Uncertainties in Measurements and Calculations of Nonelastic Cross Sections

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Objectives

Main interest: neutron interactions from 0 to ~30 MeV

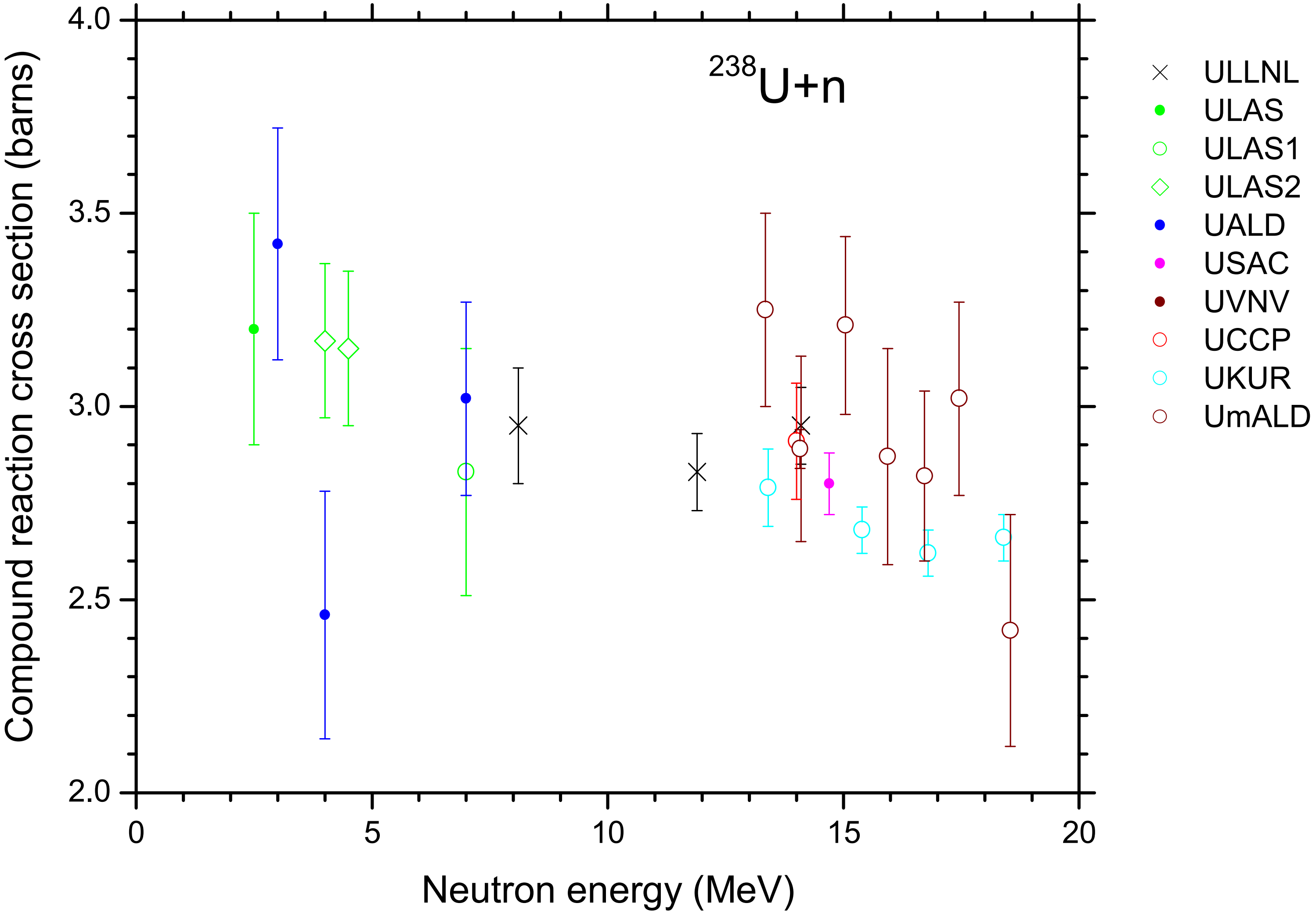
Reaction (nonelastic) cross section is most important for applications

- **Show examples of optical-model calculations of this quantity**
  - *Important case: neutrons on deformed nucleus,* ^{238}*U; also* ^{56}*Fe*
- **Show present (unsatisfactory) experimental status**
- **Recommendations for improving the measurements**

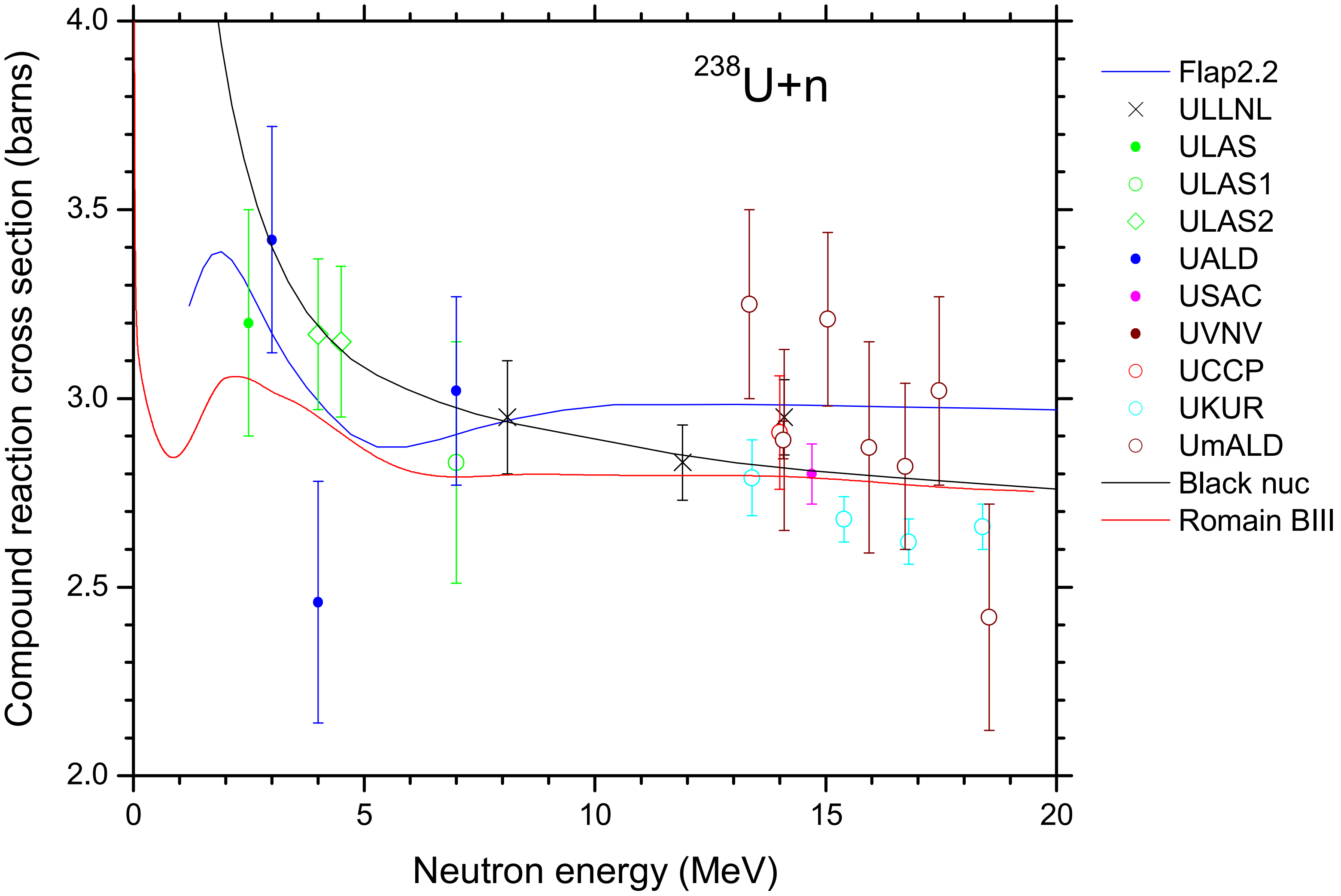
Important quantities for neutron projectiles

- **Total cross section – the sum of everything that can happen**
  - *If careful, can measure to 1%*
- **Elastic cross section**
  - *Typically measure angular distributions to 3-5%; then can integrate over solid angle if that's what's needed*
- **Nonelastic cross section – total minus (angle-integrated) elastic**
  - *Hard to measure directly (and reliably)*
- **Compound cross section – part of total that forms the compound nucleus**
  - *Important because is weighted sum of transmission coefficients*
  - *Spherical nuclei: same as nonelastic*
  - *Deformed nuclei: total – elastic – direct excitations*

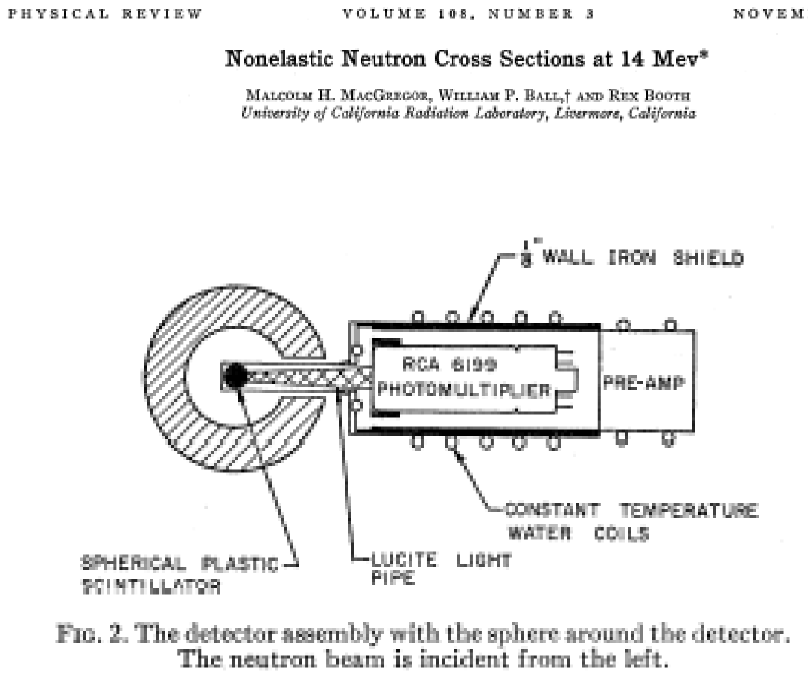
Direct measurements constrain compound cross section to only 10-20% (if believe all data)



Optical models are helpful, but they don’t really solve the problem, especially below ~7 MeV



Most direct measurements have been made by sphere-absorption technique



PHYSICAL REVIEW VOLUME 108, NUMBER 3 NOVEMBER 1, 1957

Nonelastic Neutron Cross Sections at 14 Mev*

Malcolm H. MacGregor, William P. Ball,† and Rex Booth
University of California Radiation Laboratory, Livermore, California

Fig. 2. The detector assembly with the sphere around the detector. The neutron beam is incident from the left.

LLNL made ~3% measurements that are well documented, circa 1960;

Energies were ~8, ~14, ~26 MeV

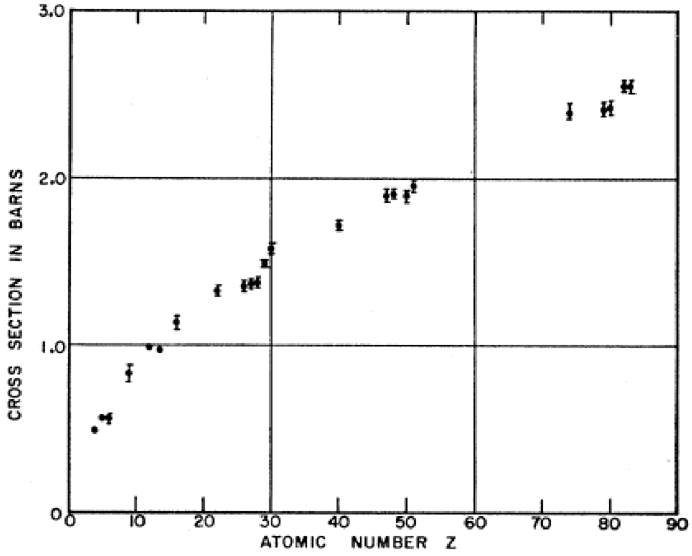


Fig. 12. Neutron nonelastic collision cross-section measurements for 23 elements at 14.2 Mev. Data from reference 1.

Compare sphere-in vs. sphere-out; result is nonelastic cross section

Extreme care required in experiment as well as absorption & multiple scattering corrections

An alternative: using Wick's limit we can improve the subtraction result $\sigma_{reac} = \sigma_{tot} - \sigma_{elas}$

Fractional deviation from Wick's limit: $\eta = \frac{\sigma_0 - \sigma_0^W}{\sigma_0^W}$

elastic cross section at 0°

Wick-limit value for 0° cross section

We define $F = \frac{\sigma_{elas}}{\sigma_0} = \frac{1}{\sigma_0} \int d\Omega \frac{d\sigma_{elas}}{d\Omega}$

These yield a new expression for the reaction cross section

$$\sigma_{reac} = \sigma_{tot} - (1+\eta) F \left(\frac{k}{4\pi} \right)^2 \sigma_{tot}^2$$

There are three independent quantities:

σ_{tot} **(experimental – correlation in 2 terms diminishes error)**

F **(experimental – need relative, not absolute cross section)**

η **(theory – we work in region where value is small, ≤3%)**

Error from total cross section greatly reduced by correlation between terms in new subtraction method

Uncertainties from the three independent input quantities are

$$\frac{\Delta\sigma_{reac}^{(1)}}{\sigma_{reac}} = \left|2 - \frac{\sigma_{tot}}{\sigma_{reac}}\right| \frac{\Delta\sigma_{tot}}{\sigma_{tot}}$$

This error can be very small!!
$\sigma_{tot}/\sigma_{reac}$ **~ 2 experimentally**
Error is exactly 0 at some energies

$$\frac{\Delta\sigma_{reac}^{(2)}}{\sigma_{reac}} = \left(\frac{\sigma_{tot}}{\sigma_{reac}} - 1\right) \frac{\eta}{1+\eta} \frac{\Delta\eta}{\eta}$$

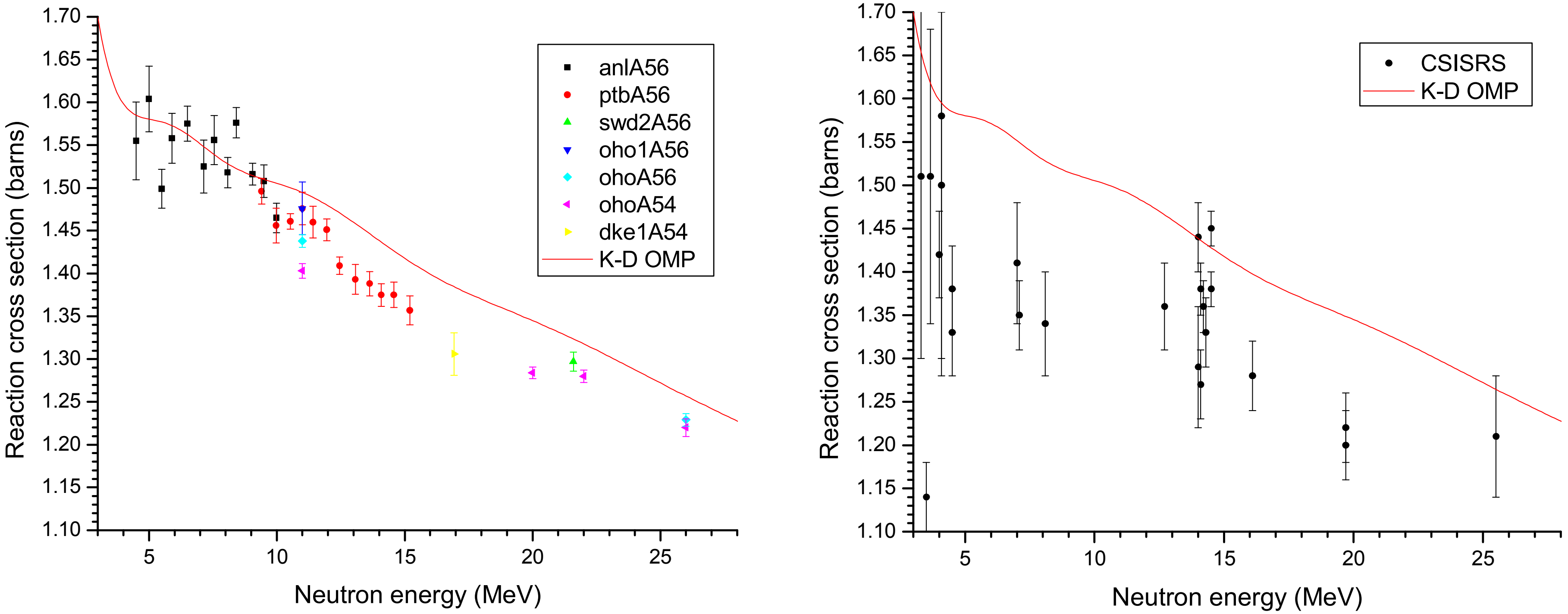
We choose fractional error $\Delta\eta/\eta$ to be 0.3 of calculated η – this is the model-dependent error

$$\frac{\Delta\sigma_{reac}^{(3)}}{\sigma_{reac}} = \left(\frac{\sigma_{tot}}{\sigma_{reac}} - 1\right) \frac{\Delta F}{F}.$$

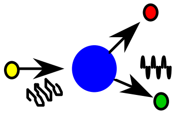
Good angular distributions yield $\Delta F/F$ in 0.01-0.02 range

These 3 errors add quadratically to get complete uncertainty

New results show significantly smaller uncertainties than those in CSISRS database for iron

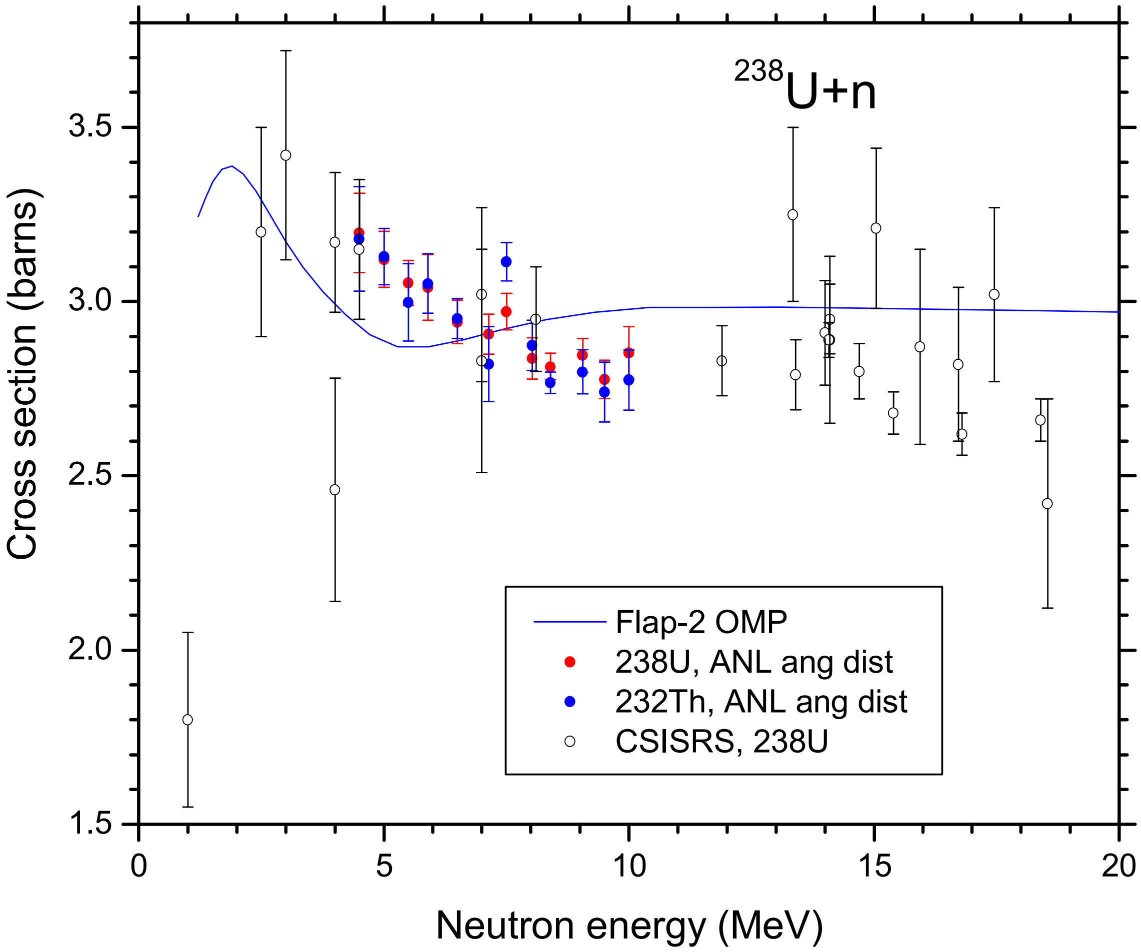


Koning-Delaroche potential slightly high (~4%) over most of energy range

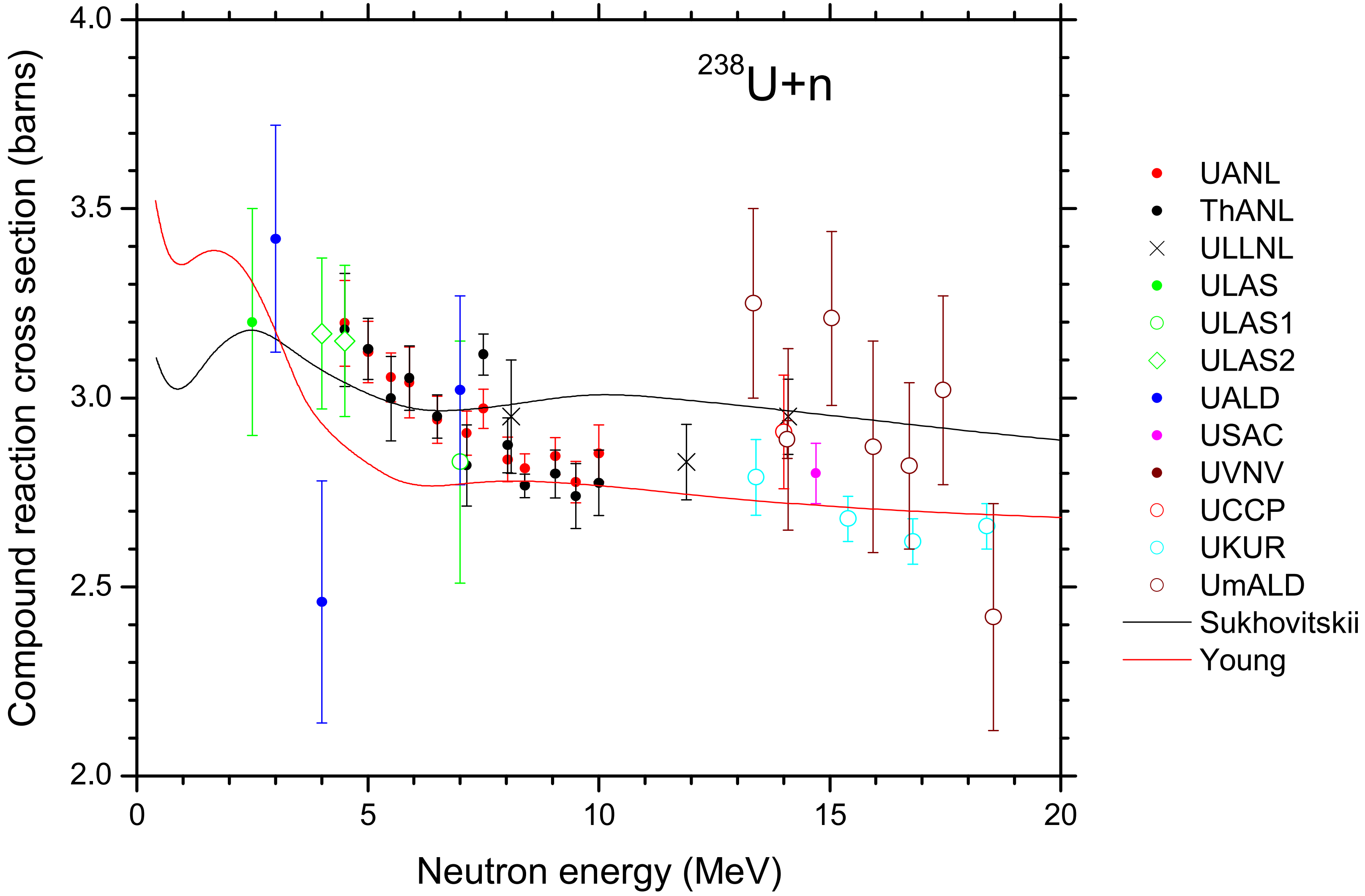


Extension to deformed actinide nuclei: ^{232}Th, ^{238}U Maps out E-dependence, 4-10 MeV

Extension involves correcting elastic for unresolved inelastic excitations to ground-state band



Neither new nor old potentials fully agree with precise data from modified subtraction technique



Conclusions and recommendations

Improved experimental data are important to provide a stable basis for testing optical model predictions of nonelastic cross sections

- **New set of careful sphere-transmission measurements should be done on selected nuclei, particularly ^{238}U, ^{232}Th**
  - *Improvements should be possible with modern techniques*
  - *Can validate the subtraction technique*

- **We should apply new subtraction technique in all cases where suitable angular distribution measurements are available**

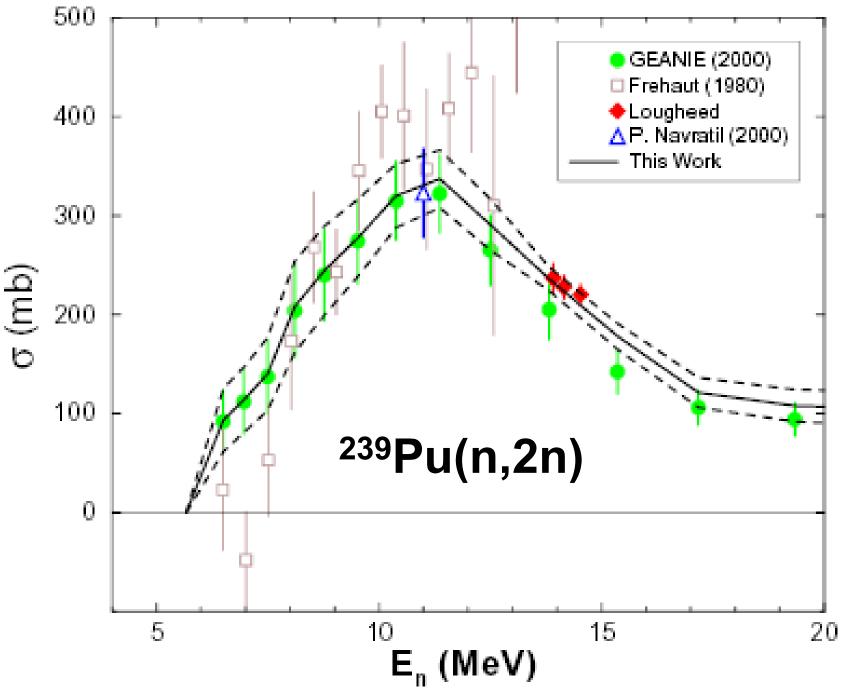
Some supporting viewgraphs begin here – show as required

Why are accurate compound cross sections important? Example: independent check on GEANIE ^{239}Pu(n,2n)

GEANIE measurement of 239Pu(n,2n) was inferred by nuclear model calculations from measured gammas

Independently determined value near 11 MeV (Navratil) agreed with GEANIE and gave confidence that nuclear modeling was accurate

Result used subtraction technique, subject to errors from difference of large numbers – need σ_{comp} accurate to ~3%

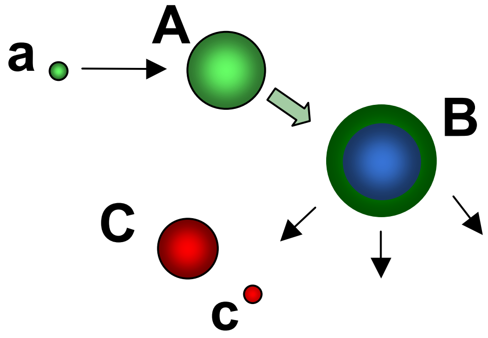


$$\sigma_{n,2n} = \sigma_{comp} - \sigma_{fiss} - \sigma_{n,n'}$$

0.31b 2.99b 2.23b 0.23b

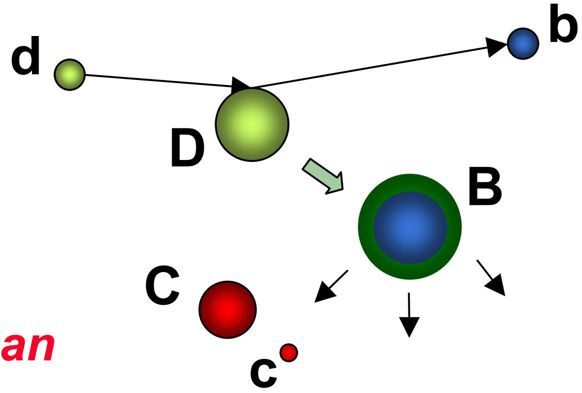
Why are accurate compound cross sections important? Example: accuracy of surrogate measurements

Want cross section for a+A→c+C going through compound nucleus B



Indirect method: make compound B by direct reaction D(d,b)B

Measure P_{cC}, the probability of obtaining c+C in coincidence with the outgoing particle b in the direct reaction



In simplest analysis, the desired cross section is just

$$\sigma_{A(a,c)C} = \sigma^{a+A}_{comp} P_{cC}$$

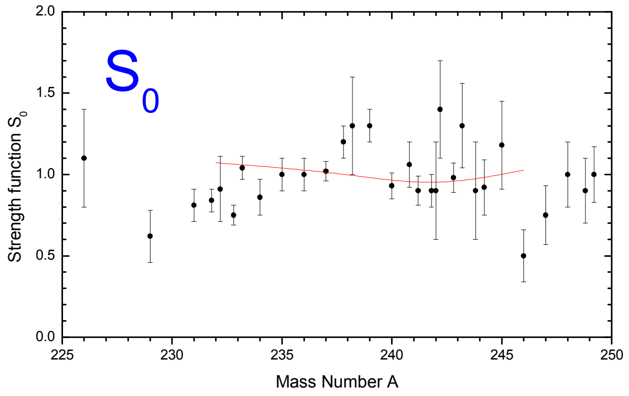
The accuracy in the desired cross section can be no better than the accuracy of the compound cross section!

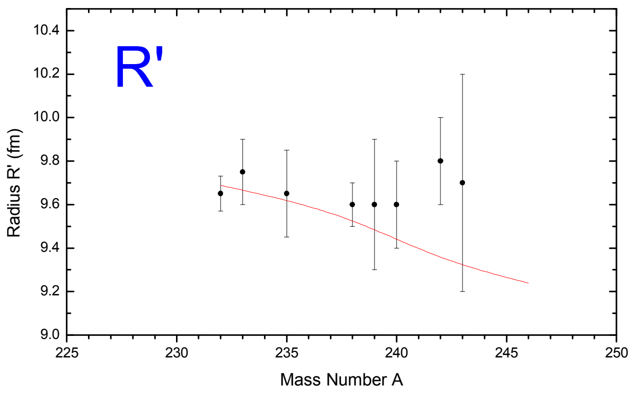
Example: the flap2.2 actinide potential

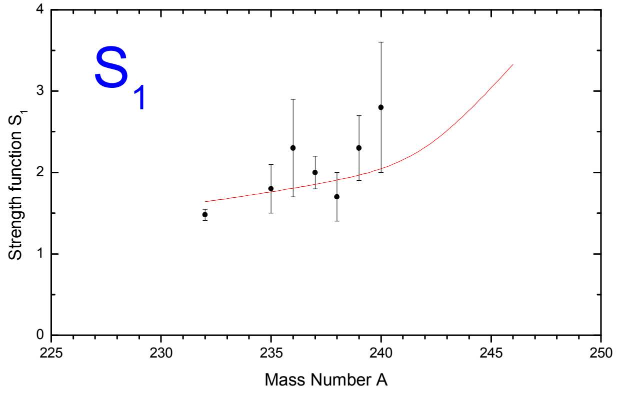
Low-energy parameters derived from energy averages in resonance region

Behavior of this potential is reasonable and typical for optical models

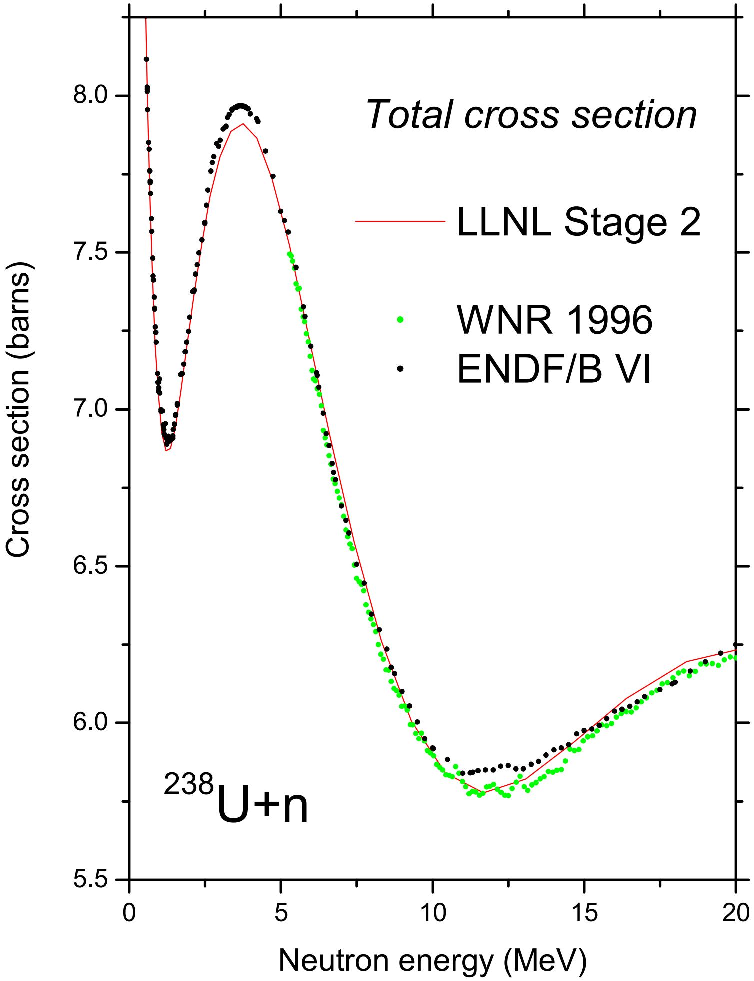
This region shows abnormal sensitivity to real potential strength – 1% change causes 40% change in S_1

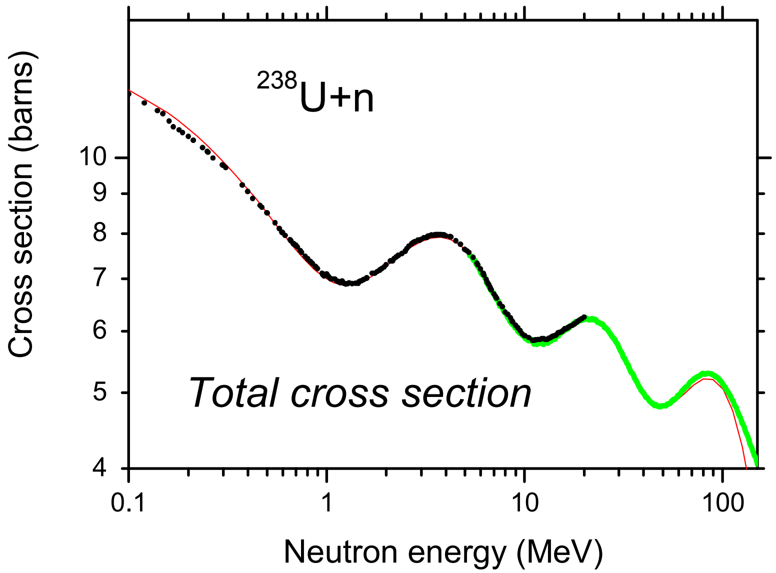


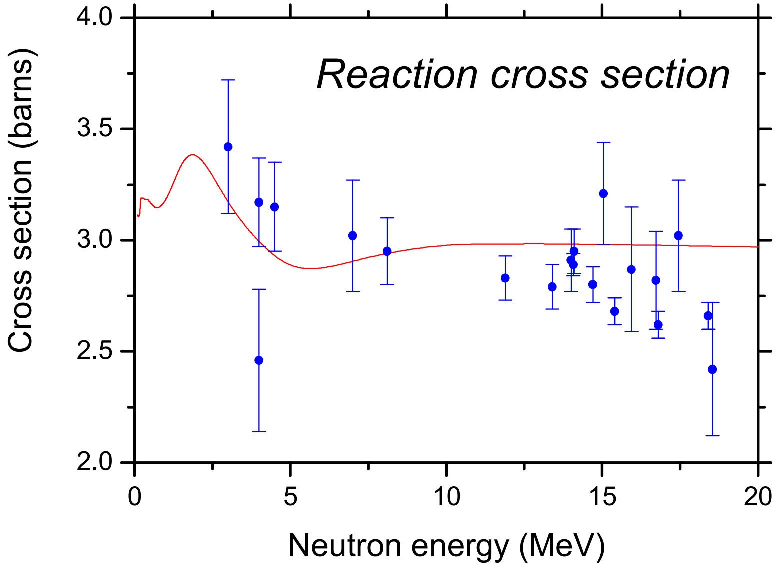




Flap2.2 fits total cross section to ~1% up to ~100 MeV and predicts reaction cross section







Putting Wick's limit to work: a new method for determining reaction (nonelastic) cross sections

Reaction cross sections are difficult to measure directly

- **Subtraction of angle-integrated elastic from total cross section**
  - *Subject to large errors from independent measurements*
- **Spherical-shell transmission measurements**
  - *Requires corrections for multiple scattering and absorption, but a few measurements have been made at the 3% level*

The subtraction technique can be greatly improved by using Wick's limit to correlate the uncertainties in the two terms

The price: a very weak model dependence

Wick's limit is key to a new method of getting compound cross section by subtraction

Wick's limit is derived from the optical theorem $\mathrm{Im} f(0^o) = \frac{k}{4\pi}\sigma_{tot}.$

Since $\sigma_0 = [\mathrm{Re} f(0^o)]^2 + [\mathrm{Im} f(0^o)]^2 \geq [\mathrm{Im} f(0^o)]^2$

Zero-degree differential cross section

we find $\sigma_0 \;\geq\; \sigma_0^W \equiv \left(\frac{k}{4\pi}\sigma_{tot}\right)^2$ **(Wick's limit)**

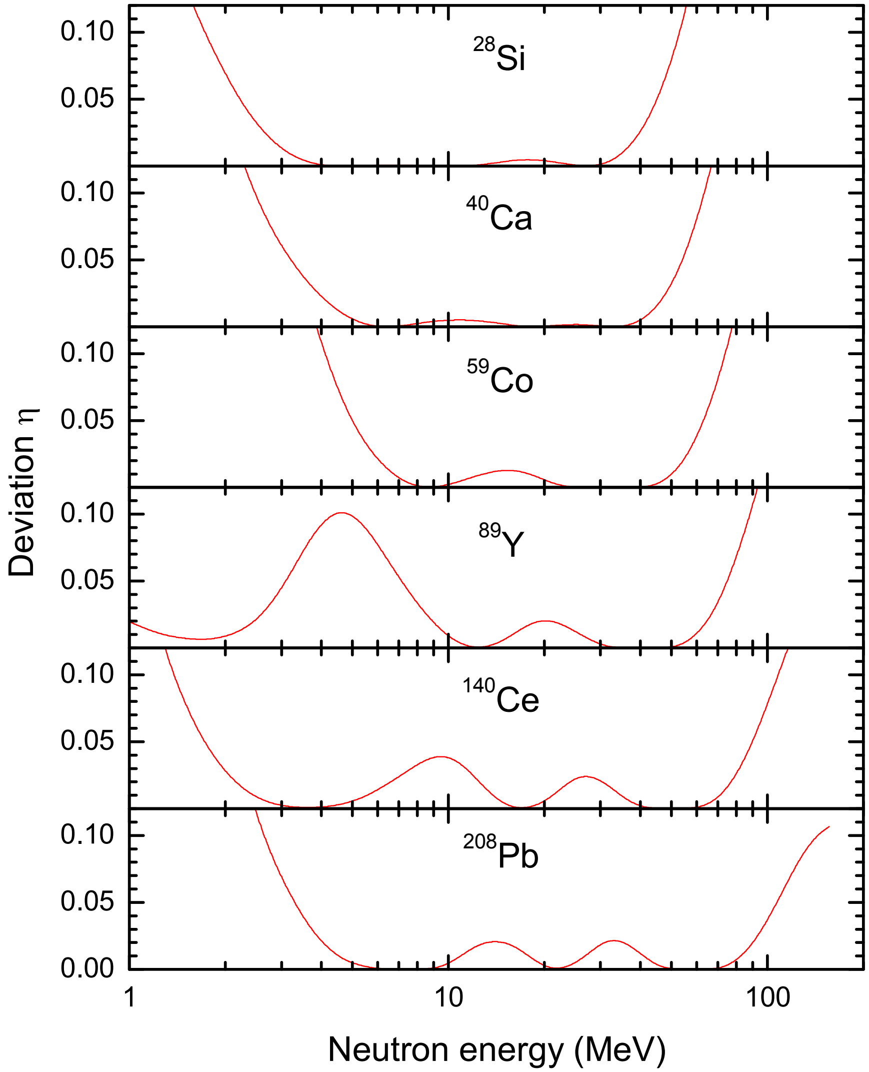
We define a fractional deviation from Wick's limit $\eta = \frac{\sigma_0 - \sigma_0^W}{\sigma_0^W}$

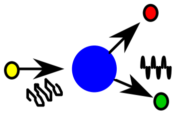
Wick's limit is useful when η is small

Optical model shows deviation of 0-deg cross section from Wick's limit is small from ~5-50 MeV

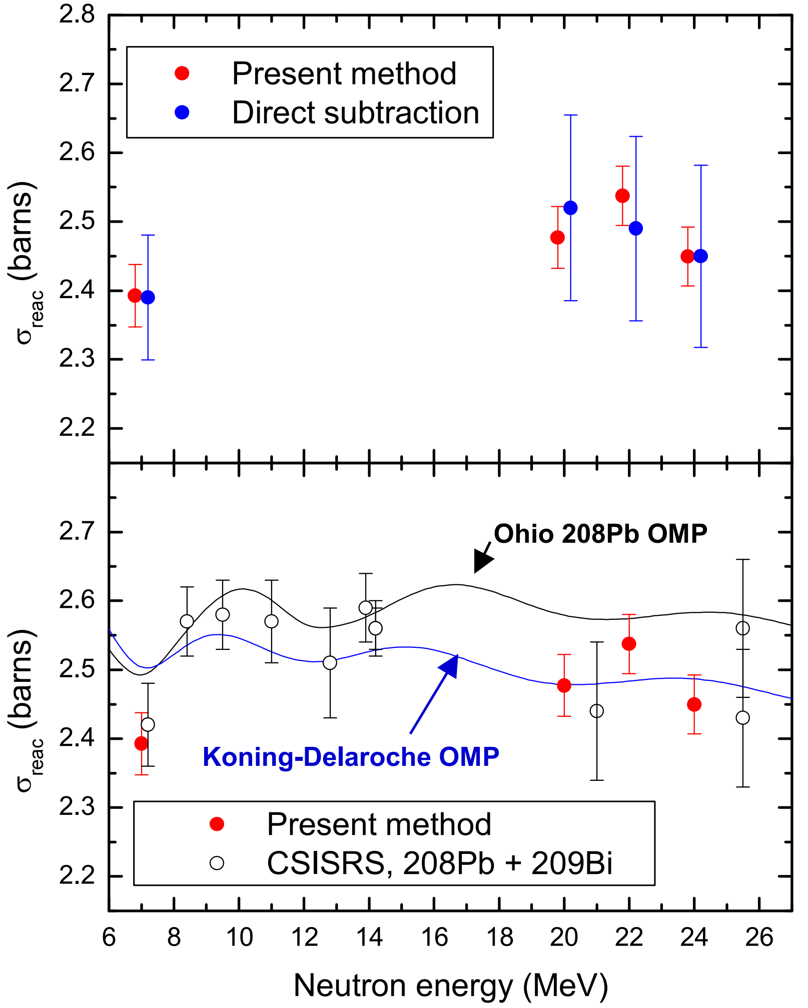
Calculations with Koning-Delaroche potential

Fractional deviation is small over wide mass as well as energy range





First test: n + ^{208}Pb



Angular distributions measured at Ohio University – Phys. Rev. C30, 796 (1984)

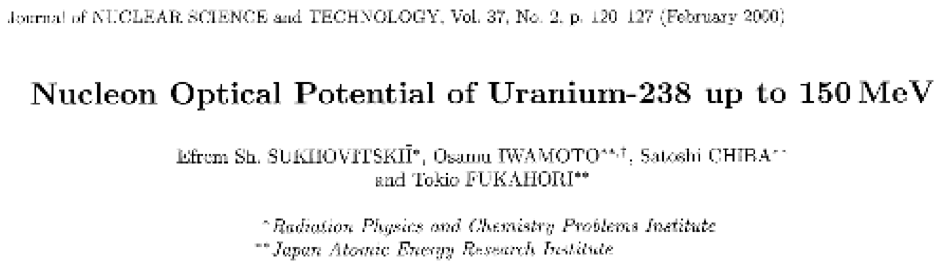
Total cross section measured with 1% uncertainties at LANSCE/WNR – Phys. Rev. C47, 237 (1993)

Reaction cross sections calculated with both direct and modified subtraction

New method reduces errors

from ~6% to ~2%

New act in town – potentials of Sukhovitskii/Chiba et al.



Journal of NUCLEAR SCIENCE and TECHNOLOGY, Vol. 37, No. 2, p. 120–127 (February 2000)

Nucleon Optical Potential of Uranium-238 up to 150 MeV

Efrem Sh. SUKHOVITSKIĬ*, Osamu IWAMOTO**,†, Satoshi CHIBA**
and Tokio FUKAHORI**

*Radiation Physics and Chemistry Problems Institute
**Japan Atomic Energy Research Institute

This potential couples 6 states, not 3 as typical

Fit to as many observables as possible

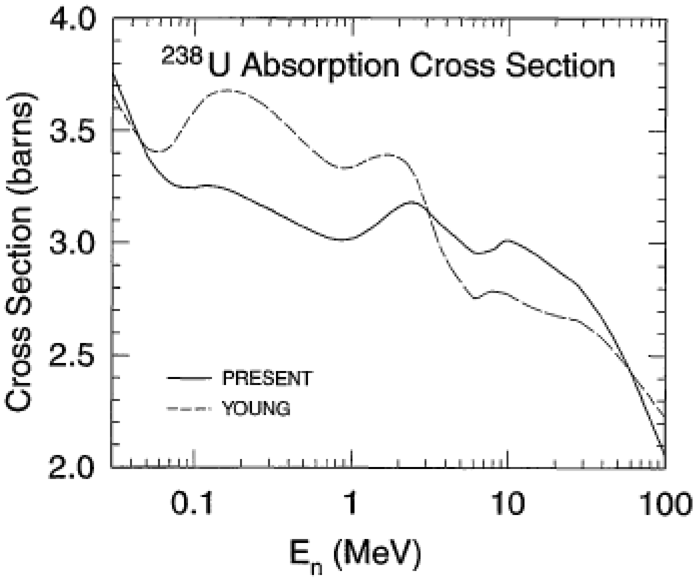
Several variations and improvements, ~1999-present

Duplicating authors' results exactly requires their coupled-channels code, which is available

Calculations lengthy if all couplings & relevant multipoles included

Some stability problems trying to duplicate results approximately with ECIS

Latest versions include isospin and dispersion corrections



An important feature –

Different behavior of compound cross section from previous potentials

Roughly comparable behavior on other observables

Need reliable measurements to test this behavior!