Workshop on Neutron Cross Section Covariances
June 24-27, 2008 - Port Jefferson, New York, USA

Application of Covariances to Fast Reactor Core Analysis

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Objective of this Talk

To inform the workshop members that how nuclear data covariance is being utilized in the field of fast reactor core analysis and design in Japan.

(Contents)

1. What is the Nuclear Data Covariances?

    → *Definition, Motivation, Methodology & tools, Files, Processing …*

2. How is the prediction accuracy of integral core parameters evaluated with nuclear data covariances?

    → *Features of fast reactor core, Target accuracy, Flow of needs, Sensitivity coefficients, Evaluation of design accuracy …*

3. Can we improve the design accuracy in virtue of nuclear data covariances?

    → *Use of integral data, E/C bias, Cross-section adjustment …*

1. What is the Nuclear Data Covariances?

Review by typical references

Definition of Covariances

<u>Donald L. Smith: "Probability, Statistics, and Data Uncertainties in Nuclear Science and Technology", OECD/NEA, 1991</u>

- ✓ Variance: $$\mu_{ii} = \mathrm{var}(x_i) = \langle (x_i - m_{0i})^2 \rangle \qquad for\ i = 1, n$$
  - Standard deviation: $$\sigma_i = std(x_i) = \sqrt{\mathrm{var}(x_i)}$$ $(m_{0i} = \langle x_i \rangle : \ mean\ value)$
- ✓ Covariance:

$$\mu_{ij} = \mathrm{cov}(x_i, x_j) = \langle (x_i - m_{0i})(x_j - m_{0j}) \rangle \qquad for\ i, j = 1, n\ with\ i \neq j$$

  - Correlations:

$$\rho_{ij} = \frac{\mu_{ij}}{\sqrt{\mu_{ii}\mu_{jj}}} = \frac{\mathrm{cov}(x_i, x_j)}{std(x_i) \times std(x_j)} \qquad where,\ -1 \leq \rho_{ij} \leq 1$$

sometimes called as "Variance-Covariance Matrix."

→ *symmetric and positive-definite characteristcs.*

Covariance of resonance parameters?

References related to Covariances (1)

Motivation to evaluate covariances

- ✓ Y. Kanda: "*Covariance Evaluation Working Group in Japan Sigma Committee*"、Nuclear Data News, Vol. 49, Oct. 1994. --- The nuclear data covariances are rather old topics in the field of nuclear data study. Why are we going to organize a working group to evaluate them? The answer is quite simple, that is, because there is the demand for covariances. ….. Since nuclear data is a kind of physics properties, it is a natural plan to evaluate the error values to be accompanied with such physical constants, in other words, covariances, and to prepare ENDF-format files for users …..

- ✓ Donald L. Smith: "*Nuclear Data Uncertainties in 2004: A Perspective,*" Int. Conf. on Nuclear Data, Santa Fe, Sep. 2004. --- Thus, the principal motivations for understanding uncertainty and developing methods that apply the tools of statistics stem mainly from practical considerations. These can be summed up by the "big three" motivators: "safety," "cost," and "reliability."

- ✓ M. Salvatores: "*Advanced fuel cycles and R&D needs in the nuclear data field ,*" Workshop on Nuclear Physics and Related Computational Science R&D for Advanced Fuel Cycles (GNEP), Maryland, Aug. 2006. --- How to meet requirements… The task to assess credible requirements requires a tight co-operation of nuclear physicists, reactor physicists and reactor system designers. A major challenge: the nuclear data covariance assessment.

References related to Covariances (2)

<u>Development of Evaluation Methodology and Tools</u>

- ✓ T. Kawano: "*Development of Tools to Evaluate Covariances*"、Nuclear Data News, Vol. 70, Nov. 2001. --- The covariance evaluation methodology adopted in JENDL is majorly classified into two categories. One is to evaluate covariances from experimental error values including systematic errors and statistic components, where the generalized least square method such as the GMA code is used. ….. The other is to use the covariance evaluation code system KALMAN which is based on the nuclear model theory …..

- ✓ T.Kawano and K.Shibata : "*Uncertainty Analysis in the Resolved Resonance Region of ^{235}U, ^{238}U and ^{239}Pu with Reich-Moore R-Matrix Theory for JENDL-3.2,*" J. of Nuclear Science and Technology, Vol.39, No.8, Aug. 2002. --- In this method, uncertainties in the total, capture, and fission cross sections are assumed, then uncertainties in the resonance parameters which reproduce the accuracy of the cross sections are estimated by means of the error propagation.

- ✓ N.M.Larson: "*SAMMY: an ORNL Tool for Generating Covariance Matrices in the Resonance Region ,*" Workshop on Nuclear Physics and Related Computational Science R&D for Advanced Fuel Cycles (GNEP), Maryland, Aug. 2006. --- Two methods for generating RPCM •Customary method, used for new evaluations−RPCM is generated by the fitting procedure automatically −Incorporates all experimental uncertainties, •Retroactive method, used when a new evaluation is not possible.

References related to Covariances (3)

Evaluated Covariances

- ✓ K.Shibata, et al.:"*JENDL-3.2 Covariance File,*" Int. Conf. on Nuclear data for Science and Technology (ND2004), Sep. 2004. --- The physical quantities for which covariances are required are cross sections, average number of emitted neutrons per fission, resolved and unresolved resonance parameters, the first order Legendre-polynomial coefficient for elastically scattered neutrons, and fission neutron spectra. Covariances were prepared for 16 nuclides: ^{1}H, ^{10}B, ^{16}O, ^{23}Na, Cr, ^{55}Mn, Fe, Ni, Zr, ^{233}U, ^{235}U, ^{238}U, ^{239}Pu, ^{240}Pu and ^{241}Pu.

- ✓ K.Shibata, et al.:"*Japanese Evaluated Nuclear Data Library Version 3 Revision-3: JENDL-3.3,*" J. of Nuclear Science and Technology, Vol.39, No.11, Nov. 2002. --- In JENDL-3.3, covariances are included for 20 nuclides, as indicated in Table 2 (^{1}H, ^{10}B, ^{11}B, ^{16}O, ^{23}Na, ^{48}Ti, V, ^{52}Cr, ^{55}Mn, ^{56}Fe, ^{59}Co, ^{58}Ni, ^{60}Ni, ^{90}Zr, ^{233}U, ^{235}U, ^{238}U, ^{239}Pu, ^{240}Pu, ^{241}Pu). The covariances for ^{48}Ti, V, and ^{59}Co were newly evaluated for JENDL-3.3.

- ✓ P. Oblozinsky: "*International effort and covariance vision,* " Workshop on Nuclear Physics and Related Computational Science R&D for Advanced Fuel Cycles (GNEP), Maryland, Aug. 2006. --- Proceed in 3 steps, adopt flexible approach, establish strong dialog with users, produce usable results in each step. 1. 1st year: Produce crude, yet reasonable covariances for all nuclei in ENDF/B-VII.0 (Chadwick's idea, LANL), make results available via ENDF/A library, establish dialog with users, release in 2007. 2. Next 2-3 years: Improve all covariances so that they are of solid quality to justify their inclusion into ENDF/B-VII.1, release in ~2010. 3. Next 4-5 years: Produce quality results, include into ENDF/B-VII.2, release ~2015.

References related to Covariances (4)

<u>Tools to Process the ENDF-format Covariances to Group-structure files</u>

- ✓ G. Chiba and M. Ishikawa: "*Revision and Application of the Covariance Data Processing Code, ERRORJ,*" Int. Conf. on Nuclear data for Science and Technology (ND2004), Santa Fe, Sep. 2004. --- ERRORJ is the only code that can process the covariance data of the Reich-Moore resolved resonance parameters and the unresolved resonance parameters in the world. Now, the new version, version 2.2, has been developed and is released with improved reliability. .... Covariance data contained in ENDF/B, JEF(F), and JENDL are processed. Large differences are observed in the covariance between these nuclear data files.

- ✓ M.E. Dunn, et al.: "*ORNL Cross-Section Covariance Processing Capabilities,* " Workshop on Nuclear Physics and Related Computational Science R&D for Advanced Fuel Cycles (GNEP), Maryland, Aug. 2006. --- PUFF-IV Module Development for AMPX code system: Complete rewrite of PUFF-III code in F90, … Processes ENDF/B Files 31, 32 and 32, Utility modules available to interface with NJOY-generated libraries…. Group averages of covariances are calculated using the above derivatives, Resolved region data can be handled—existing ENDF/B unresolved formats can be processed…

- ✓ N. Otuka, et al.: "*Covariance Analyses of Self-Shielding Factor and Its Temperature Gradient for Uranium-238 Neutron Capture Reaction,* " J. of Nuclear Science and Technology, Vol.45, No.3, Mar. 2008. --- Covariances of the self-shielding factor and its temperature gradient for the uranium-238 neutron capture reaction have been evaluated from the resonance parameter covariance matrix and the sensitivity of the self-shielding factor and its temperature gradient to the resonance parameters. ….. a new system ERRORF has been developed ….

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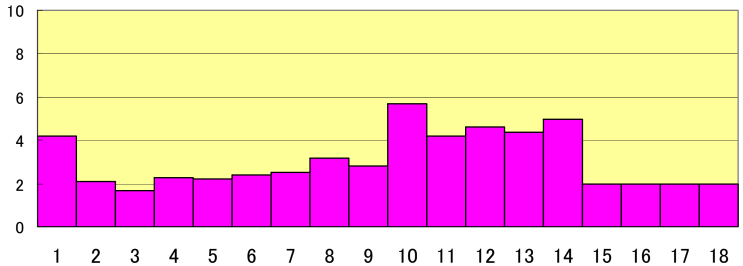
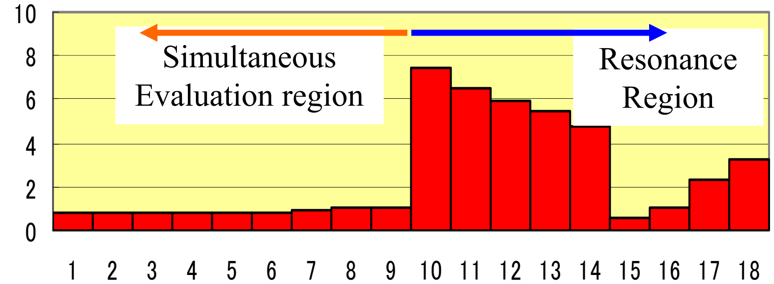
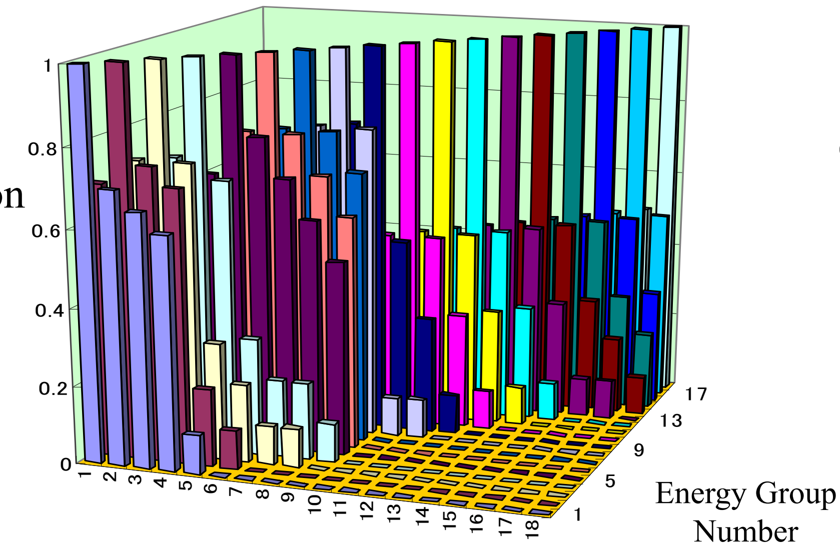
Group-structure Covariances
(Comparison of Pu-239 Fission Cross-section)

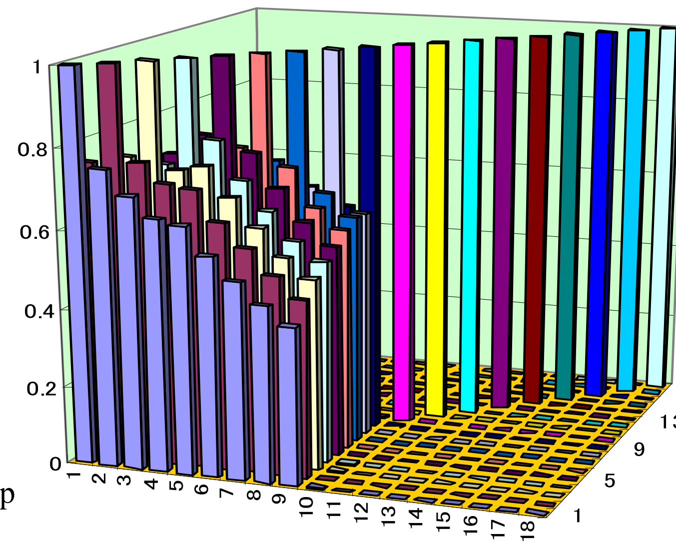
Standard Deviation (%)





Correlation Factor





Old Covariance by Rough Estimation
(1991)

New Covariance by Sigma Committee
(2000)

to JENDL-3.2

2. How is the prediction accuracy of integral core parameters evaluated with nuclear data covariances?

Always accompanied with sensitivity coefficients

Features of Nuclear Parameters in Fast Reactors

★ Complicated heterogeneous structure composed of fuel pellets, claddings, coolant, wrapper tubes, plural Pu-enrichment regions,

★ Needs to treat accurately neutron collision & absorption in wide energy range over 5-orders from several MeV of fission spectrum through a few 10 eV where neutron disappears,

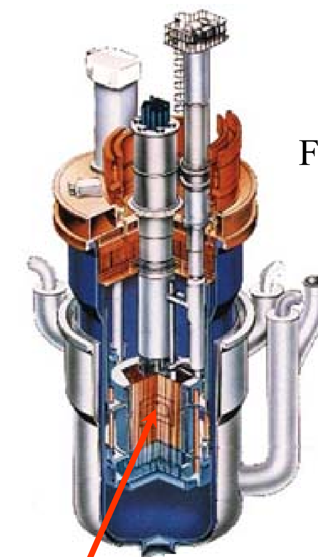
★ Major contribution to design uncertainty comes from error of physical property, that is, nuclear data, and,

★ Very high target accuracy requirement, compared with other engineering fields such as thermal hydraulics, fuel material, etc.

⬇

A. Detailed analytical modeling

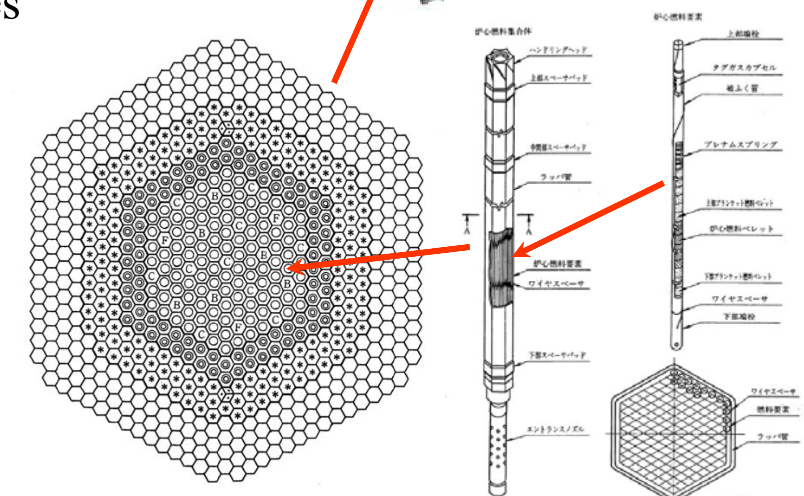
B. Use of integral experimental data



Prototype

Fast Reactor

“Monju”



Target Accuracy of Nuclear Design for FBR Core

◆ Criticality: Target* → ±0.3%Δk (1σ)

✧ Conventional uncertainty: ±1.0%Δk → This is equivalent to app. 20 fuel S/As at the core periphery. To make up for it, over-design of control rods or Pu-enrichment change in fuel manufacturing would be necessary.

◆ Power distribution: Target → ±3% (2σ)

✧ Conventional uncertainty : ± 5% → This is equivalent to design margin of 20 w/cm, which is a heavy burden for non-melting limit in the overpower accident. To cover it, over-design of safety equipment, low heat density of core by enlarging of core size, or increasing of fuel pin number could not be avoided.

◆ Doppler reactivity: Target* → ±14% (2σ)

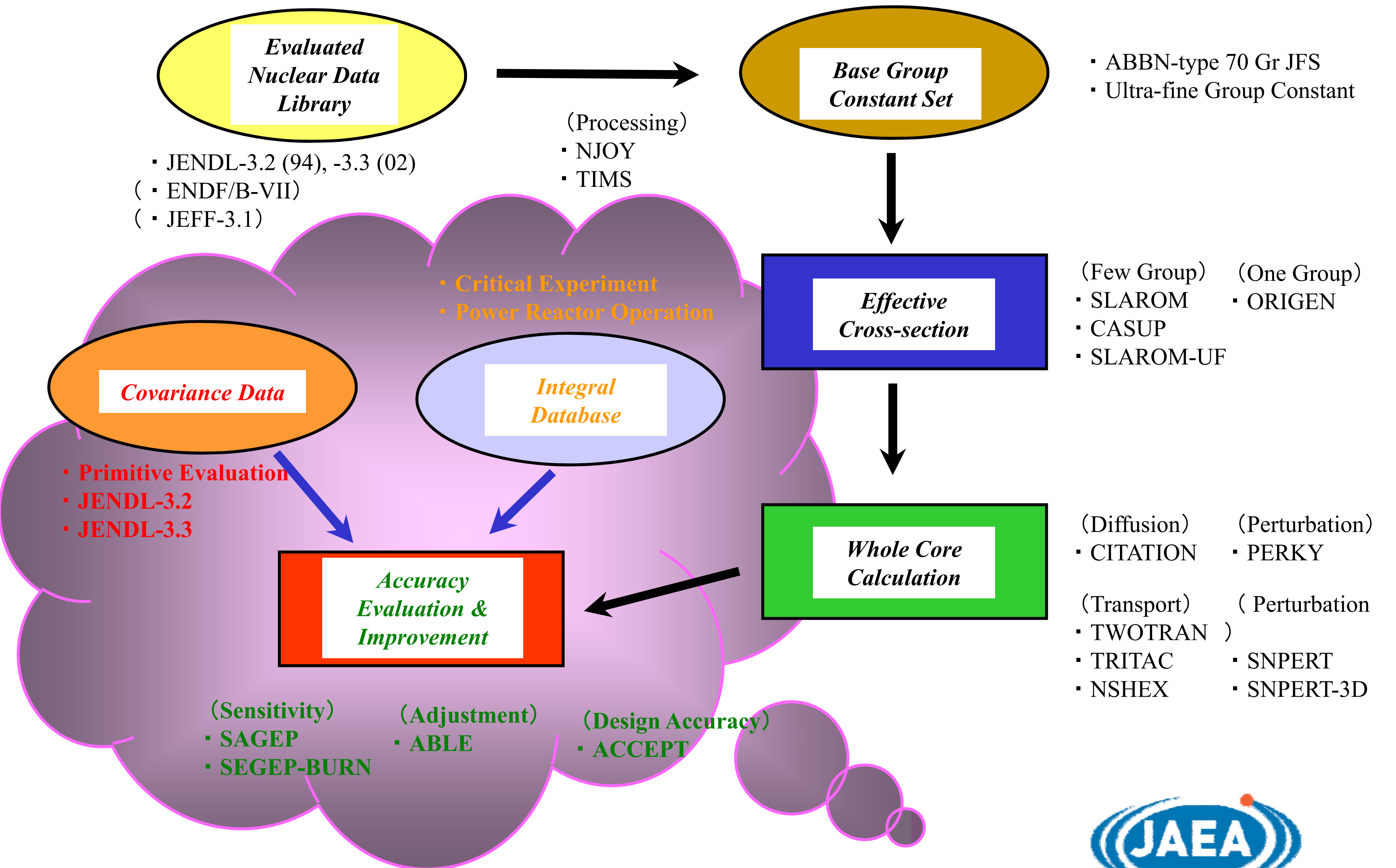
✧ Conventional uncertainty : ± 30% → Prompt feedback reactivity at accident. It affects the requirement for response time of detection & control systems.

◆ Sodium void reactivity: Target* → ±20% (2σ)

✧ Conventional uncertainty : ± 50% → This would be acceptable for small-size FBR licensing such as Monju, but the hypothetical energetics of large FBR cores might result in the impact to the integrity of the reactor vessel.

(* by Governmental Committee for R&D Program of DFBR Development (April, 1996)

Typical Flow of Fast Reactor Core Analysis



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Flow of Needs for Quantitative Uncertainty Evaluation

Step	Physical Property or Product by Using the Quantitative Error Values of B (= A)	Man or Organization who Needs the Qualitative Error Property and Quantitative Error Value of A (= B)
1	(Efficiency of Radiation Detector, Isotopic Composition of a Sample)*	Experimentalist of Nuclear Data
2	Measured Value and Document of Nuclear Data	Evaluator of Nuclear Data file
3	Evaluated Nuclear Data Library => "with Nuclear Data Covariances"	Analyst of Reactor Physics Experimentalist, Developer of Reactor Physics Analytical Modeling and Code
4	(Experimental Data of Reactor Physics)*	
5	C/E Value of Integral Experimental Analysis	Reactor Plant Design Manufacturer, Adjuster of Group-constant Set
6	Design Value of Nuclear Core Parameter of the Target Reactor	Owner of the Plant, Electricity Company
7	Accident Analysis Results of the Target Reactor Core	Licensing Authority, Public People

* means this item is not always on the flow of the error supplier and consumer.

-> Uncertainty of a property is always required by its user, NOT by its supplier.

Nuclear Data Sensitivity related to Integral Parameter

Definition

→ Change of an integral core parameter, *R*, per unit change of nuclear data (cross-sections), σi,

where, i : nuclide, reaction, number of group energy

$$S_i = \frac{\dfrac{dR}{R}}{\dfrac{d\sigma_i}{\sigma_i}}$$

Calculational method

Method 1: Change a cross-section, and calculate the change of the integral core parameter

→ A huge amount of calculation cases (nuclide number * reaction number * energy group number + χ, β = several 1,000 or more.)

→ Risk of numeric rounding error or non-linearity, if the changing of a cross-section is inappropriate.

Method 2 : Application of generalized perturbation theory

→ Calculate the generalized (adjoint) neutron flux, only once.

(※ L.N.Usachev: "Theory for the Breeding Ratio and for Other Number Ratios Pertaining to Various Reactor Processes, J. of Nuclear Energy Parts A/B, Vol.18, pp.571-583, 1964.)

Sensitivity Coefficient by Generalized Perturbation Theory (1/2)

- Conventional perturbation： Only treat the change of keff with core perturbation.

- Generalized Perturbation： Extend the target to reaction rate and reactivity.

＜History＞

- Usachev (1964) －Derive the theory for the ratio of reaction rates,
- Gandini (1967) －Extend the theory to reactivity.
- Stacey (1972) －Develop the numerical solution with use of Neumann series expansion,
- Mitani & Kuroi (1972) －Mechanism from the viewpoint of generation-wise importace,
- Hara & Takeda (1984) －Develop and open the SAGEP code to public.

★ Change of cross-sections → Affect the core parameter directly

＋ Contribution of (adjoint) flux changes indirecty

Diffusion equation: $$-\nabla\cdot D(\vec{r},E)\nabla\phi(\vec{r},E)+\Sigma_a(\vec{r},E)\phi(\vec{r},E)+\int dE'\Sigma_s(\vec{r},E\rightarrow E')\phi(\vec{r},E)-\int dE'\Sigma_s(\vec{r},E'\rightarrow E)\phi(\vec{r},E')$$
$$-\frac{\chi(E)}{k_{eff}}\int dE'\upsilon(E')\Sigma_f(\vec{r},E')\phi(\vec{r},E') \equiv B\phi = 0$$

Adjoint equation： $B^*\phi^* = 0$

Sensitivity Coefficient by Generalized Perturbation Theory (2/2)

- Parameter = Ratio of reaction rate:

$$R \equiv \frac{[\Sigma_1\phi]}{[\Sigma_2\phi]} = \frac{\iint d\vec{r}dE\Sigma_1(\vec{r},E)\phi(\vec{r},E)}{\iint d\vec{r}dE\Sigma_2(\vec{r},E)\phi(\vec{r},E)}$$

Sensitivity :

$$S \equiv \frac{\frac{dR}{R}}{\frac{d\sigma}{\sigma}} = \sigma\frac{d(\ln R)}{d\sigma} = \left\{ \frac{\left[\frac{d\Sigma_1}{d\sigma}\phi\right]}{[\Sigma_1\phi]} - \frac{\left[\frac{d\Sigma_2}{d\sigma}\phi\right]}{[\Sigma_2\phi]} + \frac{\left[\Sigma_1\frac{d\phi}{d\sigma}\right]}{[\Sigma_1\phi]} - \frac{\left[\Sigma_2\frac{d\phi}{d\sigma}\right]}{[\Sigma_2\phi]} \right\}\sigma$$

Generalized adjoint flux Γ* :

$$B^*\Gamma^* = \frac{\Sigma_1\phi}{[\Sigma_1\phi]} - \frac{\Sigma_2\phi}{[\Sigma_2\phi]}$$

Balance equation after perturbation :

$$(B + dB)(\phi + d\phi) = 0$$

$$\rightarrow \quad S \equiv \frac{\frac{dR}{R}}{\frac{d\sigma}{\sigma}} = \left\{ \frac{\left[\frac{d\Sigma_1}{d\sigma}\phi\right]}{[\Sigma_1\phi]} - \frac{\left[\frac{d\Sigma_1}{d\sigma}\phi\right]}{[\Sigma_1\phi]} + \left[\frac{d\phi}{d\sigma}B^*\Gamma^*\right] \right\}\sigma = \left\{ \frac{\left[\frac{d\Sigma_1}{d\sigma}\phi\right]}{[\Sigma_1\phi]} - \frac{\left[\frac{d\Sigma_1}{d\sigma}\phi\right]}{[\Sigma_1\phi]} - \left[\Gamma^*\frac{dB}{d\sigma}\phi\right] \right\}\sigma$$

- Parameter = Reactivity :

$$R \equiv \frac{[\phi^* H_1\phi]}{[\phi^* H_2\phi]} = \frac{\iint d\vec{r}dE\phi^* H_1(\vec{r},E)\phi(\vec{r},E)}{\iint d\vec{r}dE\phi^* H_2(\vec{r},E)\phi(\vec{r},E)}$$

Generalized adjoint flux Γ* :

$$B^*\Gamma^* = \frac{H_1^*\phi^*}{[\phi H_1^*\phi^*]} - \frac{H_2^*\phi^*}{[\phi H_2^*\phi^*]}$$

Generalized normal flux Γ:

$$B\Gamma = \frac{H_1\phi}{[\phi^* H_1\phi]} - \frac{H_2\phi}{[\phi^* H_2\phi]}$$

$$\rightarrow \quad S \equiv \frac{\frac{dR}{R}}{\frac{d\sigma}{\sigma}} = \left\{ \frac{\left[\phi^*\frac{dH_1}{d\sigma}\phi\right]}{[\phi^* H_1\phi]} - \frac{\left[\phi^*\frac{dH_2}{d\sigma}\phi\right]}{[\phi^* H_2\phi]} - \left[\Gamma^*\frac{dB}{d\sigma}\phi\right] - \left[\Gamma\frac{dB^*}{d\sigma}\phi^*\right] \right\}\sigma$$

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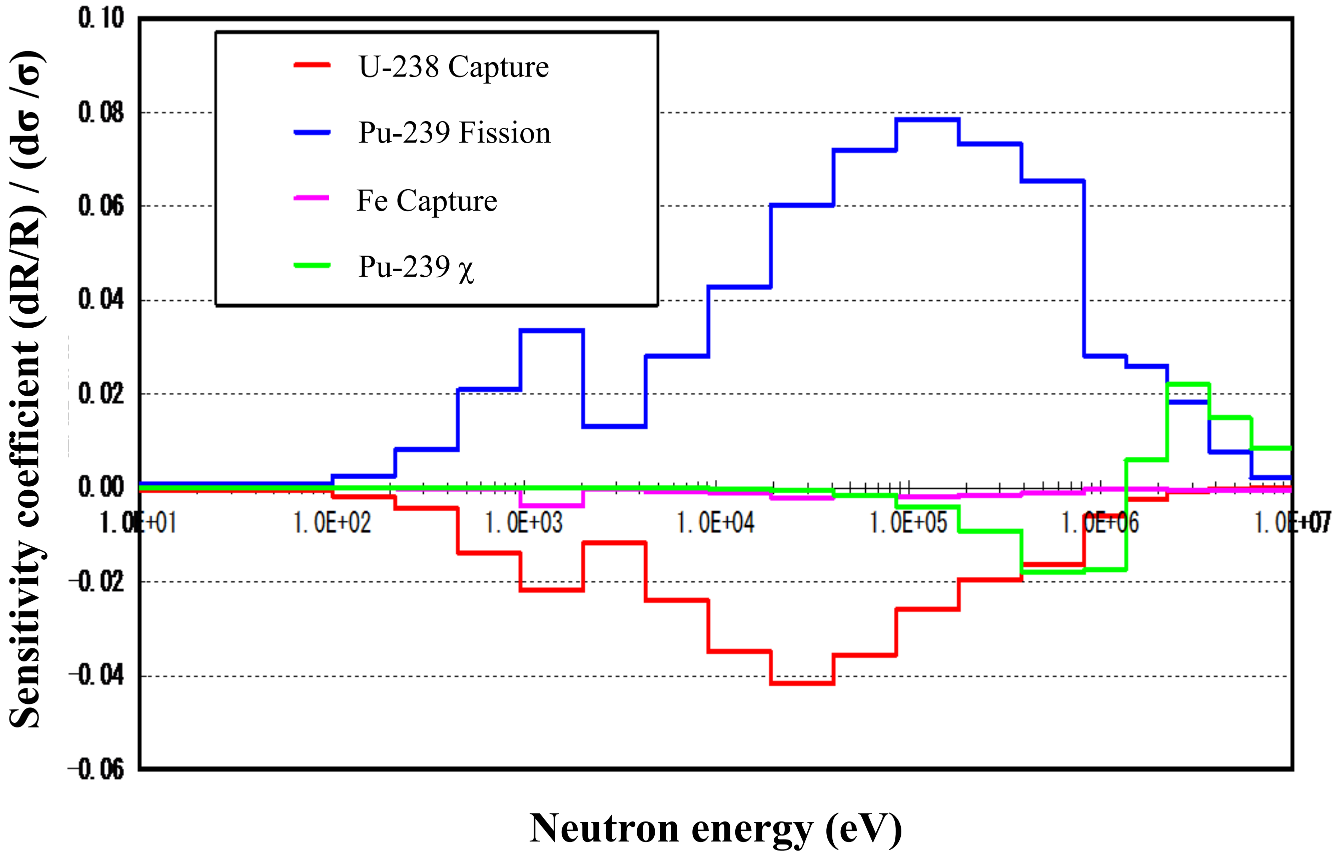
Sensitivity for Criticality (ZPPR-9 Core)

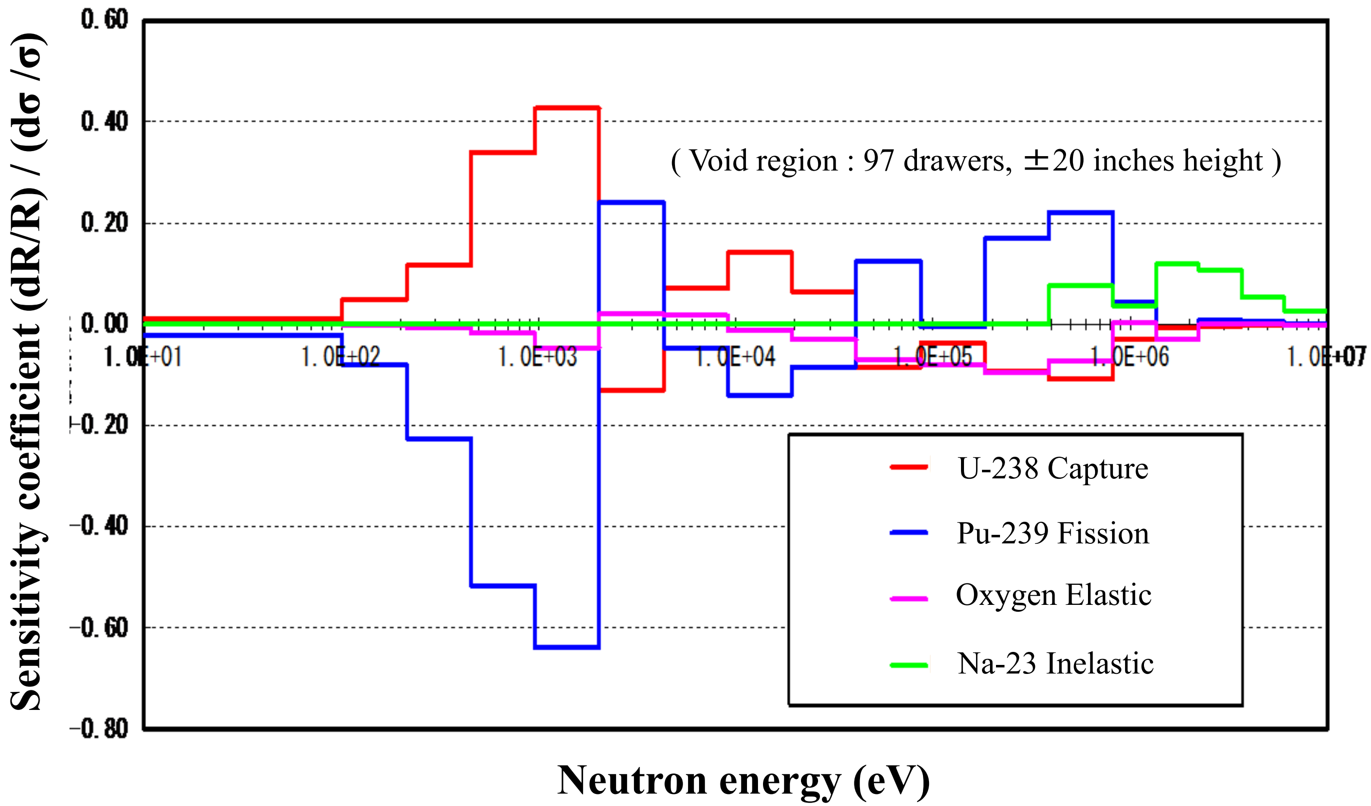
- Pu-239 fission and U-238 capture reactions have large sensitivity, but other nuclides and reactions are not negligible.
- Sensitivity of fission spectrum has positive and negative depending on energy regions, due to its normalized feature.



Sensitivity for Sodium Void Reactivity (ZPPR-9 Core)

- Huge sensitivity around a few keV region which corresponds to the giant resonance peak of sodium.

- Elastic and inelastic scattering also have large sensitivity, since they affect neutron energy spectrum and neutron leakage.



Sensitivity Coefficient of Doppler Reactivity

- Conventional sensitivity method treats only **infinitely-diluted cross-sections.**
  → Impossible to evaluate temperature characteristics.
- Doppler reactivity : $R = \dfrac{1}{k_{eff,low}} - \dfrac{1}{k_{eff,high}}$
- Relationship of effective cross-sections with temperature : $\sigma_{eff,high} \approx \left[f_{low} + \left(\dfrac{df}{dT} \right) \Delta T \right] \sigma_{\infty,low} = (1 + f' \Delta T) \sigma_{eff,low}$

  ↓

  where, $f' \equiv \dfrac{1}{f_{low}} \left(\dfrac{df}{dT} \right) \equiv \alpha$
- Introduction of **pseudo-cross-sections, *df/dT (=f')*** :

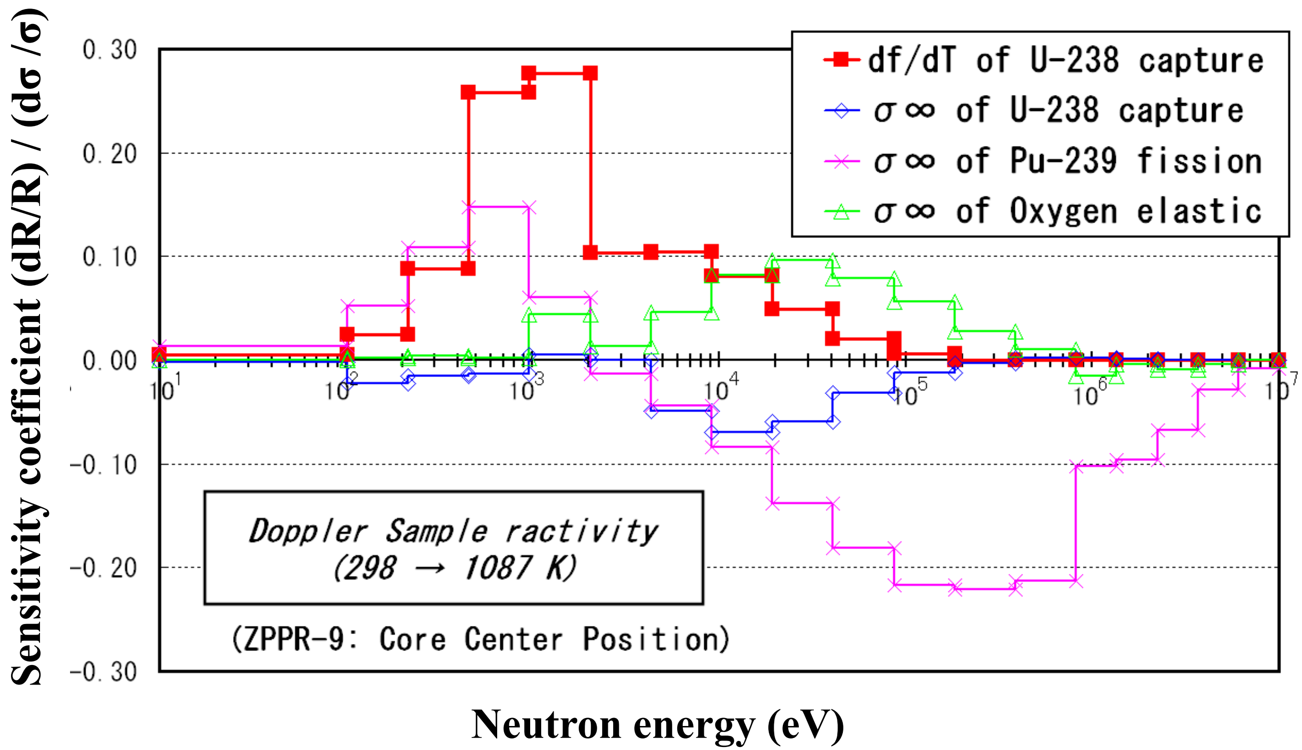
$$S_{f'} \equiv \frac{dR/R}{df'/f'} = \left(\frac{\sigma_{eff,high} - \sigma_{eff,low}}{\sigma_{eff,high}} \right) \times \frac{1}{R} \times \frac{S_{k_{eff},high}}{k_{eff,high}}$$

where, $S_{k_{eff},high} = \dfrac{dk_{eff,high} / k_{eff,high}}{d\sigma_{\infty,high} / \sigma_{\infty,high}}$

(Merits) 1) Easily calculated from the sensitivity of criticality, S_{keff},
2) No influence to the self-shielding factors at room temperature.

Sensitivity for Sample Doppler Reactivity (ZPPR-9 Core)

- The df/dT of U-238 capture has largely positive sensitivity at keV energy region,
- Sensitivity of Pu-239 fission is negative, since it increases the denominator of perturbation,
- There is also a certain sensitivity to space-related reactions, since it also has the characteristics of local sample reactivity.



Generalized Perturbation Theory for Burnup characteristics

- Needs: 1) Use of power reactor data such as burnup reactivity loss or composition changes of fuel nuclides
  2) Evaluation of FBR nuclear design accuracy

- Net sensitivity: $$S(\sigma_x^g) = \frac{dR/R}{d\sigma_x^g/\sigma_x^g} = \frac{\sigma_x^g}{R} \times \left\{ S_D + S_N + S_\phi + S_{\phi^*} + S_P \right\}$$

$$S_D = \sum_{i=1}^{I} \left[\int_t^{t_{i+1}} dt \frac{\partial R}{\partial \sigma_x^g} \right]_{E,V}$$: Direct term

$$S_N = \sum_{i=1}^{I} \int_t^{t_{i+1}} dt \left[N^* \frac{\partial M}{\partial \sigma_x^g} N \right]_{E,V}$$: Atomic number density term

$$S_\phi = \sum_{i=1}^{I+1} \left[\Gamma_i^* \frac{\partial B}{\partial \sigma_x^g} \phi_i \right]_{E,V}$$: Normal flux term

$$S_{\phi^*} = \sum_{i=1}^{I+1} \left[\Gamma_i \frac{\partial B}{\partial \sigma_x^g} \phi_i^* \right]_{E,V}$$: Adjoint flux term

$$S_P = \sum_{i=1}^{I+1} \left[P_i^* \frac{\partial P_i}{\partial \sigma_x^g} \right]_{E,V}$$: Power normalization term

where,

$$\frac{\partial}{\partial t} N(t) = M \times N(t)$$: Burnup equation

$$P_i = \int_{E,V} dE dV \left[\kappa \sigma_f N \phi_i \right]$$: Reactor power

P^* : Adjoint power

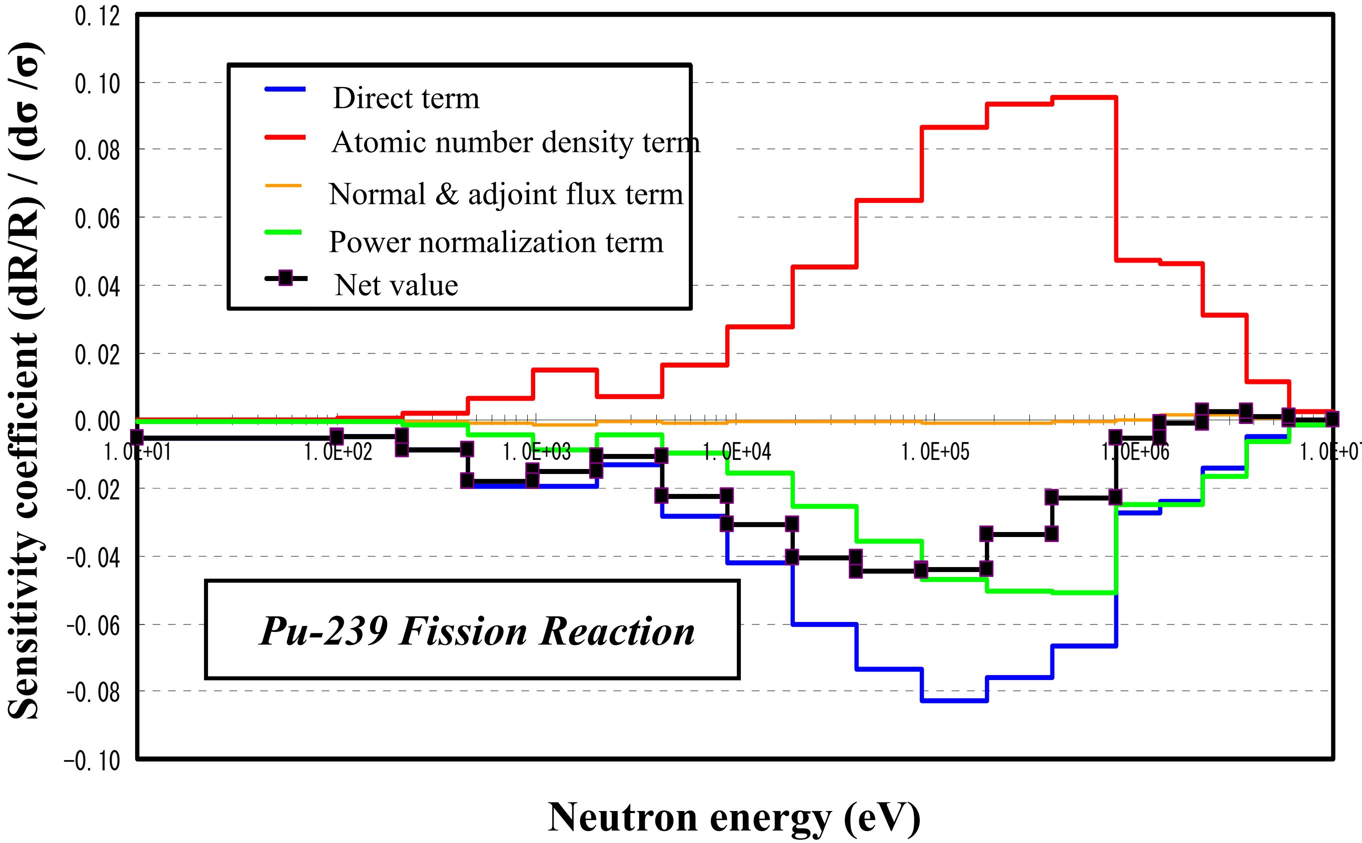
N_i^* : Adjoint atomic number density

(※M.L.Williams: "Development of Depletion Perturbation Theory for Coupled Neutron/Nuclide Fields," Nuclear Science and Engineering 70, pp.20-36, 1979.)

Sensitivity for Burnup Reactivity Loss (JOYO Mk-I Core)

- Direct term is negative because it increases Denominator
- Atomic number density term is positive since it acceralates the decrease of Pu-239.
- Power normalization term is negative because it lower the neutron flux level.

Net sensitivity of Pu-239 fission is slightly negative because of these cancellation.



Evaluation of Design Accuracy (I)

※ T.Takeda, et al.: "Prediction Uncertainty Evaluation Methods of Core Performance Parameters in Large Liquid-Metal Fast Breeder Reactors," NSE 103, pp.157-165, 1989

(Case: No use of integral information)

- Design nominal value: $Rc^{*(2)}(T_0) = Rc^{(2)}(T_0)$
- Design error (variance): $V[Rc^{*(2)}(T_0)] = \underline{G^{(2)} M G^{(2)t}} + Vm^{(2)}$

$$\begin{pmatrix} S_1 & S_2 & \cdots & S_n \end{pmatrix} \begin{pmatrix} M_{11} & M_{12} & \cdots & M_{1n} \\ M_{21} & M_{22} & \cdots & M_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ M_{n1} & M_{n2} & \cdots & M_{nn} \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \end{pmatrix}$$

where, $\boldsymbol{T_0}$: Original group constant set.

Rc: Analytical value of nuclear parameter *R*.

*****: Design nominal value, i.e., best estimated.

(2): Target design core.

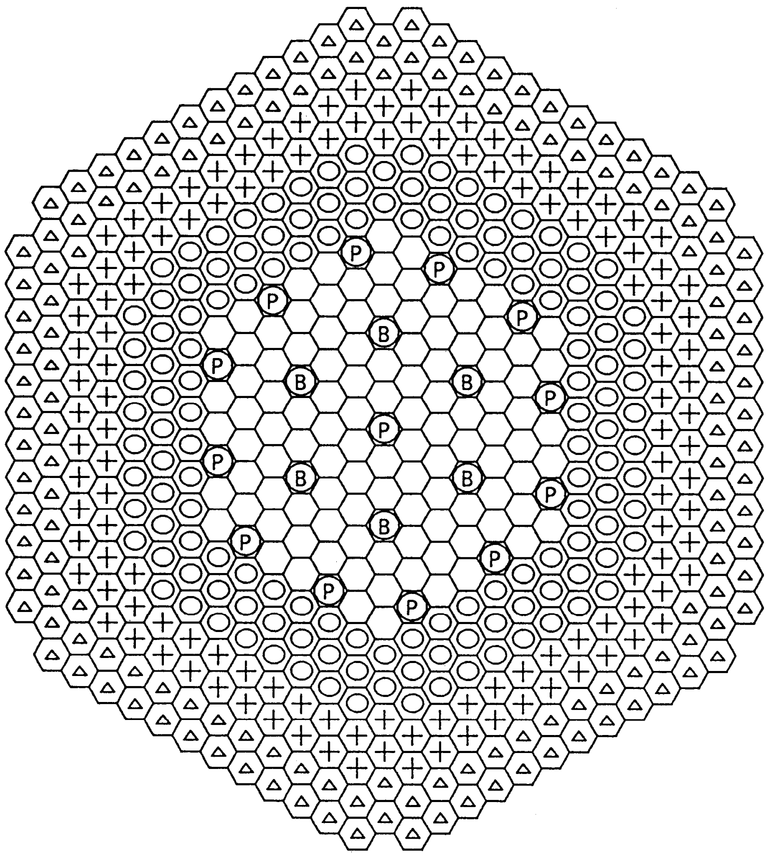
G: Sensitivity coefficient defined by (dR/R)／(dσ/σ).

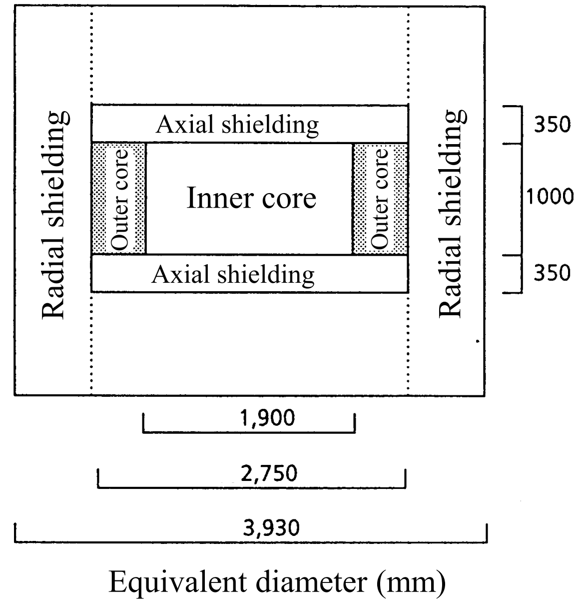
M: Covariance (error with correlation) of T_0

Vm: Analytical modeling error of *R*, including manufacturing errors.

(If the number of R is more than 2 ?)

Evaluation of Design Accuracy for a 600 MWe-class FBR Core





Inner core	108
Outer core	138
Stainless steel	126
B_4C shield	150
Primary control rod	13
Backup control rod	6
Total	541

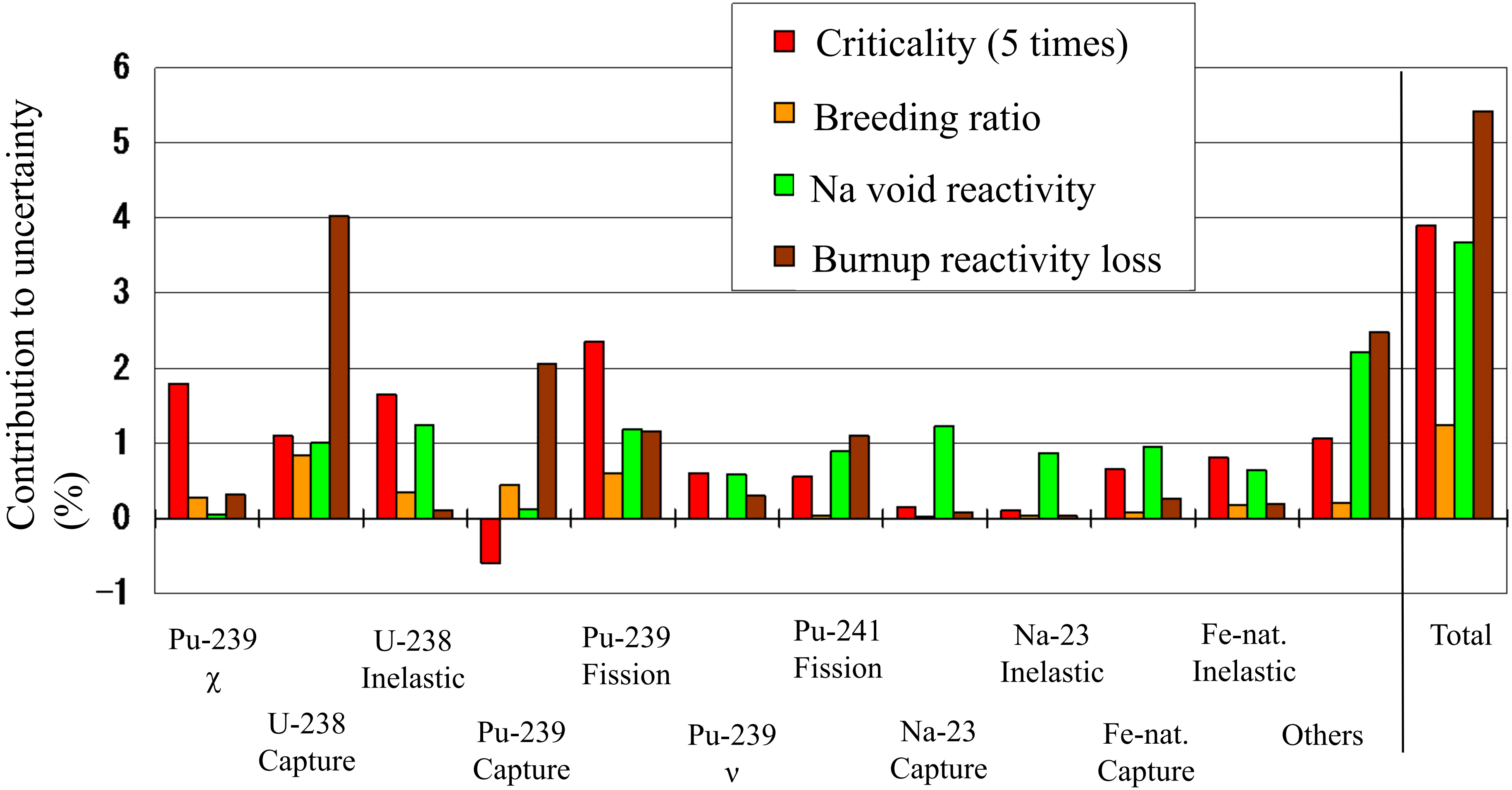
Core structure of a 600 MWe-class FBR

Nuclear Design Accuracy of a 600 MWe-class FBR

*1σ value (Non-diagonal terms show correlation factors.)

Design method	No use of integral information Total : % unit. (( ) : contribution from nuclear data covariances.)					
Nuclear parameter	**Criticali-ty**	**Breeding ratio**	**Power distribution**	**Doppler reactivity**	**Na void Reactivity**	**Burnup reactivity loss**
Criticality (End of equilibrium cycle)	0.79 (0.78) %					
Breeding ratio (C28/F49 reaction rate ratio)	-0.63	1.4 (1.1) %			(Symmetry matrix)	
Power distribution (Outer core region)	-0.54	0.64	1.2 (1.1) %			
Doppler reactivity (Whole core region)	-0.35	0.26	0.24	6.3 (6.2) %		
Na void Reactivity (Whole core region)	-0.03	0.08	0.09	0.10	4.7 (3.7) %	
Burnup reactivity loss	0.26	-0.66	-0.38	0.03	-0.10	5.5 (5.4) %

Nuclide-wise Contribution to Nuclear Design Accuracy of a 600 MWe-class FBR



3. Can we improve the design accuracy in virtue of nuclear data covariances?

Addition of integral information

Evaluation of Design Accuracy (II)

※ T.Kamei and T.Yoshida: "Error due to Nuclear Data Uncertainties in the Prediction of Large Liquid-Metal Fast Breeder Reactor Core Performance Parameters," NSE 84, pp.83-97, 1983

(← Comment from J.J.Wagshal and Y.Yeivin, NSE 86, pp.121-124, 1984)

(Case: E/C-bias method)

● Design nominal value :

$$Rc^{*(2)}(T_0) = Rc^{(2)}(T_0) \times \frac{Re^{(m)}}{Rc^{(m)}(T_0)}$$

● Design error (variance) :

$$V[Rc^{*(2)}(T_0)] = \Delta G M \Delta G^t + \underline{Ve^{(m)}} + \Delta Vm$$

Additional error

where, ***(m)*** : Mockup critical experiment, only one in principle.

Ve : Error of mockup experiment , including manufacturing errors.

$\Delta G = G^{(2)} - G^{(m)}$: Difference of sensitivity between Target design core and mockup experiment. -> The error included only in the mockup experiment is introduced.

$\Delta Vm = Vm^{(m)} + Vm^{(2)} - Vm^{(m2)} - Vm^{(m2)t}$: Non-correlation part between the analytical error of the mockup experiment and that of the target design core. -> a part of the analytical error is cancelled by the E/C bias method.

Integral Data for Fast Reactors (1/4)

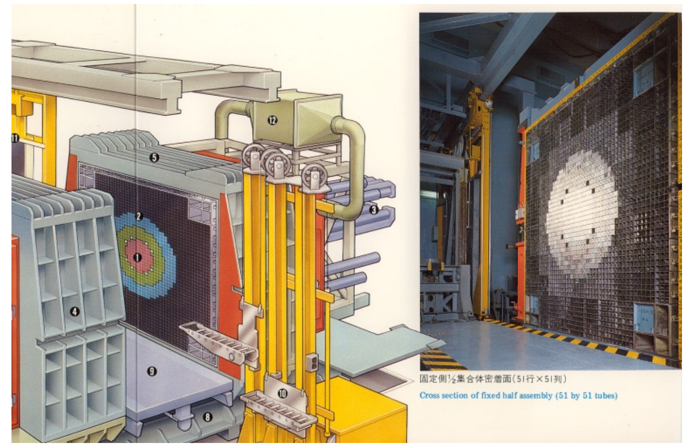
- **JUPITER Critical Experiment**
  - ✓ Cooperative study of DOE and JNC in 1978～1988, using ZPPR facility at ANL, USA.
  - ✓ The largest FBR mockup experiment in history, 4,600 – 8,500 liters.
  - ✓ Various core concepts, sizes, and structures:
    - ◆ 600～800MWe-class two-region homogeneous cores,
    - ◆ 650MWe-class radially-heterogeneous cores,
    - ◆ 650MWe-class axially-heterogeneous cores,
    - ◆ and, 1000MWe-class homogeneous cores with enriched uranium regions.
  - ✓ Many kinds of measured parameters.

↓

As-built experimental information is available for the public.

ZPPR Critical Assembly（ANL）

Integral Data for Fast Reactors (2/4)

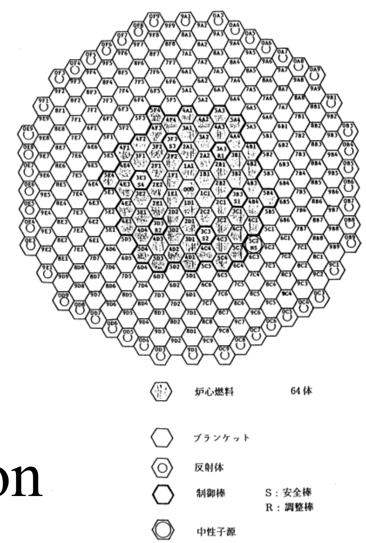


FCA Critical Experiment

- ✓ Fast Critical Assembly at JAEA, Japan.
- ✓ To simulate small FBR cores with plutonium and enriched uranium fuels.
  - ◆ FCA ⅩⅦ-1 Core（1993）-650 liters.
  - ◆ FCA Ⅹ-1 Core（1982）-130 liters.

Experimental Reactor JOYO

- ✓ First Japanese FBR (1st Criticality in 1977) ―Burnup, pin-wrapper structure.
  - ◆ Mixed one-region plutonium and enriched uranium core with 240 liter-size.
  - ◆ Criticality, fuel-blanket replacement reactivity, and burnup reactivity were adopted.



JOYO Mk-I Core
（JAEA）

（Minimum critical core）

As-built experimental information is available for the public.

Integral Data for Fast Reactors (3/4)

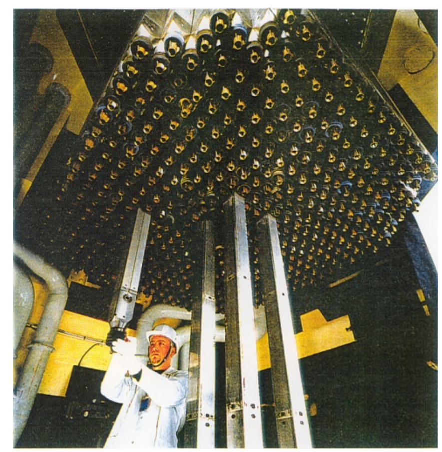
■ BFS-1, 2 Critical Experiment

- ✓ Fast Critical Assembly at IPPE, Obninsk, Russia.
  - ◆ BFS-58-1-I1 Core（1996） －Uranium-free region in core center and enriched uranium region in periphery.
  - ◆ BFS-62-1 ～5, 66-1 Core（1999 ～ 2002） －Three enriched uranium region core to study Pu-disposition in BN-600 core.
  - ◆ BFS-67, 69, 66-2 Core（1993 ～ 2003） －10 kg of NpO_2 loading cores in central MOX region with weapon-grade Pu, high enriched Pu, and degraded Pu.

BFS-2
Critical Assembly
（IPPE）

■ MASURCA Critical Experiment

- ✓ Fast Critical Assembly at CEA, Cadarache, France.
  - ◆ ZONA2B Core（1996） －a 380 liter-size MOX fuel core with reflectors, which simulated Pu-burner.



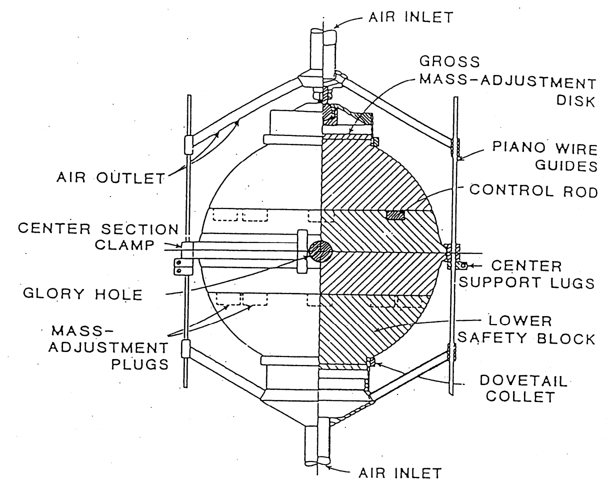
MASURCA
Critical Assembly
（CEA）

Integral Data for Fast Reactors (4/4)

- **Los Alamos Small Core Experiment**
  - ✓ Sphere-shaped cores of approx. ten centimeter in diameter with metallic fuel consisted of Pu-239, or degraded Pu, or U-235.
    - ◆ FLATTOP-Pu, FLATTOP-25, JEZEBEL, JEZEBEL-Pu, GODIVA

↓

Benchmark models have already opened.



Jezebel under operating conditions. The nearly spherical assembly is supported by lightweight clamps and guides.

Radius 6.3cm
Mass 17kg Pu
LA – 9685 – H Unclassified

Los Alamos Small Core (JEZEBEL)

- **Other Experiments**
  - ✓ **ZEBRA** (MOZART program, UK) - a 550 liter-sized one-region MOX core as a clean benchmark.
  - ✓ **SEFOR** (General Electric, USA) - a 20MWt fast power reactor core fueled with mixed PuO_2-UO_2 and cooled with sodium.

JAEA

Evaluation of Design Accuracy (II)

※ T.Takeda, et al.: "Prediction Uncertainty Evaluation Methods of Core Performance Parameters in Large Liquid-Metal Fast Breeder Reactors," NSE 103, pp.157-165, 1989

Case: Cross-section adjustment method

● Design nominal value :

$$Rc^{*(2)}(T') = Rc^{(2)}(T') = Rc^{(2)}(T_0) + G^{(2)}(T' - T_0)$$

● Design error (variance) :

$$V[Rc^{*(2)}(T_0)] = \underline{G^{(2)} M' G^{(2)t}} + Vm^{(2)} - N Vm^{(12)} - Vm^{(12)t} N^t$$

where, ***(1)*** : A set of critical experiments.

T' : Adjusted group constant set

M' : Covariance of T'.

$$N = G^{(2)} M G^{(1)t} [G^{(1)} M G^{(1)t} + Ve^{(1)} + Vm^{(1)}]^{-1}$$

Theory of Cross-section Adjustment

※ J.B.Dragt, et al.: "Methods of Adjustment and Error Evaluation of Neutron Capture Cross Sections; Application to Fission Product Nuclides," NSE 62, pp.117-129, 1977

- Based on the Bayes theorem, i.e., the conditional probability estimation method
  → To maximize the posterior probability that a cross-section set, ***T***, is true, under the condition that the information of integral experiment, ***Re***, is obtained.

$$J(T) = (T-T_0)^t M^{-1}(T-T_0) + [Re-Rc(T)]^t[Ve+Vm]^{-1}[Re-Rc(T)]$$

  Minimize the function ***J(T)***. → $dJ(T)/dT = 0$

- The adjusted cross-section set ***T'***, and its uncertainty (covariance), ***M'*** *(Algebra)*

$$T' = T_0 + MG^t[GMG^t+Ve+Vm]^{-1}[Re-Rc(T_0)]$$

$$M' = M - MG^t[GMG^t+Ve+Vm]^{-1}GM$$

✓If $GMG^t << Ve+Vm$, $T' ≒ T_0$ and $GM'G^t ≒ GMG^t$

✓If $GMG^t >> Ve+Vm$, $GM'G^t ≒ Ve+Vm$

✓If $GMG^t ≒ Ve+Vm$, $GM'G^t ≒ 1/2 \times GMG^t$

- Prediction error induced by the cross-section errors

  Before adjustment: GMG^t After adjustment: $GM'G^t$

Where,
T_0 : Cross-section set before adjustment
M : Covariance before adjustment
Re : Measured values of integral experiments
Rc : Analytical values of integral experiments
Ve : Experimental errors of integral experiments
Vm : Analytical modeling errors of integral experiments
G : Sensitivity coefficients, $(dR/R)/(d\sigma/\sigma)$

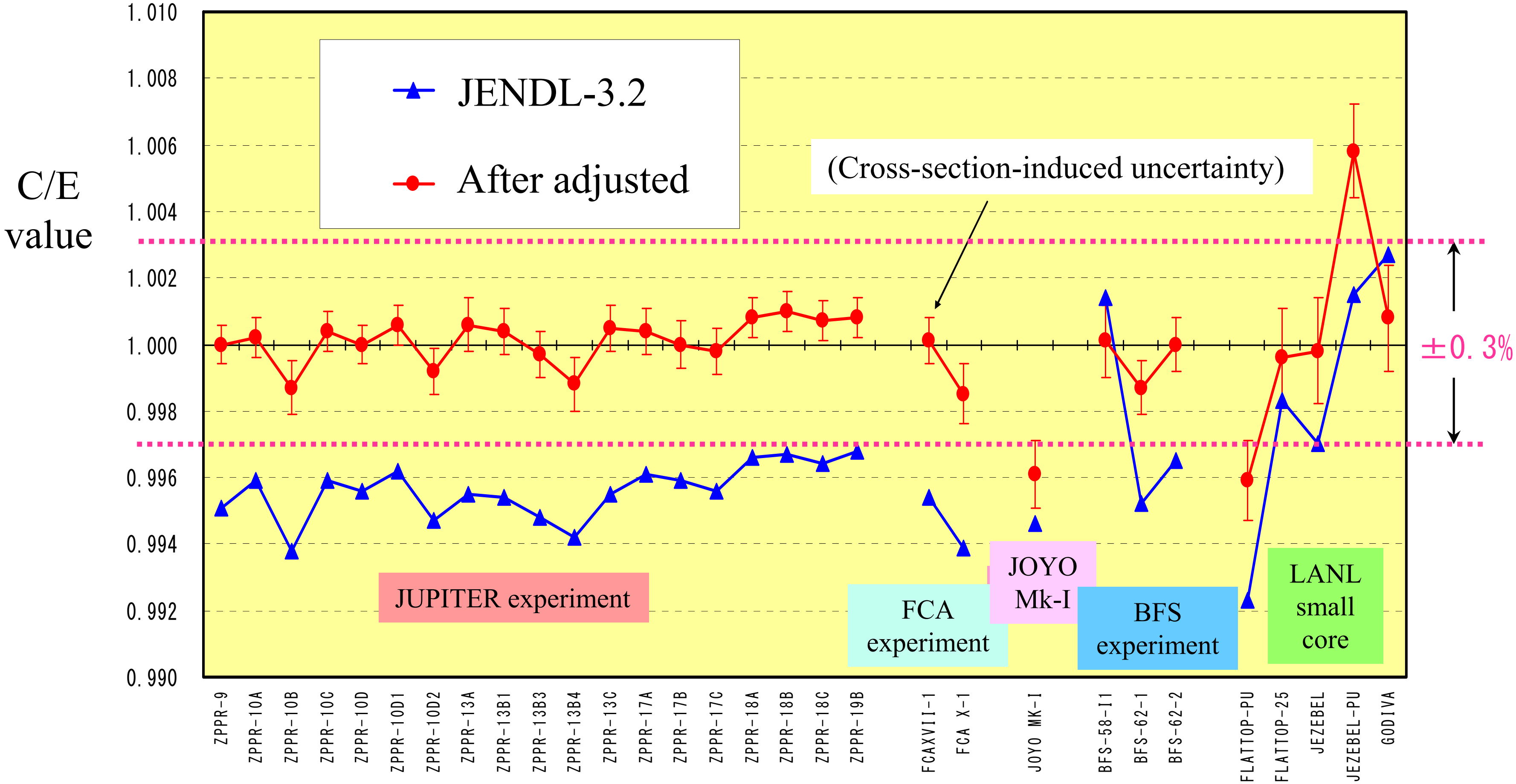
Determination of Experimental and Analytical uncertainties

- Experimental uncertainty
  - Follows the evaluation by experimenters like ANL.
- Analytical modeling uncertainty
  - Assumes it is proportional to the sensitivity against the degree of modeling detail,
  - Absolute value was decided to make the ratio of the chi-square value to the freedom approx. unity.
- Elimination of abnormal data
  - Excludes if the deviation of C/E value from unity is three times larger than the total uncertainty value.

(confidence level : 1σ)

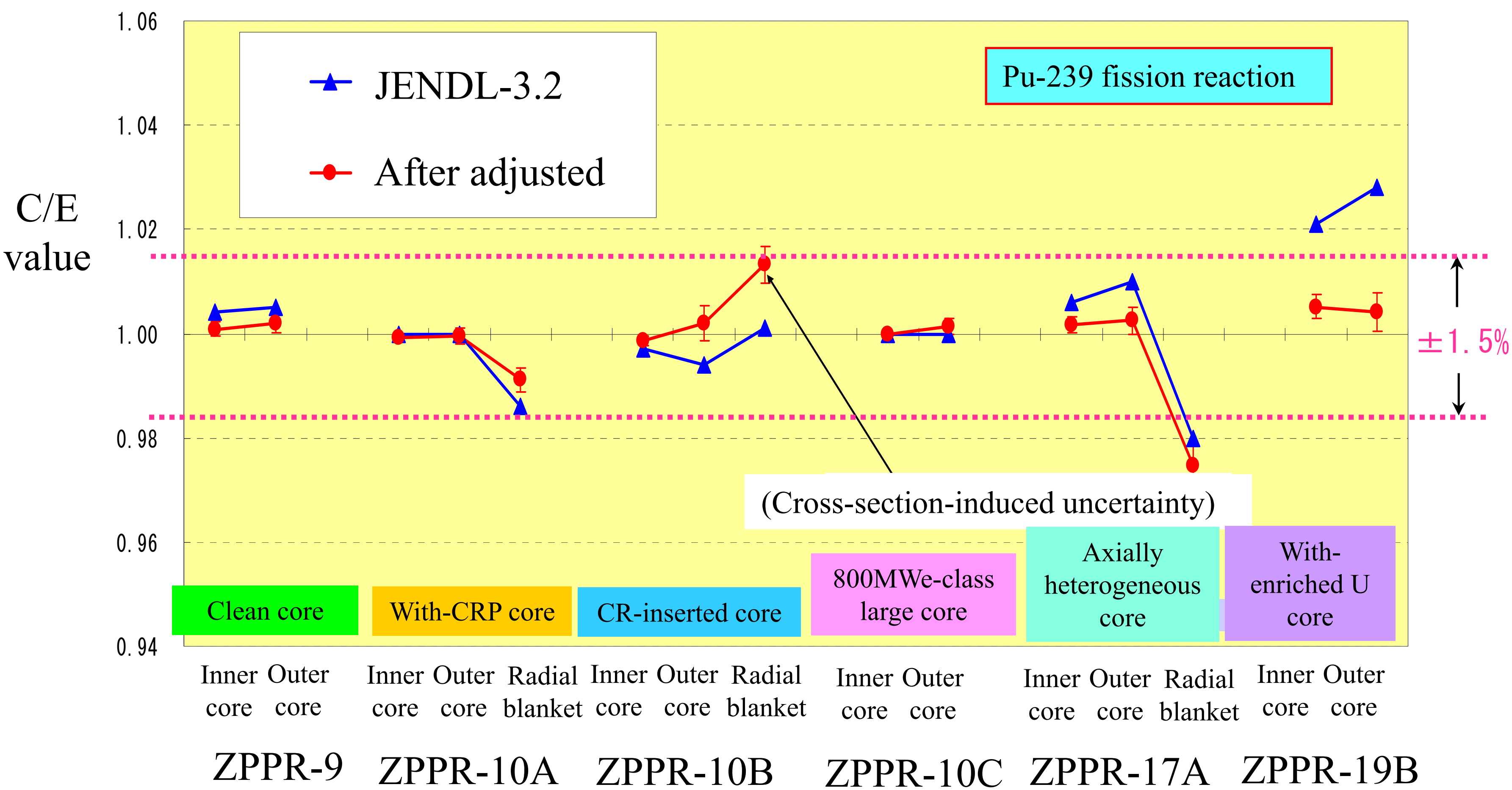
Core Parameter		Experimental uncertainty	Analytical Modeling uncertainty
Criti-cality	JUPITER, FCA, etc.	0.04%	0.17%
	Los Alamos	0.1～0.18%	0.15%
F28/F49 Ratio		2.5%	1.1%
F25/F49、C28/F49 Ratio		2.2%	0.55%
F49 Distribution		1.0%	0.6～1.2%
Control Rod Worth		1.2%	1.3%
Sodium Void Reactivity		2%	5.5～8.8%
Doppler Reactivity		2.0～3.0%	5.0～6.6%

Analytical Results (1/4) - Criticality -



- The C/E values of criticality after adjusted are within ±0.3% Δk, except several small cores.
- The good performance is not only for Pu-fuel cores, but enriched-U fuel cores.

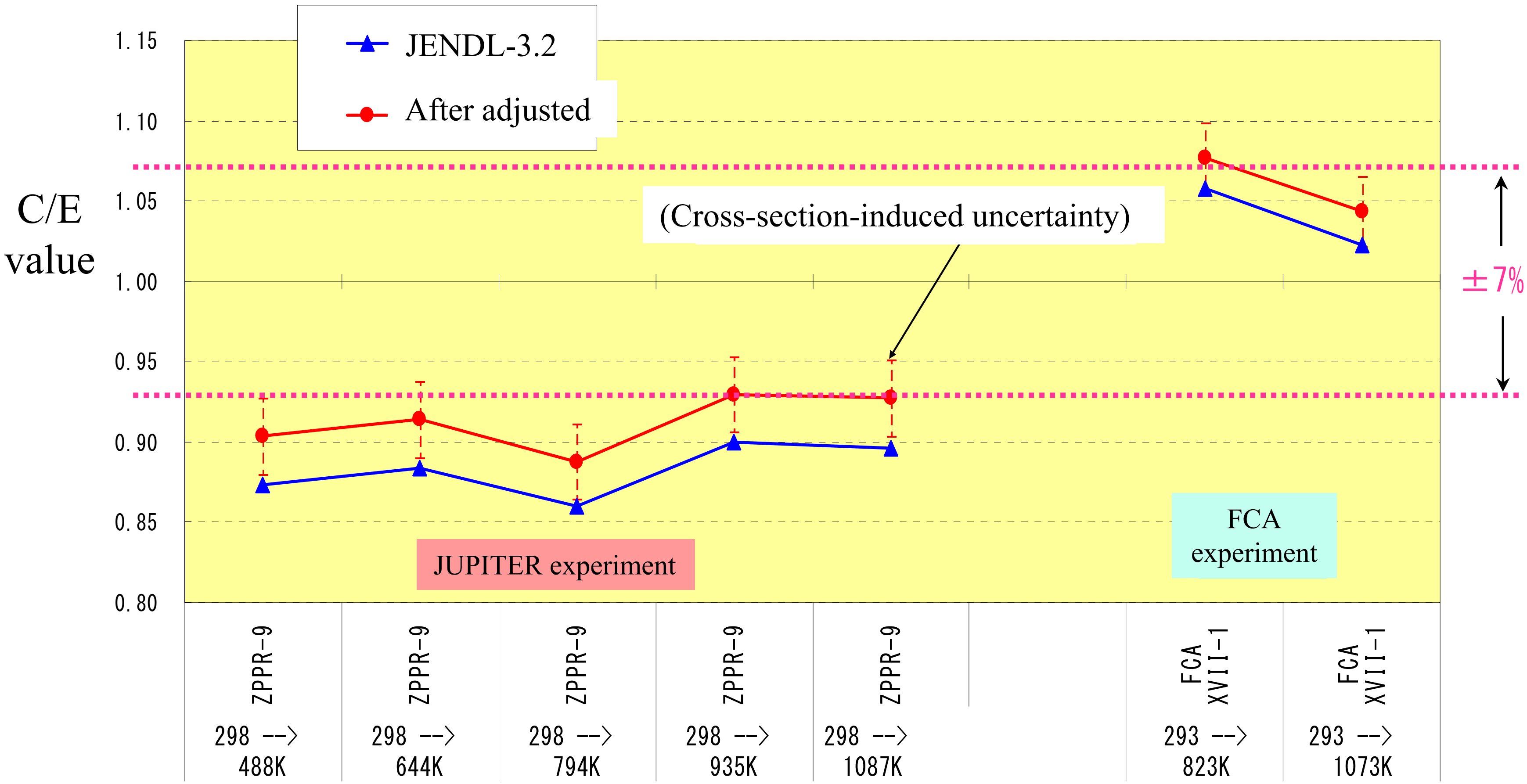
Analytical Results (2/4) - Power Distribution -



<u>JUPITER experiment series</u>

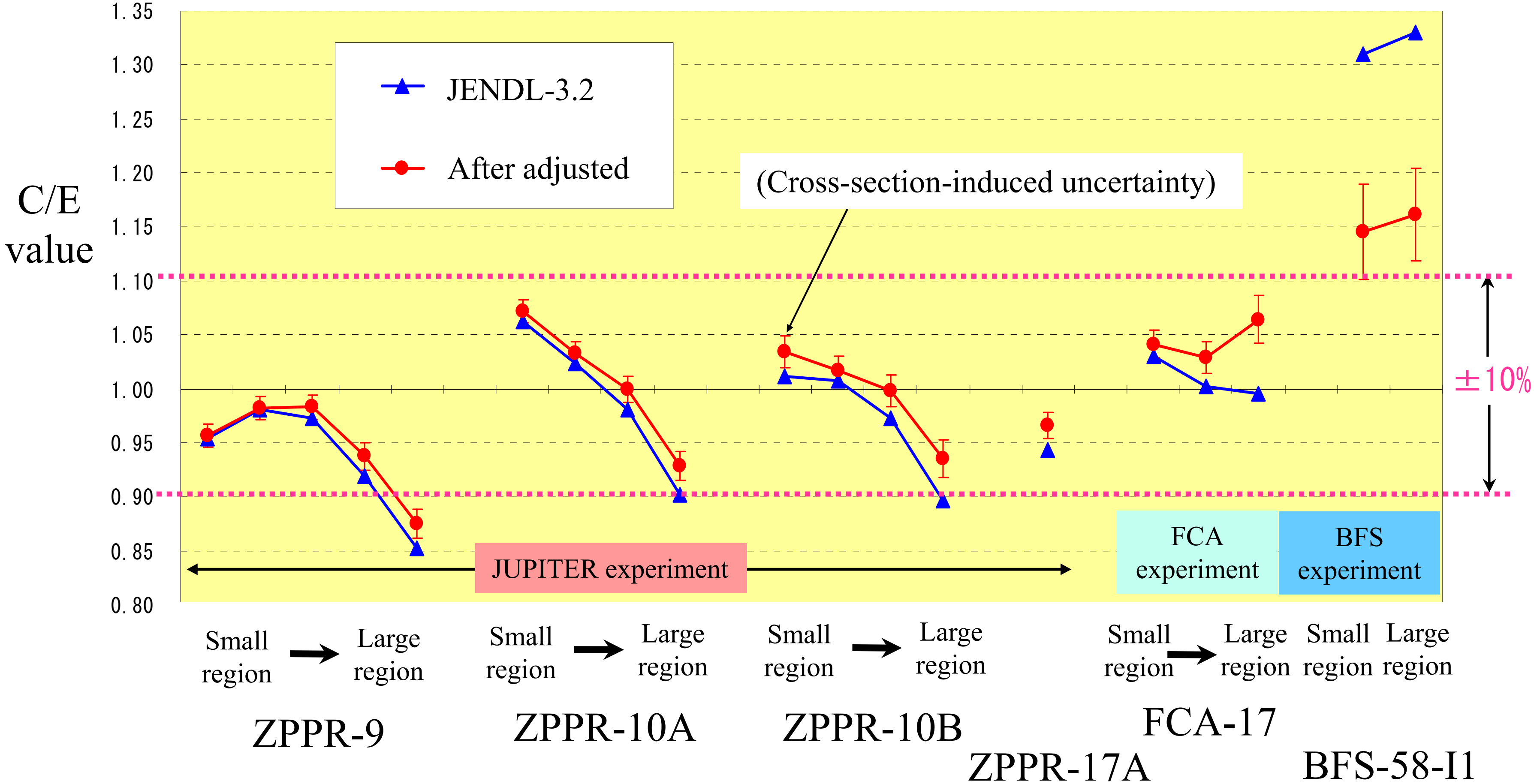
- The C/E values of reaction rate distribution after adjusted are sufficiently smaller than ±1.5% in the core fuel region.
- There is room for improvement for the blanket region.

Analytical Results (3/4) - Doppler Reactivity -



- It seems Re-investigation is needed for the accuracy of sample Doppler reactivity measurements.

Analytical Results (4/4) - Sodium Void Reactivity -



- The effect of adjustment is small for JUPITER and FCA experiment, and the C/E values are within app. ±10%.
- The discrepancy of C/E values from 1.0 may be caused by something, besides nuclear data.

Design Accuracy Improvement of a 600 MWe-class FBR

(*1σ value)

Design method / Nuclear parameter	No use of integral information	E/C-bias method	Cross-section adjustment method
Criticality (End of equilibrium cycle)	0.79	0.37	**0.16**
Breeding ratio (C28/F49 reaction rate ratio)	1.4	2.3	**0.7**
Power distribution (Outer core region)	1.2	0.8	**0.7**
Doppler reactivity (Whole core region)	6.3	5.7	**4.2**
Na void Reactivity (Whole core region)	4.7	5.6	**3.3**
Burnup reactivity loss	5.5	7.4	**3.3**

JAEA

Concluding Remarks

- Though the importance of nuclear data covariance was recognized from the beginning of nuclear data use, the national project of Japan to develop the covariance data was launched in 1989. First, it was only a primitive estimation by the statistics of experimental data scattering around JENDL. Later, some refined evaluation tools such as the KALMAN system were developed, and JENDL-3.2 and 3.3 has been equipped with the covariance of important isotopes for FBR application. In March of 2010, the JENDL-4 library will be released, one of whose major objectives is the expansion of its covariance data.

- People know the recent activity of OECD/NEA/WPEC, SG-26, one of the conclusions is the next SG establishment to survey the cross-section adjustment methodology.

We have gulped down the apple in the garden of Eden. What we should do now is to prepare the reliable nuclear-data covariance, and to use it for the reactor nuclear design to keep its accountability to public.