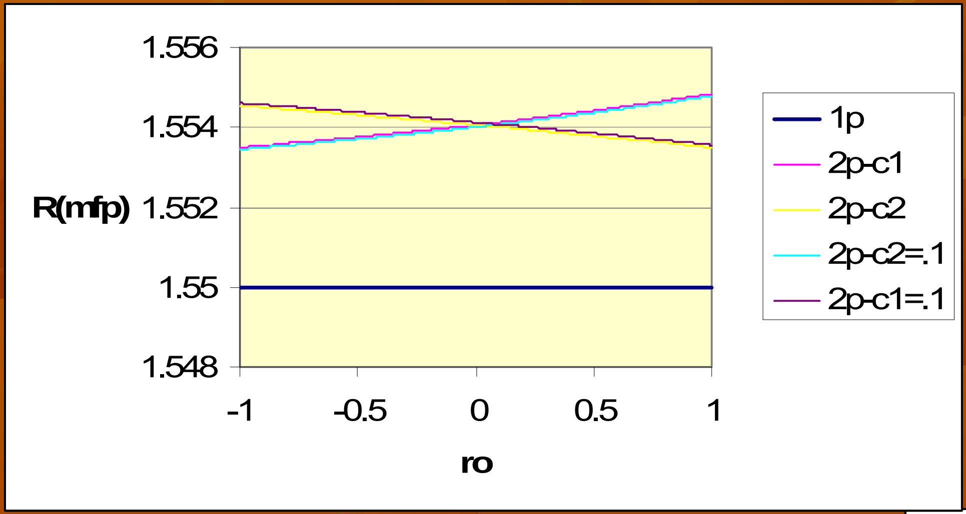


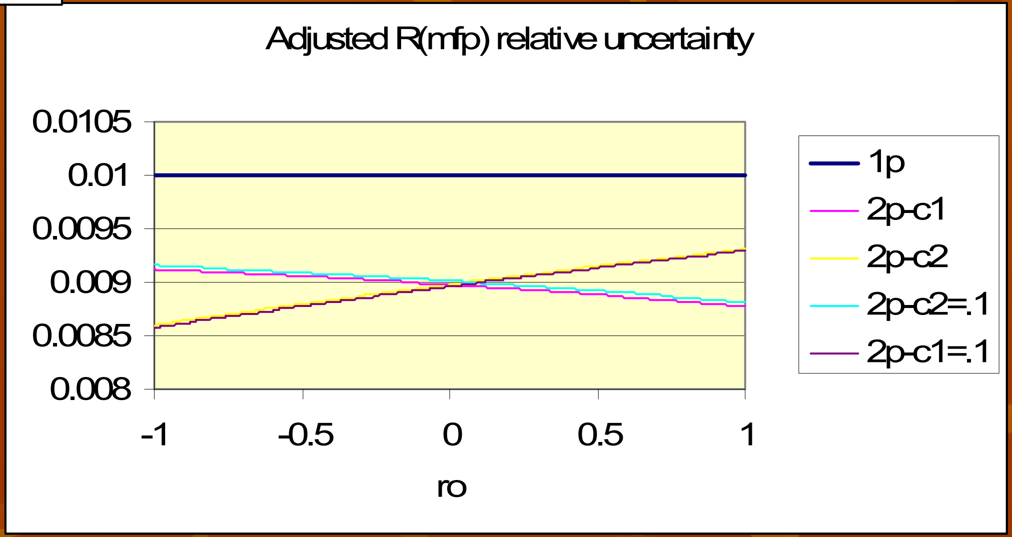
Adjusted collision multiplication, c



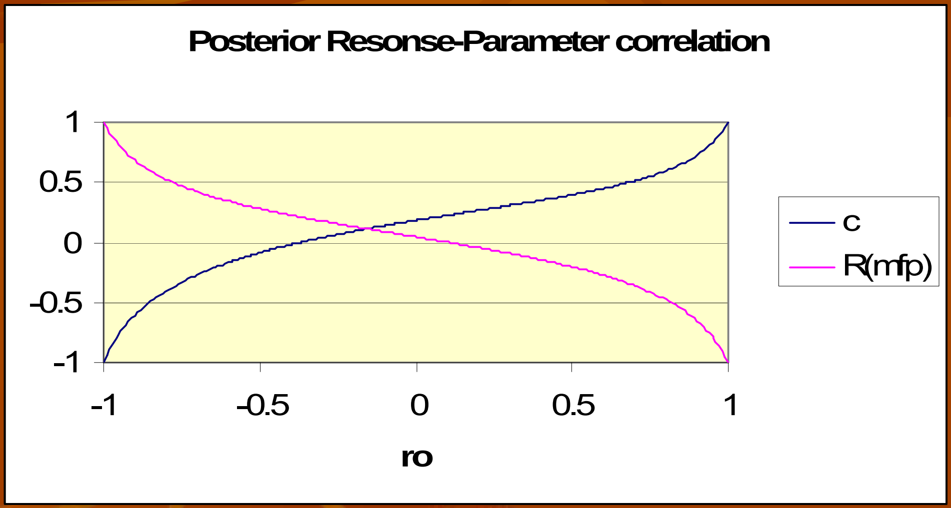


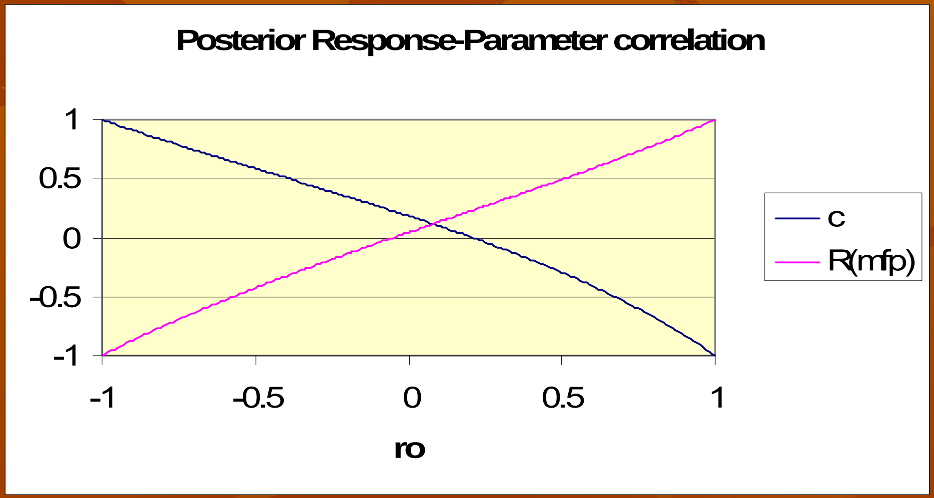
Adjusted R(m.f.p.)





Adjusted Response-Parameter Correlation Coefficient





Summary and Conclusions

- One-response/one-parameter case may lead to wrong conclusions
- With more than one parameter, regardless of the response-parameters correlation:
  - Response's uncertainty reduced
  - Parameters' uncertainty reduced
  - Adjustment is worth its while and is recommended