



# Cross Section Uncertainties in the Thermal and Resonance Regions

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# Background

- ❑ Increasing interest in uncertainties of neutron resonances required for advanced reactor systems.
- ❑ This information is available in the Atlas of Neutron Resonances.
- ❑ In the Atlas, resonance information reproduces the cross sections and resonance integrals, but not their uncertainties.
- ❑ Present study initiated to achieve consistencies between measured and calculated uncertainties.

# Background

- ❑ The Subgroup 26 of the International Working Party on evaluation cooperation is interested in 53 materials comprising 19 actinides and 34 structural, coolant and moderator materials.
- ❑ Out of these NNDC produced covariances for 35 materials.
- ❑ In the present project, emphasis is placed on the uncertainties of the thermal capture (fission) cross sections for the following.
- ❑  $^{19}\text{F}$ ,  $^{23}\text{Na}$ ,  $^{28}\text{Si}$ ,  $^{52}\text{Cr}$ ,  $^{55}\text{Mn}$ ,  $^{56}\text{Fe}$ ,  $^{57}\text{Fe}$ ,  $^{58}\text{Ni}$ ,  $^{90}\text{Zr}$ ,  $^{91}\text{Zr}$ ,  $^{92}\text{Zr}$ ,  $^{94}\text{Zr}$ ,  $^{166}\text{Er}$ ,  $^{167}\text{Er}$ ,  $^{168}\text{Er}$ ,  $^{170}\text{Er}$ ,  $^{206}\text{Pb}$ ,  $^{207}\text{Pb}$ ,  $^{208}\text{Pb}$ ,  $^{209}\text{Bi}$ ,  $^{233}\text{U}$ ,  $^{234}\text{U}$ ,  $^{236}\text{U}$ ,  $^{237}\text{Np}$ ,  $^{238}\text{Pu}$ ,  $^{240}\text{Pu}$ ,  $^{241}\text{Pu}$ ,  $^{242}\text{Pu}$ ,  $^{241}\text{Am}$ ,  $^{242\text{m}}\text{Am}$ ,  $^{243}\text{Am}$ ,  $^{242}\text{Cm}$ ,  $^{243}\text{Cm}$ ,  $^{244}\text{Cm}$ ,  $^{245}\text{Cm}$ .

# Methodology

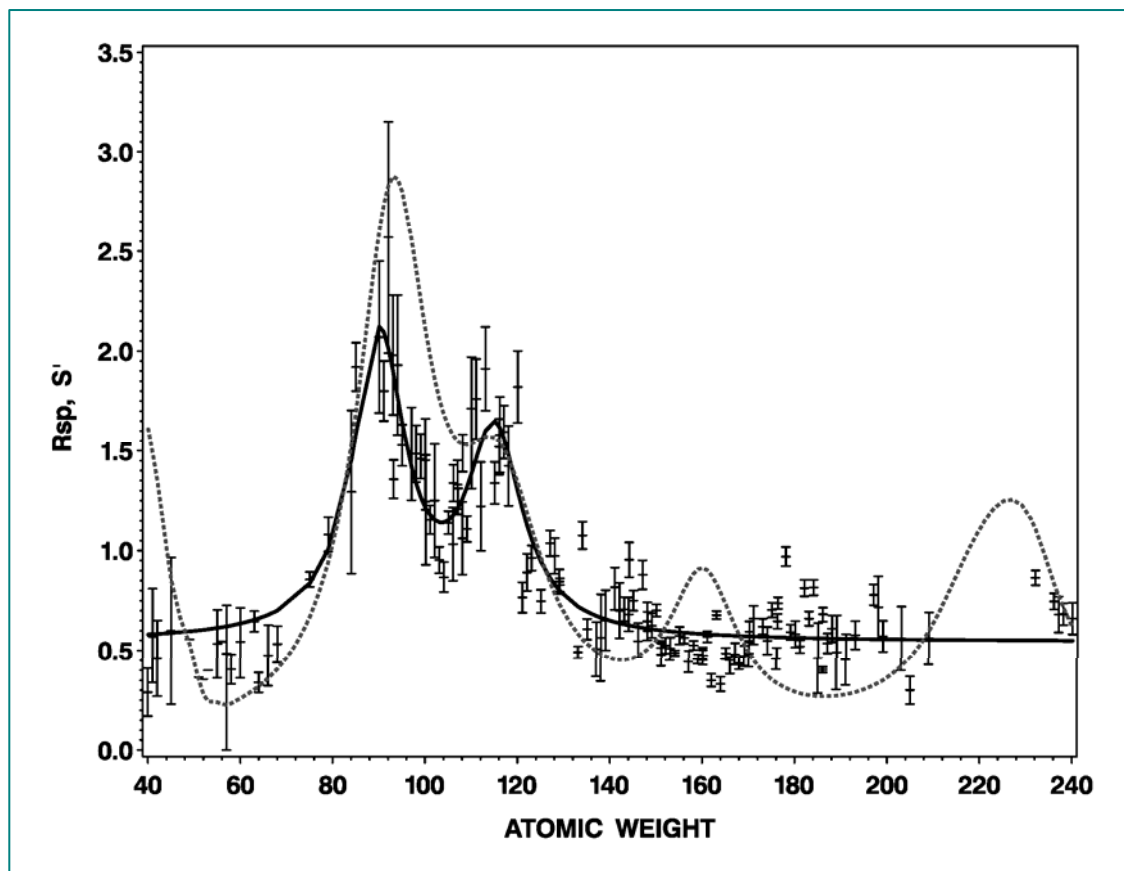
- ❑ Inspect the Atlas electronic files . Fill in the missing information, such as  $\Gamma_\gamma$  for s- and p-wave resonances and estimated uncertainties.
- ❑ Convert the Atlas file to ENDF format by applying PTANAL program; Random assignments of spins to follow  $(2J+1)$  law.
- ❑ Run PSY325 to calculate the contributions from the various resonances to the thermal cross section and resonance integral with their uncertainties read from the Atlas file.
- ❑ Compare with the uncertainty of the measured thermal cross section. Carry out changes in the uncertainties of RP, iterate the procedure until conversion is achieved.

# Methodology

- Attribute reported uncertainty to that of bound levels and/or low-lying positive energy resonances.

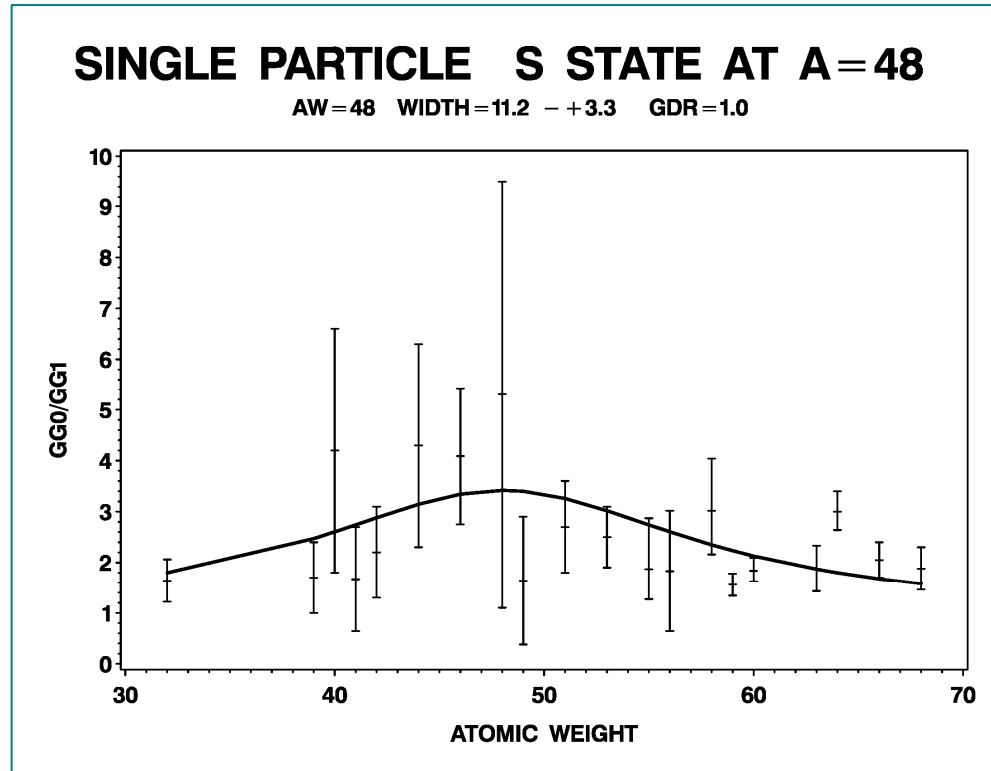
# $\Gamma_{\gamma_1} / \Gamma_{\gamma_0}$ and $S_1$ Versus Mass Number

## Systematics



# $\Gamma_{\gamma_0} / \Gamma_{\gamma_1}$ Versus Mass Number

## Systematics



# Methodology

$$\left(\Delta\sigma_{\gamma}\right)^2 = \left(\frac{\partial\sigma_{\gamma}}{\partial\Gamma_n}\Delta\Gamma_n\right)^2 + \left(\frac{\partial\sigma_{\gamma}}{\partial\Gamma_{\gamma}}\Delta\Gamma_{\gamma}\right)^2 + \left(\frac{\partial\sigma_{\gamma}}{\partial\Gamma_f}\Delta\Gamma_f\right)^2 + \left(\frac{\partial\sigma_{\gamma}}{\partial E_o}\Delta E_o\right)^2$$

$$\left(\Delta\sigma_f\right)^2 = \left(\frac{\partial\sigma_f}{\partial\Gamma_n}\Delta\Gamma_n\right)^2 + \left(\frac{\partial\sigma_f}{\partial\Gamma_{\gamma}}\Delta\Gamma_{\gamma}\right)^2 + \left(\frac{\partial\sigma_f}{\partial\Gamma_f}\Delta\Gamma_f\right)^2 + \left(\frac{\partial\sigma_f}{\partial E_o}\Delta E_o\right)^2$$



# Methodology

$$\left(\Gamma_\gamma, \Gamma_n\right) = 2 \left( \frac{\partial \sigma_\gamma}{\partial \Gamma_n} \Delta \Gamma_n \right) \left( \frac{\partial \sigma_\gamma}{\partial \Gamma_\gamma} \Delta \Gamma_\gamma \right) \rho\left(\Gamma_\gamma, \Gamma_n\right)$$

$$\left(\Gamma_f, \Gamma_\gamma\right) = 2 \left( \frac{\partial \sigma_f}{\partial \Gamma_f} \Delta \Gamma_f \right) \left( \frac{\partial \sigma_f}{\partial \Gamma_\gamma} \Delta \Gamma_\gamma \right) \rho\left(\Gamma_f, \Gamma_\gamma\right)$$

# $^{55}_{25}\text{Mn}$ Thermal Cross Sections and Resonance Properties

$^{55}_{25}\text{Mn}$

$^{55}_{25}\text{Mn}$

## THERMAL CROSS SECTIONS

$\sigma_{\gamma}^0 = 13.36 \pm 0.05 \text{ b}$   
 $\sigma_s = 2.06 \pm 0.03 \text{ b}$   
 $\sigma_{\text{coh}} = 1.734 \pm 0.017 \text{ b}$   
 $\sigma_{\text{incoh}} = 0.6 \pm 0.2 \text{ b}$   
 $b_{\text{coh}} = -3.750 \pm 0.018 \text{ fm}$   
 $b_{\text{coh}(+)} = -2.36 \pm 0.03 \text{ fm}$   
 $b_{\text{coh}(-)} = -5.69 \pm 0.03 \text{ fm}$   
 $R' = 4.5 \pm 0.4 \text{ fm}$

## RESONANCE PROPERTIES

$D_0 = 2.42 \pm 0.15 \text{ keV}$   
 $D_1 = 1.1 \pm 0.1 \text{ keV}$   
 $S_0 = 4.2 \pm 0.8$   
 $S_1 = 0.31 \pm 0.05$   
 $\langle \Gamma_{\gamma 0} \rangle = 0.75 \pm 0.15 \text{ eV}$   
 $\langle \Gamma_{\gamma 1} \rangle = 0.40 \pm 0.10 \text{ eV}$   
 $I_{\gamma} = 13.4 \pm 0.5 \text{ b}$

# $^{55}\text{Mn}$ Resonance Parameters

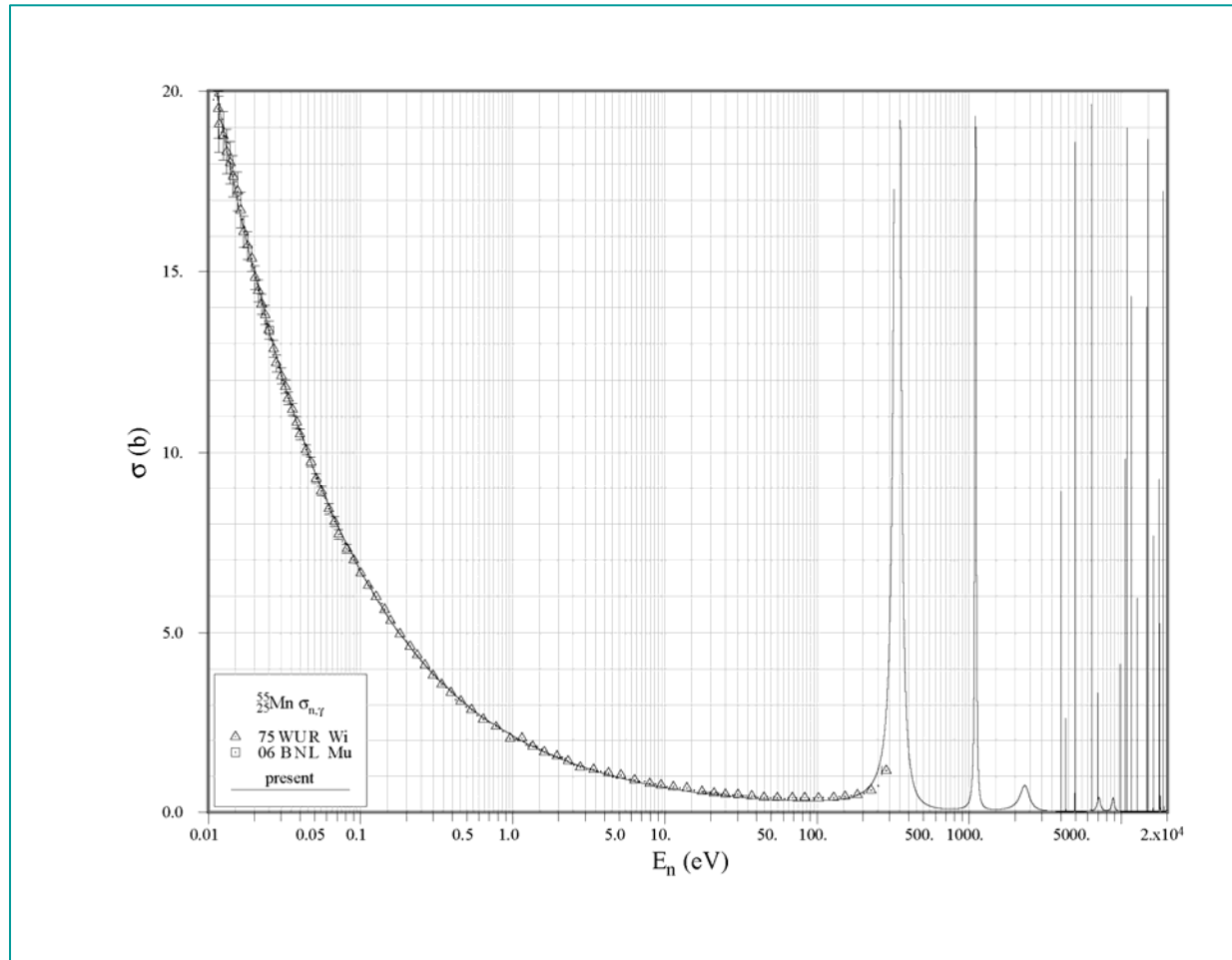
RESONANCE PARAMETERS						
$I^\pi = 5/2^-$		$\%Abn = 100$		$S_n = 7270.45 \pm 0.13 \text{ keV}$		
$\sigma_\gamma(+)= 1.75 \text{ b}$		$\sigma_\gamma(-) = 8.28 \text{ b}$		$\sigma_\gamma(B) = 3.33 \text{ b}$		
$E_0$ (keV)	J	$l$	$2g\Gamma_n$ (eV)	$\Gamma_\gamma$ (eV)	$2g\Gamma_n^0$ (eV)	$2g\Gamma_n^1$ (eV)
-1.615	3	0		(0.75)	49.217	
-0.2028	2	0		(0.75)	0.0805	
0.3373 $\pm$ 0.0010	2	0	18.3 $\pm$ 0.4	0.31 $\pm$ 0.02	0.998 $\pm$ 0.022	
1.099 $\pm$ 0.002	3	0	18.0 $\pm$ 0.8	<sup>a</sup> 0.435 $\pm$ 0.100	0.543 $\pm$ 0.024	
2.327 $\pm$ 0.005	3	0	460 $\pm$ 24	0.34 $\pm$ 0.13	9.45 $\pm$ 0.49	
4.000 $\pm$ 0.008	[2]		0.00124 $\pm$ 0.00014			0.00387 $\pm$ 0.00044
4.302 $\pm$ 0.009	1		0.00384 $\pm$ 0.00024			0.0108 $\pm$ 0.0007
4.939 $\pm$ 0.010	1		0.932	0.290 $\pm$ 0.013		2.12
6.330 $\pm$ 0.006	1		0.232 $\pm$ 0.011			0.365 $\pm$ 0.017
6.960 $\pm$ 0.006	1		0.0073 $\pm$ 0.0011			0.0100 $\pm$ 0.0015
7.102 $\pm$ 0.006	2	0	332 $\pm$ 8	1.03 $\pm$ 0.08	3.94 $\pm$ 0.09	
8.815 $\pm$ 0.007	3	0	432 $\pm$ 12	0.82 $\pm$ 0.08	4.60 $\pm$ 0.13	
9.789 $\pm$ 0.008	1		0.014 $\pm$ 0.001			0.0115 $\pm$ 0.0008
10.615 $\pm$ 0.009	1		0.0432 $\pm$ 0.0018			0.0315 $\pm$ 0.0013
10.895 $\pm$ 0.009	1		0.24 $\pm$ 0.02			0.17 $\pm$ 0.01
11.629 $\pm$ 0.010	1		0.067 $\pm$ 0.003			0.043 $\pm$ 0.002
12.688 $\pm$ 0.010	1		1.1 $\pm$ 0.1	0.323 $\pm$ 0.035		0.61 $\pm$ 0.06
14.705 $\pm$ 0.012	1		0.096 $\pm$ 0.003			0.043 $\pm$ 0.001
14.948 $\pm$ 0.012	1		0.358 $\pm$ 0.020			0.157 $\pm$ 0.009
16.167 $\pm$ 0.013	1		0.704 $\pm$ 0.056			0.275 $\pm$ 0.022
17.725 $\pm$ 0.015	1		0.0700 $\pm$ 0.0048			0.0238 $\pm$ 0.0016
17.802 $\pm$ 0.015	3	0	12.8 $\pm$ 2.3	0.740 $\pm$ 0.032	0.096 $\pm$ 0.017	
17.990 $\pm$ 0.015	2	0	54 $\pm$ 5	0.47 $\pm$ 0.02	0.40 $\pm$ 0.04	
18.812 $\pm$ 0.016	1		1.43 $\pm$ 0.24			0.447 $\pm$ 0.075
19.127 $\pm$ 0.016	[2]		0.0113 $\pm$ 0.0017			0.00344 $\pm$ 0.00052
20.225 $\pm$ 0.017	1		0.36 $\pm$ 0.02			0.101 $\pm$ 0.006
20.488 $\pm$ 0.017	[2]		0.0038 $\pm$ 0.0020			0.0010 $\pm$ 0.0006

Note: on basis of the present resonance parameters, 85% of thermal neutron capture takes place in channel spin J=2.

<sup>a</sup> A measured radiative width of 310 meV results in a calculated resonance capture integral of 11.7 barns, which is at variance with a measured weighted average value of 13.4 $\pm$ 0.5 b.

# $^{55}\text{Mn}$ Capture Cross Section

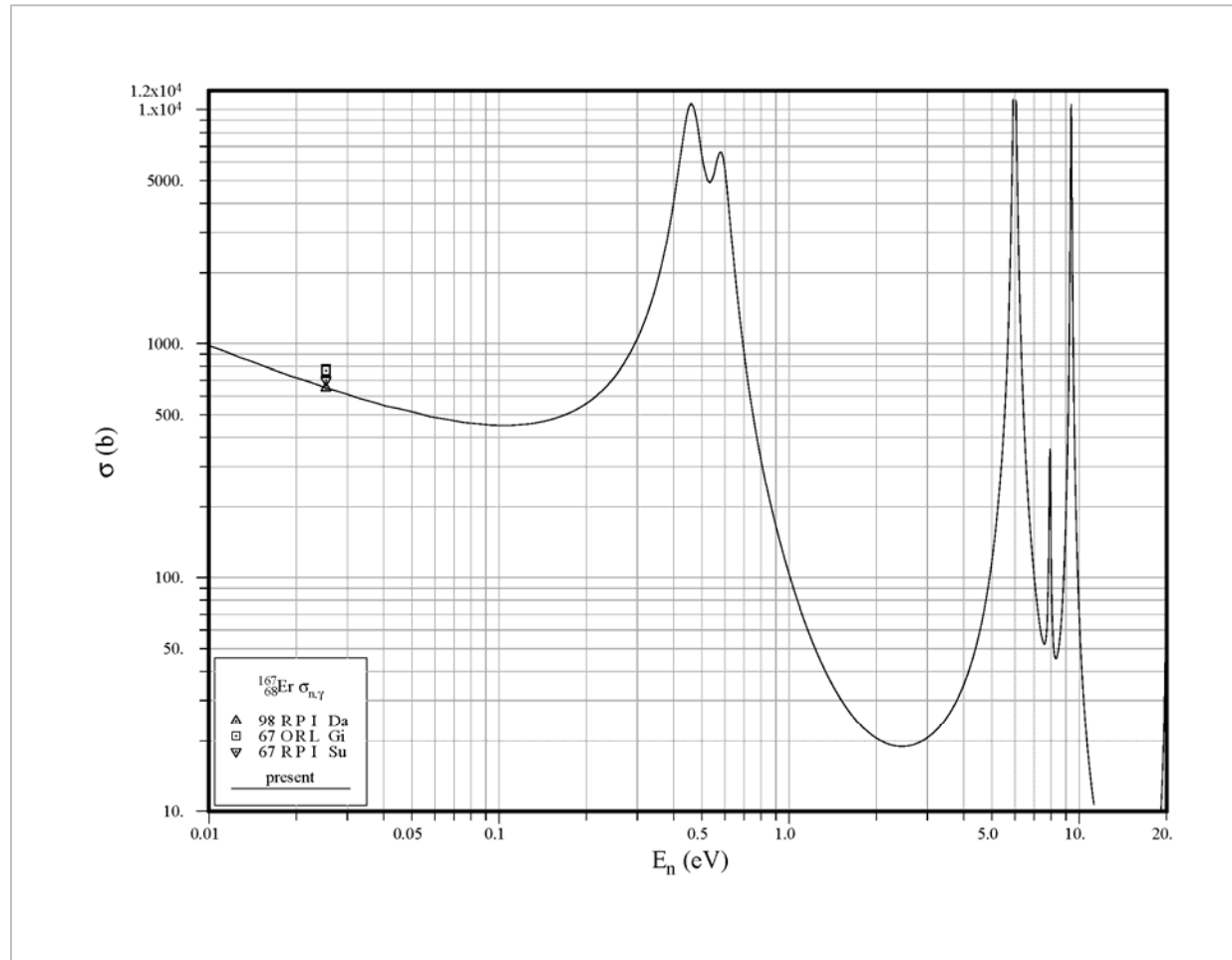
$$\sigma_{\gamma} = 13.36 \pm 0.05 \text{ (0.37\%)} \text{ b}$$



# $^{55}\text{Mn}$ Uncertainty Adjustments

$E_0(\text{keV})$	$2g\Gamma_n(\text{eV})$	$\Gamma_\gamma(\text{eV})$	$\sigma_\gamma\text{-b}$
-16.15	6255(0, 0)	0.790, 0.75(0, 0)	0.32
-0.202	1.15(0, 0)	0.790, 0.75(0, 0)	3.28
0.337	18.3(0.2, 2.2)	0.400, 0.31(0, 6.5)	7.44
1.099	18(0.4, 4)	0.488, 0.435(0, 23)	1.50
2.327	460(0.02, 5.2)	0.340, 0.40(0, 38)	0.17

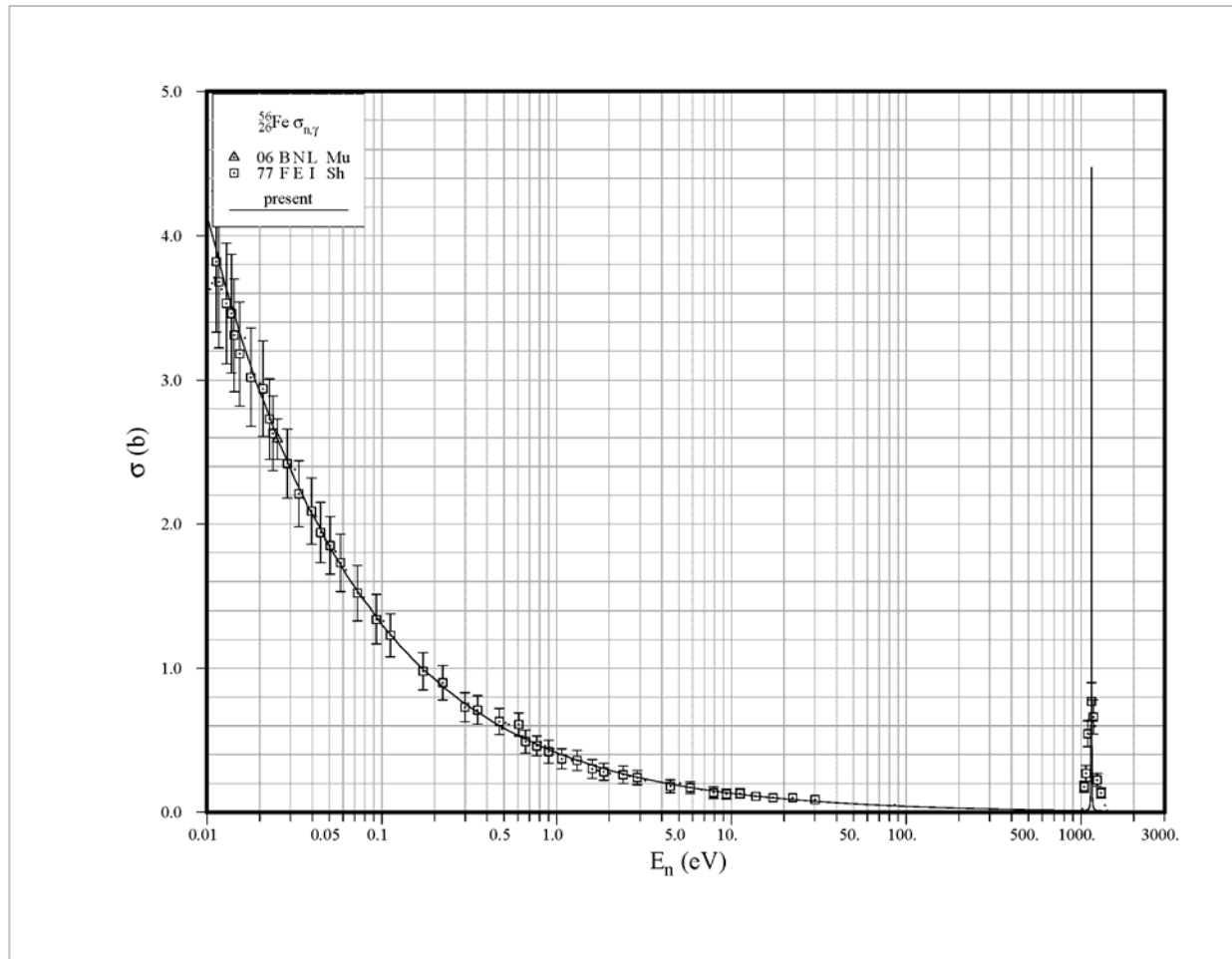
# $^{167}\text{Er}$ Capture Cross Section



$$^{167}\text{Er} \quad \sigma_{\gamma} = 649 \pm 8 \text{ b}$$

$E_0(\text{eV})$	$2g\Gamma_n \text{ (mV)}$	$\Gamma_{\gamma} \text{ (mV)}$	$\sigma_{\gamma} \text{ (b)}$
-23.6	117 (3.3, 0)%	88.0 (11, 0)%	7.9
0.460	0.3031 (3.3, 3.3)%	87.12 (0.4, 0.2 )%	423
0.584	0.2163 (4.6)%	86.20 (0.4, 0.4)%	161

# $^{56}\text{Fe}$ Capture Cross Section





# Some Sources of Correlations

- Capture Kernel  $\frac{g\Gamma_n\Gamma_\gamma}{\Gamma}$  :  $\rho(\Gamma_n, \Gamma_\gamma)$  is Negative.
- Capture and Fission Kernels:  $\Gamma_f / \Gamma_\gamma$  is Determined.  
Hence,  $\rho(\Gamma_f, \Gamma_\gamma)$  is Positive.
- Neutron Sensitivity Correction to Capture Measurements:  
$$\Gamma_\gamma(\text{exp.}) = \Gamma_\gamma + k\Gamma_n$$
  
Hence,  $\rho(\Gamma_n, \Gamma_\gamma)$  is Positive.
- However, This May Be Signature of Valence Capture.

# Effect of $\rho(\Gamma_n, \Gamma_\gamma)$ on Uncertainties

nucleus	$\sigma_\gamma$ (b)	$\rho = 0$ $\Delta$ (b)	$\rho = -1$ $\Delta$ (b)	$\rho = +1$ $\Delta$ (b)
$^{52}\text{Cr}$	$0.86 \pm 0.02$	0.020	0.018	0.023
$^{55}\text{Mn}$	$13.36 \pm 0.05$	0.047	0.044	0.049
$^{237}\text{Np}$	$178.7 \pm 3.0$	3.0	1.6	4.0

# Conclusions

- ❑ Internal consistency between calculated and estimated uncertainties is achieved for 15 actinides and 21 coolant and structural materials.
- ❑ This was realized by re-assigning uncertainties to the parameters of the bound levels and low-energy resonances.
- ❑ If the major contribution to the thermal cross section is due to the positive energy resonances, then their uncertainties are drastically changed.
- ❑ In few cases, where the thermal cross sections are dominated by bound levels, the uncertainties of the positive energy resonances are unaffected.
- ❑ The correlations between parameters are studied and their effects on the uncertainties is determined for some cases.

# Conclusions

- Outstanding issue:
  - In certain cases, uncertainty of positive energy resonances drastically reduced.
  - Possible solution to invoke negative correlation between the bound levels and positive energy resonances.