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Use of Covariance Data to Select Experiments Relevant to Target Systems for GNEP

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Introduction

Under the Fast Reactor Campaign of Global Nuclear Energy Partnership, work (in collaboration with INL, LANL and BNL) was initiated to improve the nuclear data by a combined use of the science-based covariance data and integral experiments.

- ➡ The purpose of this activity is to precisely account for existing integral experiment data to reduce the uncertainty of reactor performance predictions.
- ➡ For the initial phase of study, the equilibrium cycle metal and oxide core configurations of the reference 1000 MWt Advanced Burner Reactor (ABR) were selected as the tentative target systems.

Selected Experiments

Main Features of the Investigated Systems.

	Fuel Type	Coolant	Core Volume [liters]	% Pu in (U+Pu+MA)	% Pu239 in Pu	% MA^(a) in (U+Pu+MA)	Blanket	Reflector
ZPR3 Assembly 53	Carbide		226	40	93	0	Yes	No
ZPR3 Assembly 54	Carbide		197	40	93	0	No	Yes
ZPPR-2	Oxide	Na	2407	15 - 22 ^(b)	87 - 87 ^(b)	0.04 - 0.06 ^(b)	Yes	Yes
ZPPR-9	Oxide	Na	4599	11 - 16 ^(b)	87 - 87 ^(b)	0.07 - 0.10 ^(b)	Yes	Yes
ZPPR-15A	Metal	Na	2505	12 - 18 ^(b)	87 - 87 ^(b)	0.12 - 0.18 ^(b)	Yes	Yes
ZPR6 Assembly 6A	Oxide	Na	3990	-	-	-	Yes	No
ZPR6 Assembly 7	Oxide	Na	3120	15 - 15 ^(b)	87 - 87 ^(b)	0.04 - 0.04 ^(b)	Yes	No
CIRANO/ZONA2A	Oxide	Na	477	27 - 25 ^(b)	79 - 92 ^(b)	0.73 - 0.07 ^(b)	Yes	No
CIRANO/ZONA2A3	Oxide	Na	425	27	79	0.73	Yes	Yes
CIRANO/ZONA2B	Oxide	Na	388	27	79	0.73	No	Yes
ABR Metal Core	Metal	Na	3532	17 - 22 ^(b)	54 - 52 ^(b)	1.5 - 2.2 ^(b)	No	Yes
ABR Oxide Core	Oxide	Na	4730	21 - 23 - 33 ^(c)	47 - 46 - 46 ^(c)	2.2 - 2.4 - 4.1 ^(c)	No	Yes

^(a) MA: Minor Actinides (Np, Am, Cm isotopes);

^(b) Inner Core - Outer Core;

^(c) Inner Core - Middle Core - Outer Core

The study will be completed with the analysis of MUSE4, COSMO, GODIVA, BIGTEN, JEZEBEL (Pu239, Pu240, U233), FLATTOP (Pu239, U235, U233) in a GNEP report by G. Aliberti, W.S. Yang, R.D. McKnight by September 30, 2008.

The approach and theoretical background

Integral experiments play an essential role in the reduction of design uncertainties related to reactor neutronics calculations. For this purpose, it is recommended that the proposed experiment shows neutronics features similar to those of the calculated reactor.

- ➡ A quantitative and synthetic measure to judge the relevance of selected experiments to the ABR systems can be based on the “representativity” concept. The approach uses a sensitivity methodology associated to selected integral parameters and based on the Generalized Perturbation Theory (GPT).
- ➡ To carry out a representativity study, sensitivity coefficients have to be first calculated for the selected integral parameters:

$$S = \frac{dP}{P} \left/ \frac{d\sigma}{\sigma} \right. \quad \sigma = \sigma_x \text{ with } x = \text{isotope, energy group, cross-section type.}$$

The approach and theoretical background

Sensitivity studies using Generalized Perturbation Theory (GPT).

- ↳ Uncertainty assessment;
- ↳ Representativity analysis.

Using the sensitivity coefficients, S_R , for each integral parameter P of the reactor R under study and the covariance matrix D, the uncertainty on the integral parameter P can be evaluated:

$$I_R^2 = S_R^+ D S_R$$

An integral experiment can be conceived in order to reduce the uncertainty I_R^2 . If S_E is the sensitivity matrix associated with this experiment, a “representativity factor” defined as:

$$r_{RE} = \frac{(S_R^+ D S_E)}{\sqrt{[(S_R^+ D S_R)(S_E^+ D S_E)]}}$$

can be introduced to quantify the similarity between the reactor and the selected experiment.

It can be shown that the uncertainty on the reference parameter P is reduced by: $I'_R^2 = I_R^2 \cdot (1 - r_{RE}^2)$

- ↳ From this expression it is clear that the experiment should be conceived in such a way that the sensitivity matrices S_E and S_R are as similar as possible, i.e. r_{RE}^2 should be as close to 1 as possible.

Computational tools

- ↳ Four response parameters were evaluated in this study: the core multiplication factor, spectral indices of U238 fission to Pu239 fission and of U238 capture to Pu239 fission at the core center, and coolant void reactivity coefficient. For the spectral indices, only the indirect effect of the sensitivity coefficients has been considered, since the direct effect, which essentially dominates the sensitivity profiles with the U238 and Pu239 reactions (fission or capture), is of a minor interest.
- ↳ For the present analysis, cross-sections were generated using the MC²-2 code and the ENDF/B-VII nuclear data in a 33 energy group structure. Sensitivity coefficient calculations were performed in diffusion theory using the VARI3D code. Flux, adjoint flux, and generalized adjoint flux were calculated with the finite difference diffusion theory option of the DIF3D code. Previous studies demonstrated that for the kind of systems under investigation, the transport and diffusion approaches show generally non-negligible differences in the absolute values of calculated integral parameters, but no significant difference is observed on the sensitivity coefficients.

Uncertainty Analysis

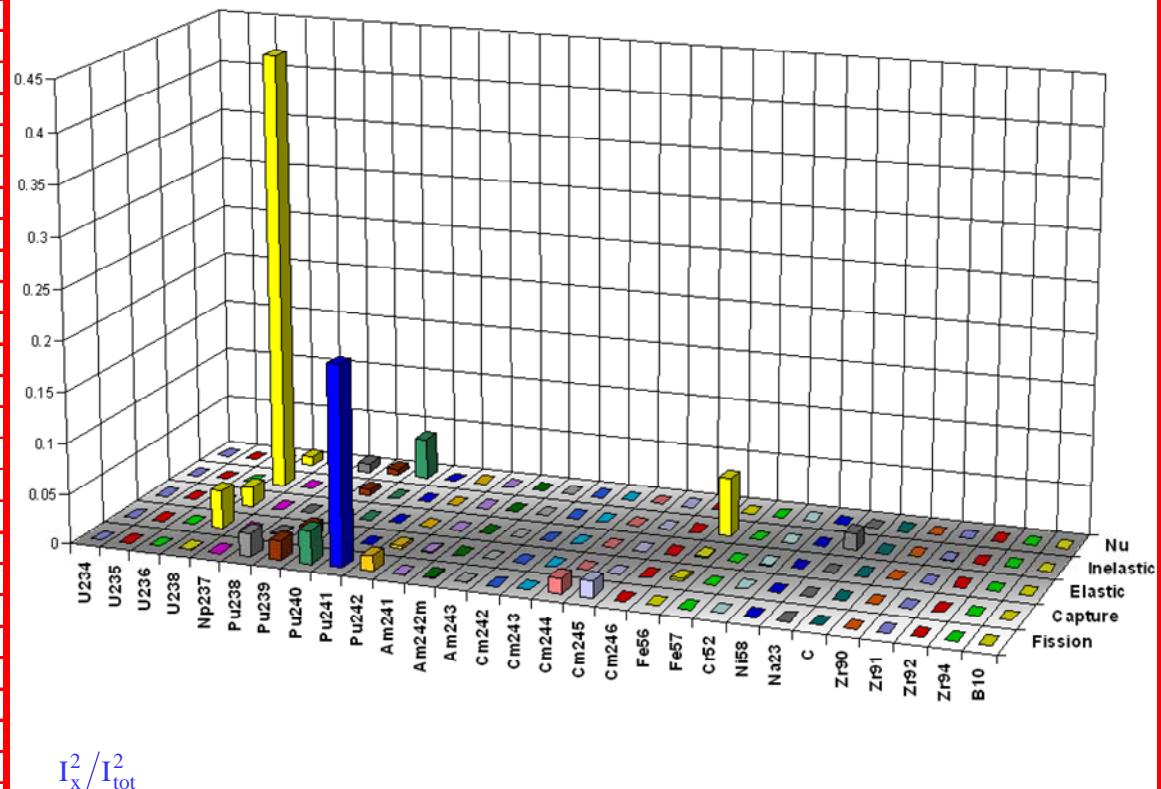
“Diagonal” and “Full BOLNA” Total Uncertainties. Values are in %.

	k_{eff}		$\frac{\langle \sigma_{\text{fiss,U}^{238}} \Phi \rangle}{\langle \sigma_{\text{fiss,Pu}^{239}} \Phi \rangle}$		$\frac{\langle \sigma_{\text{capt,U}^{238}} \Phi \rangle}{\langle \sigma_{\text{fiss,Pu}^{239}} \Phi \rangle}$		Void Coefficient	
	Diag	Full	Diag	Full	Diag	Full	Diag	Full
ZPR3-53	0.70	0.88	2.22	3.13	0.82	1.06	-	-
ZPR3-54	0.87	1.15	2.27	3.28	0.95	1.26	-	-
ZPPR2	0.70	0.92	4.71	5.90	0.74	1.06	172.34	212.55
ZPPR9	0.92	1.25	5.58	7.18	0.79	1.14	10.22	13.32
ZPPR15A	0.72	1.00	5.18	6.10	0.85	1.16	8.18	9.66
ZPR6-6A	1.23	2.39	-	-	-	-	9.58	10.46
ZPR6-7	0.78	1.04	4.65	5.81	0.74	1.03	79.92	104.50
ZONA2A	0.66	0.90	3.74	4.91	0.93	1.38	8.92	10.41
ZONA2A3	0.58	0.80	3.70	4.88	0.92	1.37	7.17	8.45
ZONA2B	0.53	0.74	3.66	4.83	0.91	1.35	5.99	7.13
ABR Metal Core	0.95	1.47	5.19	6.33	0.94	1.34	10.94	13.10
ABR Oxide Core	0.95	1.44	4.75	5.47	0.70	0.99	6.66	7.82

Uncertainty profiles

Uncertainties BOLNA Full (%) for k_{eff} of ABR Metal Fuel.

Isotope	σ_{fiss}	σ_{cant}	σ_{el}	σ_{inel}	v	Total
U234	-	-	-	-	-	-
U235	-	0.01	-	-	-	0.01
U236	-	-	-	-	-	-
U238	0.04	0.28	0.21	0.97	0.14	1.04
Np237	0.02	0.01	-	0.01	-	0.02
Pu238	0.22	0.03	-	-	0.13	0.26
Pu239	0.21	0.16	0.03	0.11	0.11	0.31
Pu240	0.27	0.20	0.01	0.01	0.30	0.45
Pu241	0.65	0.03	-	0.01	0.02	0.65
Pu242	0.18	0.07	-	0.02	0.04	0.20
Am241	0.04	0.03	-	0.01	0.01	0.05
Am242m	0.05	-	-	-	-	0.05
Am243	0.02	0.02	-	0.02	0.01	0.03
Cm242	0.01	-	-	-	-	0.01
Cm243	0.01	-	-	-	-	0.01
Cm244	0.18	0.02	-	-	0.03	0.18
Cm245	0.18	0.01	-	-	0.02	0.19
Cm246	0.02	-	-	-	-	0.02
Fe56	-	0.07	0.05	0.35	-	0.36
Fe57	-	-	-	0.01	-	0.01
Cr52	-	0.01	0.04	0.01	-	0.04
Ni58	-	-	-	-	-	-
Na23	-	-	0.01	0.19	-	0.19
C	-	-	-	-	-	-
Zr90	-	0.01	0.01	0.04	-	0.04
Zr91	-	0.01	-	0.01	-	0.01
Zr92	-	0.01	-	0.02	-	0.02
Zr94	-	0.01	-	0.03	-	0.03
B10	-	-	0.01	-	-	0.01
Total	0.83	0.40	0.22	1.06	0.38	1.47

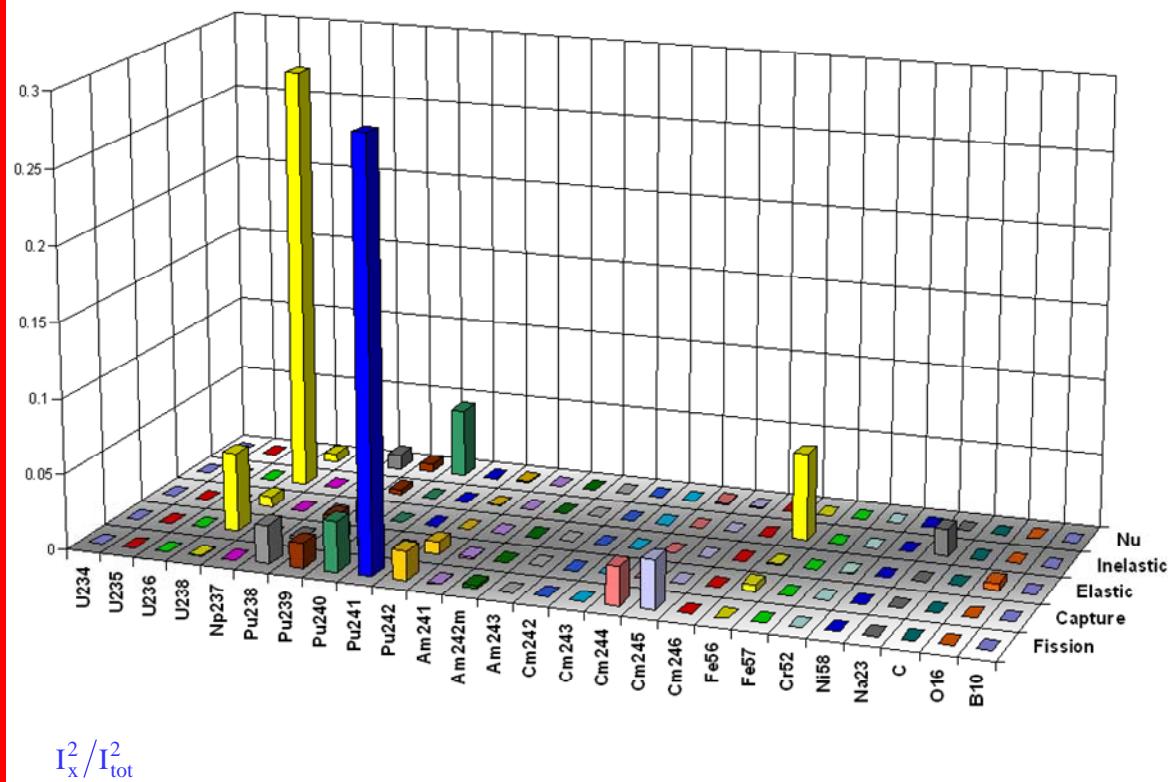


$$I_x^2 / I_{\text{tot}}$$

Uncertainty profiles

Uncertainties BOLNA Full (%) for k_{eff} of ABR Oxide Fuel.

Isotope	σ_{fiss}	σ_{capt}	σ_{el}	σ_{inel}	v	Total
U234	-	-	-	-	-	-
U235	-	0.01	-	-	-	0.01
U236	-	-	-	-	-	-
U238	0.03	0.33	0.12	0.76	0.11	0.85
Np237	0.01	0.01	-	-	-	0.02
Pu238	0.23	0.05	-	0.01	0.14	0.27
Pu239	0.18	0.19	0.02	0.09	0.11	0.30
Pu240	0.26	0.25	0.01	0.01	0.31	0.48
Pu241	0.77	0.04	-	-	0.03	0.77
Pu242	0.20	0.13	-	0.02	0.04	0.24
Am241	0.05	0.05	-	0.01	0.01	0.07
Am242m	0.06	-	-	-	-	0.06
Am243	0.02	0.03	-	0.02	0.01	0.04
Cm242	0.01	-	-	-	-	0.01
Cm243	0.01	-	-	-	-	0.01
Cm244	0.23	0.03	-	-	0.05	0.24
Cm245	0.26	0.02	-	-	0.02	0.26
Cm246	0.03	0.01	-	-	-	0.03
Fe56	-	0.09	0.04	0.35	-	0.36
Fe57	-	0.01	-	0.01	-	0.01
Cr52	-	0.01	0.02	0.01	-	0.02
Ni58	-	-	-	-	-	-
Na23	-	-	0.02	0.19	-	0.19
C	-	-	-	-	-	-
O16	-	-	0.10	0.03	-	0.10
B10	-	-	-	-	-	-
Total	0.96	0.49	0.16	0.86	0.38	1.44



Uncertainty Analysis

Main Cross-Section Contributors to k_{eff} “Full BOLNA” Uncertainty. Values are in %.

	ZPR3-53	ZPR3-54	ZPPR-2	ZPPR-9	ZPPR-15A	ZPR6-6A	ZPR6-7	CIRANO ZONA2A	CIRANO ZONA2A3	CIRANO ZONA2B	ABR Metal Core	ABR Oxide Core
Total ^(a)	0.88	1.15	0.92	1.25	1.00	2.39	1.04	0.90	0.80	0.74	1.47	1.44
U235 σ_{capt}	- ^(b)	-	-	-	-	2.18	-	-	-	-	-	-
U235 v	-	-	-	-	-	0.55	-	-	-	-	-	-
U238 σ_{capt}	0.29	0.17	0.46	0.56	0.43	0.59	0.51	0.30	0.26	0.21	0.28	0.33
U238 σ_{el}	0.30	-	0.23	0.25	0.28	-	0.26	0.27	0.21	0.16	0.21	-
U238 σ_{inel}	0.13	0.12	0.55	0.96	0.68	0.37	0.69	0.33	0.32	0.31	0.97	0.76
U238 v	0.13	-	0.14	0.17	0.15	-	0.15	0.13	0.12	0.11	0.14	-
Pu238 σ_{fiss}	-	-	-	-	-	-	-	-	-	-	0.22	0.23
Pu239 σ_{fiss}	0.22	0.25	0.24	0.23	0.25	-	0.23	0.22	0.23	0.23	0.21	0.18
Pu239 σ_{capt}	0.54	0.61	0.31	0.29	0.25	-	0.31	0.27	0.27	0.27	0.16	0.19
Pu239 v	0.12	0.13	0.14	0.14	0.14	-	0.14	0.13	0.12	0.12	-	-
Pu240 σ_{fiss}	-	-	0.06	-	0.07	-	-	0.12	0.13	0.13	0.27	0.26
Pu240 σ_{capt}	-	-	0.07	-	-	-	-	0.11	0.12	0.12	0.20	0.25
Pu240 v	-	-	0.07	-	0.08	-	-	0.15	0.15	0.16	0.30	0.31
Pu241 σ_{fiss}	-	-	0.14	0.10	0.08	-	0.12	0.16	0.17	0.17	0.65	0.77
Pu242 σ_{fiss}	-	-	-	-	-	-	-	-	-	-	0.18	0.20
Cm244 σ_{fiss}	-	-	-	-	-	-	-	-	-	-	0.18	0.23
Cm245 σ_{fiss}	-	-	-	-	-	-	-	-	-	-	0.18	0.26
Fe56 σ_{el}	-	0.28	-	-	-	-	-	0.06	0.10	0.13	-	-
Fe56 σ_{inel}	-	0.09	0.15	0.16	0.26	-	0.16	0.07	0.07	0.07	0.35	0.35
Na23 σ_{inel}	-	-	0.08	0.11	0.09	-	0.10	-	-	-	0.19	0.19
C σ_{el}	0.45	0.84	-	-	-	-	-	-	-	-	-	-
O16 σ_{el}	-	-	0.13	-	-	-	-	0.52	0.39	0.29	-	-

^(a) Overall uncertainty (this is not the total of the components listed in the table);

^(b) Uncertainty component null or not important;

Uncertainty Analysis

Main Cross-Section Contributors to the “Full BOLNA” Uncertainty on the Ratio of U238 Fission to Pu239 Fission. Values are in %.

	ZPR3-53	ZPR3-54	ZPPR-2	ZPPR-9	ZPPR-15A	ZPR6-7	CIRANO ZONA2A	CIRANO ZONA2A3	CIRANO ZONA2B	ABR Metal Core	ABR Oxide Core
Total	3.13	3.28	5.90	7.18	6.10	5.81	4.91	4.88	4.83	6.33	5.47
U238 σ_{capt}	0.24	-	-	-	-	0.58	-	-	-	-	-
U238 σ_{el}	0.44i ^(a)	-	0.60i	0.70i	0.78i	0.61i	-	-	-	0.76i	0.57i
U238 σ_{inel}	2.16	2.23	5.48	6.92	5.55	5.41	4.69	4.65	4.59	5.90	4.65
Pu239 σ_{capt}	0.62	0.61	-	-	-	-	-	-	-	-	-
Pu239 σ_{inel}	0.71	0.73	-	-	-	-	0.67	0.67	0.67	-	-
Fe56 σ_{inel}	-	-	1.47	1.16	2.49	1.44	0.84	0.84	0.86	2.12	2.01
Na23 σ_{inel}	-	-	0.64	-	-	0.64	0.58	0.58	0.57	0.85	0.90
C σ_{el}	2.03	2.13	-	-	-	-	-	-	-	-	-
C σ_{inel}	0.46	0.45	-	-	-	-	-	-	-	-	-
O16 σ_{el}	-	-	1.34	1.31	-	1.26	0.70	0.59	0.54	-	1.72

^(a) Imaginary uncertainty component due to cross-correlations among isotopes or reactions in the dispersion matrix associated to sensitivity coefficients of opposite signs.

Uncertainty Analysis

Main Cross-Section Contributors to the “Full BOLNA” Uncertainty on the Ratio of U238 Capture to Pu239 Fission. Values are in %.

	ZPR3-53	ZPR3-54	ZPPR-2	ZPPR-9	ZPPR-15A	ZPR6-7	CIRANO ZONA2A	CIRANO ZONA2A3	CIRANO ZONA2B	ABR Metal Core	ABR Oxide Core
Total	1.06	1.26	1.06	1.14	1.16	1.03	1.38	1.37	1.35	1.34	0.99
U238 σ_{capt}	0.11	-	0.32	0.38	0.44	0.34	0.32	0.30	0.28	0.44	0.31
U238 σ_{el}	0.12i	-	0.11i	0.12i	0.16i	0.11i	-	0.16	0.18	0.12i	0.09i
U238 σ_{inel}	0.52	0.59	0.88	1.01	0.90	0.86	1.13	1.11	1.08	1.03	0.73
Pu239 σ_{capt}	0.30	0.32	0.17	0.14	0.18	0.17	0.30	0.30	0.29	0.15	0.13
Pu239 σ_{inel}	0.24	0.28	0.17	0.14	0.25	0.18	0.43	0.43	0.44	0.26	0.13
Pu240 σ_{capt}	-	-	-	-	-	-	-	0.11	0.11	0.17	0.15
Fe56 σ_{inel}	-	-	0.21	0.15	0.34	0.21	0.17	0.19	0.22	0.30	0.27
Na23 σ_{el}	-	-	0.16	0.13	0.24	0.16	0.22	0.23	0.23	0.45	0.25
Na23 σ_{inel}	-	-	0.25	0.20	0.22	0.25	0.30	0.30	0.30	0.28	0.26
C σ_{el}	0.83	1.02	-	-	-	-	-	-	-	-	-
O16 σ_{el}	-	-	0.22	0.18	-	0.19	0.22	0.20	0.20	-	0.26

Uncertainty Analysis

Main Cross-Section Contributors to the Void Reactivity “Full BOLNA” Uncertainty. Values are in %.

	ZPPR-2	ZPPR-9	ZPPR-15A	ZPR6-6A	ZPR6-7	CIRANO ZONA2A	CIRANO ZONA2A3	CIRANO ZONA2B	ABR Metal Core	ABR Oxide Core
Total	212.55	13.32	9.66	10.46	104.50	10.41	8.45	7.13	13.10	7.82
U235 σ_{fiss}	-	-	-	2.59	-	-	-	-	-	-
U235 σ_{capt}	-	-	-	5.89	-	-	-	-	-	-
U238 σ_{capt}	70.16	4.62	5.20	2.89	34.65	1.34	1.71	1.85	4.57	1.67
U238 σ_{el}	70.73	3.40	3.02	3.70	39.44	3.30	2.44	1.70	2.62	0.93
U238 σ_{inel}	103.92	7.82	2.82	5.29	60.85	2.15	2.30	1.93	3.84	2.24
U238 v	18.67	-	-	1.24	8.67	1.06	0.83	0.62	-	-
Pu239 σ_{fiss}	60.48	3.11	2.16	-	27.70	1.44	1.43	1.39	1.41	1.23
Pu239 σ_{capt}	28.86	1.45	1.82	-	11.80	1.02	0.97	0.91	1.55	-
Pu239 σ_{inel}	13.03	1.00	0.71	-	6.62	-	-	-	-	-
Pu239 v	16.26	0.83	0.92	-	7.23	-	-	0.48	-	-
Pu240 σ_{fiss}	-	-	-	-	-	-	0.86	0.89	1.58	0.83
Pu240 v	-	-	-	-	-	-	0.80	0.83	1.39	-
Pu241 σ_{fiss}	22.02	0.80	-	-	9.43	-	0.68	0.67	2.82	2.84
Cm244 σ_{fiss}	-	-	-	-	-	-	-	-	1.22	0.65
Fe56 σ_{capt}	36.85	1.72	1.60	1.76	17.20	-	-	-	-	1.08
Fe56 σ_{el}	14.92	-	0.60	-	-	-	1.38	1.70	1.14	-
Fe56 σ_{inel}	20.53	-	1.11	1.11	8.18	-	0.80	0.92	1.46	-
Na23 σ_{el}	34.52	2.17	1.99	2.65	17.28	2.15	2.24	2.25	1.13	0.99
Na23 σ_{inel}	108.20	7.66	5.50	1.59	49.67	2.13	2.05	1.84	9.88	6.07
O16 σ_{el}	57.32	0.83	-	0.87	7.52	8.61	6.19	4.65	-	0.70

Uncertainty Analysis

Uncertainty Breakdown by Energy Group (Use of “Full BOLNA”). Values are in %.

	k_{eff}									Void
	ZPR3 Assembly 54		ZPPR-2		ZPR6 Assembly 6A	CIRANO ZONA2A	ABR Oxide Core		ABR Metal Core	
	E [MeV]	C σ_{el}	Pu239 σ_{capt}	U238 σ_{inel}	U238 σ_{capt}	U235 σ_{capt}	O16 σ_{el}	Pu241 σ_{fiss}	Fe56 σ_{inel}	Na23 σ_{inel}
19.6	0.06	-	0.05	-	0.01i	0.09	0.02i	0.06i	1.47	
6.07	0.27	0.01	0.40	-	0.05i	0.47	0.07	0.13i	2.67	
2.23	0.25	0.02	0.35	0.02	0.08	0.14	0.18	0.31	3.62	
1.35	0.43	0.06	0.11	0.05	0.29	0.10	0.28	0.20	8.66	
4.98E-01	0.46	0.08	0.02	0.06	0.63	0.08	0.33	-	0.42	
1.83E-01	0.28	0.08	0.04	0.09	1.01	0.04	0.45	-	-	
6.74E-02	0.18	0.12	0.01	0.10	1.05	0.04	0.27	-	-	
2.48E-02	0.18	0.21	-	0.41	1.09	0.06	0.20	-	-	
9.12E-03	0.17	0.54	-	0.12	0.98	0.04	0.16	-	-	
2.03E-03	0.06	0.06	-	0.10	0.11	0.03	0.13	-	-	
4.54E-04	0.03	0.05	-	0.03	0.01	0.02i	0.05	-	-	
2.26E-05	0.01i	-	-	-	-	0.01i	-	-	-	
4.00E-06	-	-	-	-	-	-	-	-	-	
5.40E-07	-	-	-	-	-	-	-	-	-	
1.00E-07	-	-	-	-	-	-	-	-	-	
Total	0.84	0.61	0.55	0.46	2.18	0.52	0.77	0.35	9.88	

Representativity Analysis

Representativity between Experiments (E) and ABR Metal Core (R) with respect to k_{eff} .
Uncertainty Values are in %.

	Use of the Unity Matrix		Use of “Diagonal BOLNA”		Use of “Full BOLNA”	
	$I_R^{(a)} = 35.45$		$I_R = 0.95$		$I_R = 1.47$	
	r_{RE}	$(I_R)'^{(a)}$	r_{RE}	$(I_R)'$	r_{RE}	$(I_R)'$
ZPR3-53	0.768	22.71	0.303	0.91	0.437	1.32
ZPR3-54	0.707	25.08	0.055	0.95	0.070	1.47
ZPPR-2	0.938	12.32	0.751	0.63	0.785	0.91
ZPPR-9	0.929	13.08	0.791	0.58	0.799	0.88
ZPPR-15A	0.978	7.38	0.822	0.54	0.819	0.84
ZPR6-6A	0.113	35.22	0.300	0.91	0.187	1.44
ZPR6-7	0.924	13.55	0.773	0.60	0.796	0.89
ZONA2A	0.964	9.37	0.562	0.79	0.662	1.10
ZONA2A3	0.964	9.38	0.625	0.74	0.710	1.03
ZONA2B	0.960	9.90	0.655	0.72	0.727	1.01

I_R : Uncertainty for the Reactor; r_{RE} : representativity factor; $(I_R)'$: Reduced Uncertainty for the Reactor;

^(a) Parameter uncertainty one would get using a variance matrix with a standard deviation of 100% for all cross-sections. These uncertainty values are of course not realistic.

Representativity Analysis

Representativity between Experiments (E) and ABR Metal Core (R) with respect to the parameter P. Use of “Full BOLNA” only. Uncertainty Values are in %.

$P = \frac{\langle \sigma_{\text{fiss}, \text{U}238} \Phi \rangle}{\langle \sigma_{\text{fiss}, \text{Pu}239} \Phi \rangle}$		$P = \frac{\langle \sigma_{\text{capt}, \text{U}238} \Phi \rangle}{\langle \sigma_{\text{fiss}, \text{Pu}239} \Phi \rangle}$		P = Void Coefficient	
$I_R = 6.33$		$I_R = 1.34$		$I_R = 13.10$	
	r_{RE}	$(I_R)'$	r_{RE}	$(I_R)'$	r_{RE}
ZPR3-53	0.653	4.80	0.438	1.20	-
ZPR3-54	0.662	4.75	0.436	1.21	-
ZPPR-2	0.964	1.68	0.917	0.54	0.754
ZPPR-9	0.960	1.78	0.901	0.58	0.810
ZPPR-15A	0.991	0.84	0.969	0.33	0.874
ZPR6-6A	-	-	-	-	-0.335 ^(a)
ZPR6-7	0.966	1.65	0.921	0.52	0.747
ZONA2A	0.967	1.62	0.918	0.53	-0.384
ZONA2A3	0.970	1.55	0.923	0.52	-0.470
ZONA2B	0.972	1.49	0.926	0.51	-0.508

^(a) Negative values are due to sensitivities of opposite sign for the reactor and the experiment.

Representativity Analysis

Representativity between Experiments (E) and ABR Oxide Core (R) with respect to the Parameter P.
Use of “Full BOLNA” only. Uncertainty Values are in %.

$P = k_{\text{eff}}$		$P = \frac{\langle \sigma_{\text{fiss}, \text{U}^{238}} \Phi \rangle}{\langle \sigma_{\text{fiss}, \text{Pu}^{239}} \Phi \rangle}$		$P = \frac{\langle \sigma_{\text{capt}, \text{U}^{238}} \Phi \rangle}{\langle \sigma_{\text{fiss}, \text{Pu}^{239}} \Phi \rangle}$		$P = \text{Void Coefficient}$		
$I_R = 1.44$		$I_R = 5.47$		$I_R = 0.99$		$I_R = 7.82$		
	r_{RE}	$(I_R)'$	r_{RE}	$(I_R)'$	r_{RE}	$(I_R)'$	r_{RE}	$(I_R)'$
ZPR3-53	0.435	1.30	0.607	4.35	0.446	0.89	-	-
ZPR3-54	0.119	1.43	0.616	4.31	0.440	0.89	-	-
ZPPR-2	0.735	0.98	0.980	1.08	0.960	0.28	0.803	4.67
ZPPR-9	0.726	0.99	0.955	1.62	0.929	0.37	0.827	4.39
ZPPR-15A	0.740	0.97	0.937	1.90	0.911	0.41	-0.314	7.43
ZPR6-6A	0.171	1.42	-	-	-	-	0.775	4.94
ZPR6-7	0.742	0.96	0.979	1.12	0.959	0.28	0.799	4.70
ZONA2A	0.599	1.15	0.951	1.70	0.934	0.36	-0.427	7.08
ZONA2A3	0.654	1.09	0.946	1.77	0.930	0.36	-0.484	6.85
ZONA2B	0.681	1.05	0.945	1.79	0.932	0.36	-0.499	6.78

Conclusions



- ↳ Work was initiated to improve the nuclear data by a combined use of the science-based covariance data and integral experiments. The metal and oxide core configurations of a reference ABR were selected as the tentative target systems.
- ↳ To identify the most relevant experiments to the selected target systems, a representativity study was performed. In this approach, the similarity between the target system and a selected experiment in connection with the response parameter of interest is quantitatively evaluated by comparing the sensitivity profiles of the response parameter with respect to nuclear data in the two systems, filtered by the estimated covariance data. The estimated covariance data play a critical role, since the representativity factors employed in the comparison are dominated by the sensitivity components that correspond to the cross-sections of significant uncertainties.
- ↳ It has been found that the ZPPR-2, ZPPR-9, ZPPR-15A, ZPR6 Assembly 7 and the CIRANO experiments show a quite good similarity with both the metal and oxide core ABRs; only the coolant void reactivity coefficients shows significant discrepancy because of different fuel to coolant loading ratios. However, with respect to k_{eff} , even the experiment providing the strongest representativity is not enough to bring the initial k_{eff} uncertainty below the desired accuracy of 0.3% for both ABRs. Degradation in the representativity factors is then observed when the ZPR3 experiments are compared with the ABR cores. As expected from the analysis of the sensitivity profiles, the ZPR6 Assembly 6A loaded with uranium oxide fuel offers poor chances to be associated to the reference ABR cores.

BOLNA Variance Matrix (Fissile Isotopes): U235, U238 and Pu239 (with nu-bar corrected) from ORNL; Cm246 from ANL; BNL Most Recent Evaluations

Gr	E [MeV]	Th232					U233					U234					U235					U236					U238					Cm245		Cm246									
		v	σ _f	σ _{inel}	σ _{el}	σ _{capt}	σ _{n,2n}	v	σ _f	σ _{inel}	σ _{el}	σ _{capt}	σ _{n,2n}	v	σ _f	σ _{inel}	σ _{el}	σ _{capt}	σ _{n,2n}	v	σ _f	σ _{inel}	σ _{el}	σ _{capt}	σ _{n,2n}	v	σ _f	σ _{inel}	σ _{el}	σ _{capt}	σ _{n,2n}	v	σ _f	σ _{inel}	σ _{el}	σ _{capt}	σ _{n,2n}						
1	19.6	2.11	4.85	1.39	17.4	10	0.84	3.98	10.79	2.51	57.01	18.44	12.9	32.41	1.6	56.74	34.85	0.89	0.5	21.73	9.6	61.13	20.35	14.64	32.82	1.01	48.32	18.49	1.26	0.57	29.28	13.3	21.41	5.32	9.64	18.11	5	40					
2	6.07	2.1	3.77	1.22	25.67	0	0.25	6.44	4.48	4.02	61.4	0	23.52	29.45	2.06	26.11	0	0.69	0.47	6.79	4.15	36.99	8.86	26.9	7.07	0.89	47.92	0	1.17	0.55	19.75	14.5	13.5	0	2.91	30.96	5	40					
3	2.23	2.1	3.94	4	3.41	0	0.22	7.26	31.58	3	38.88	0	13.83	21.15	4.88	15.91	0	0.56	0.48	6.41	4.54	19.14	0	28.88	15.45	3.84	36	0	1.34	0.6	20.58	18.7	6.05	0	2.85	44.17	5	40					
4	1.35	2.15	4.41	1.8	1.89	0	0.18	7.26	40.46	1.88	30.35	0	37.97	14.61	2	11.99	0	0.55	0.46	7.55	3.56	16.1	0	31.97	37.2	0.65	15.1	0	1.3	2.91	11.56	5.35	2.27	0	2.95	49.43	5	40					
5	4.98e-1	95.17	12.58	1.65	1.24	0	0.18	7.26	14.29	2.49	10.62	0	37.97	25.97	2.75	13.98	0	0.61	0.5	11.32	2.87	22.13	0	31.97	38.02	3.34	14.25	0	2	5.26	4.19	1.92	1.41	0	3.01	37.22	5	40					
6	1.83e-1	46.92	23.6	1.18	1.14	0	0.19	7.26	0	3.16	8.94	0	31.61	31.49	1.6	12.92	0	0.66	0.53	15.01	2.38	30.64	0	16.47	34.14	1.43	10.53	0	2	5.14	10.96	2.12	1.67	0	3.01	47.45	5	40					
7	6.74e-2	0	0	0.48	1.4	0	0.18	7.26	0	4.06	13.29	0	22.84	0	5.3	22.72	0	0.66	0.5	14.72	2.63	32.89	0	7.62	0	0.76	7.84	0	2	5.14	11.12	3.76	1.64	0	3.01	26.53	5	40					
8	2.48e-2	0	0	0.31	1.46	0	0.21	7.26	0	4.19	13.52	0	19.41	0	4.86	19.16	0	0.66	0.58	50	3.24	34.03	0	6.68	0	0.46	7.04	0	2	50.31	0	1.52	9.43	0	3.01	13.47	5	40					
9	9.12e-3	0	0	0.48	2.8	0	0.18	7.26	0	9.04	14.19	0	13.69	0	3.58	11.53	0	0.66	3.18	48.48	5.16	33.92	0	3.5	0	0.46	3.89	0	2	214.6	0	0.67	3.11	0	3.01	13.18	5	40					
10	2.04e-3	0	0	1.53	3.35	0	0.3	10.35	0	6.17	7.92	0	14.97	0	6.94	1.86	0	0.66	0.77	0	2.07	4.56	0	3.3	0	2.26	1.24	0	2	9.69	0	0.72	2.1	0	3.01	13.03	5	40					
11	4.54e-4	0	0	3.82	2.21	0	0.25	4.49	0	6	4.28	0	5.11	0	23.1	2.42	0	0.66	0.44	0	1.33	0.63	0	1.37	0	5.96	1.2	0	2	2.38	0	2.39	1.71	0	3.01	8.66	5	40					
12	2.26e-5	0	0	1.53	3.28	0	0.14	2.23	0	5.61	3.02	0	17.49	0	17.1	1.38	0	0.69	0.62	0	1.52	0.65	0	2.57	0	2.35	0.44	0	2	5.82	0	5.97	1.03	0	3.01	3.89	5	40					
13	4.00e-6	0	0	0.86	1.9	0	0.13	5.03	0	4.16	9.06	0	21.85	0	2.32	9.34	0	0.69	0.4	0	1.78	1.36	0	15.79	0	4.77	3.47	0	2	51.89	0	0.82	2.45	0	3.01	6.21	5	40					
14	5.40e-7	0	0	0.81	1.31	0	0.13	1.46	0	2.14	2.59	0	24.67	0	2.02	3.08	0	0.71	0.3	0	3.42	1.55	0	19.58	0	4.91	3.44	0	2	55.19	0	0.92	1.66	0	3.01	5.12	5	40					
15	1.00e-7	0	0	0.79	1.25	0	0.13	1.03	0	5.34	4.23	0	24.81	0	1.98	2.93	0	0.71	0.25	0	4.9	1.73	0	19.86	0	4.89	3.58	0	2	55.42	0	0.94	1.64	0	3.01	3.82	5	40					
Np237		Pu238					Pu239					Pu240					Pu241					Pu242					Cm245		Cm246														
Gr	E [MeV]	v	σ _f	σ _{inel}	σ _{el}	σ _{capt}	σ _{n,2n}	v	σ _f	σ _{inel}	σ _{el}	σ _{capt}	σ _{n,2n}	v	σ _f	σ _{inel}	σ _{el}	σ _{capt}	σ _{n,2n}	v	σ _f	σ _{inel}	σ _{el}	σ _{capt}	σ _{n,2n}	v	σ _f	σ _{inel}	σ _{el}	σ _{capt}	σ _{n,2n}	σ _{inel}	σ _{el}	σ _{capt}	σ _{n,2n}								
1	19.6	1.94	5.58	42.85	2.39	41.47	9.51	2.17	25.2	24.56	0.92	50.91	58.36	0.5	0.63	23.06	6.94	37.08	8.53	1.09	9.56	37.11	2.34	52.16	54.09	0.45	24.09	25.15	4.45	55.39	39.68	0.9	37.24	26.25	0.8	78.47	51.7	85.1	4.81	50	10		
2	6.07	2.19	7.9	6.54	3.7	36.48	0	5.3	20.53	5.69	0.82	28.81	0	0.17	0.69	22.18	9.36	37.8	4.34	2.65	4.8	9.65	5.19	32.47	0	0.27	14.16	19.47	3.74	54.1	33.43	2.21	15.1	3.27	0.51	22.72	0	13.01	7.25	50	10		
3	2.23	1.47	7.63	22.35	4.12	17.62	0	5.39	33.82	28.65	6.24	21.55	0	0.17	0.89	19	10.3	26.56	0	2.69	5.65	10.09	5.42	19.74	0	0.27	21.26	18.38	4.39	38.41	0	2.24	21.42	29.28	3.34	16.12	0	40.92	7.45	50	10		
4	1.35	0.66	5.82	28.0	3.62	10.34	0	7	17.11	44.92	6	9.74	0	0.12	0.64	29.01	10.2	18.18	0	3.74	5.82	7.79	4.76	16.28	0	0.28	16.62	19.78	5.38	31.66	0	3.11	18.98	59.78	3.68	12.48	0	54.47	4.75	50	10		
5	4.98e-1	0.6	5.79	44.99	3.47	5.79	0	7	17.11	43.09	9.52	12.31	0	0.19	0.68	34.01	5.66	11.55	0	4.81	3.91	9.78	5.53	14.29	0	0.29	13.54	20.92	5.16	20.51	0	4.01	18.63	37.99	1.73	24.05	0	81.84	9.94	50	10		
6	1.83e-1	0.6	5.79	54.97	4.07	2.08	0	6.5	8.78	20.81	4.1	16.62	0	0.54	0.85	46.06	3.98	9.04	0	4.81	5.7	42.55	5.76	13.79	0	0.29	19.87	30.09	4.69	11.29	0	4.01	32.07	19	1.78	32.28	0	94.96	15	50	10		
7	6.74e-2	0.6	5.79	36.27	4.37	6.66	0	6	11.91	0	5.26	22.14	0	0.58	0.72	40.04	2.37	10.12	0	4.81	7.45	48.58	5.11	31.31	0	0.29	8.74	37.51	3.92	4.43	0	4.01	33.06	0	1.59	37.26	0	94.77	13.67	55	10		
8	2.48e-2	0.6	5.79	0	4.48	5.25	0	5.5	11.2	0	4.85	18.03	0	0.58	0.96	28.52	2.16	7.39	0	4.81	7.45	0	5.05	10.21	0	0.29	11.29	0	9.14	7.79	0	4.01	33.19	0	1.42	38.63	0	0	18.14	0	10	10	
9	9.12e-3	0.6	5.79	0	3.93	5.25	0	5	7.47	0	4.43	9.6	0	0.65	0.62	8.64	4.04	15.46	0	4.81	8.01	0	2.08	4.35	0	0.29	10.44	0	9.29	7.73	0	4.01	13.23	0	1.59	38.45	0	0	18.23	0	10	10	
10	2.04e-3	0.6	5.77	0	2.44	5.54	0	4.5	4.26	0	4.35	3.69	0	0.2	1.2	0	0.74	1.39	0	4.81	21.62	0	1.26	1.47	0	0.29	12.68	0	10.9	7.74	0	4.01	5.89	0	3.76	1.7	0	0	18.11	0	10	10	
11	4.54e-4	0.6	7.54	0	2.41	1.7	0	4	8.09	0	20.0	4.66	0	0.2	1.24	0	1.2	1.25	0	4.81	4.72	0	1.64	1.63	0	0.29	19.38	0	10.8	7.43	0	4.01	1.96	0	2.29	2.23	0	0	0	15.1	0	40	10
12	2.26e-5	0.6	4.64	0	2.31	0.55	0	3.5	18.98	0	10.1	9.12	0	0.2	0.47	0	0.24	0.61	0	4.81	8.91	0	3.25	5.5	0	0.29	4.21	0	10.6	8.38	0	4.01	6.46	0	5.94	7.4	0	0	0	20.25	0	10	10
13	4.00e-6	0.6	5.58	0	2.23	0.7	0	3	4.57	0	5.79	3.71	0	0.2	1.43	0	0.3	1.22	0	4.81	1.22	0	0.48	0.44	0	0.29	26.83	0	11.4	6.37	0	4.01	7.6	0	4.68	3.78	0	0	0	21.78	0	10	10
14	5.40e-7	0.6	14.74	0	2.18	2.41	0	2.4	4.63	0	5.1	1.39	0	0.2	0.88	0	0.44	1.36	0	4.81	29.76	0	4.58	3.23	0	0.29	2.94	0	9.91	6													

BOLNA Variance Matrix (Structural Isotopes): Only Pb204, Pb206, Pb207, Pb208 from NRG; C, B10 and He4 from ANL; BNL Most Recent Evaluations

		Pb204			Pb206			Pb207			Pb208			Bi			Fe56			Fe57			Ni58			Cr52			Zr90			He4											
Gr	E [MeV]	Oinel	Oel	Ocapt	Oinel	Oel	Ocapt	Oinel	Oel	Ocapt	Oinel	Oel	Ocapt	Oinel	Oel	Ocapt	Oinel	Oel	Ocapt	Oinel	Oel	Ocapt	Oinel	Oel	Ocapt	Oinel	Oel	Ocapt	Oinel	Oel	Ocapt	Oinel	Oel	Ocapt									
1	19.6	19.78	3.81	56.24	4.58	19.82	3.74	63.4	4.38	17.01	4	61.28	5.69	17.84	3.55	63.34	5.22	5.25	0.83	47.58	9.39	12.97	4.61	46.24	7.05	15.15	2.59	85.38	19.2	12.28	3.52	47.76	8.3	8.56	3.55	49.2	10.53	11.32	0.44	46.36	6.74	0	5
2	6.07	5.56	6.08	31.43	0	5.45	5.88	30.43	0	4.98	5.81	24.25	0	5.38	4.99	28.98	0	2.44	1.02	27.74	0	7.23	8.14	31.69	0	11.08	3.24	53.46	0	10.08	4.64	14.54	0	2.24	2.4	23.27	0	17.96	0.92	18.59	0	0	5
3	2.23	14.2	5.12	27.61	0	14.17	4.7	24.54	0	13.77	4.43	21.56	0	0	6.3	22.56	0	34.07	2.06	17.56	0	25.4	5.89	23.48	0	34.21	2.71	18.33	0	31.07	0.93	9.68	0	3.12	2.92	19.31	0	18.52	3.96	9.14	0	0	5
4	1.35	7.74	5.26	17.89	0	9.15	5.14	19.37	0	11.31	4.78	19.46	0	0	7.15	22.4	0	41.77	4.59	11.35	0	16.12	0.64	7.43	0	10.29	1.28	11.99	0	0	1.54	7.71	0	0	4.19	4.36	0	50	3.39	6.26	0	0	5
5	4.98e-1	0	2.81	14.07	0	0	2.74	15.94	0	0	2.44	16.41	0	0	4.88	21.28	0	0	2.22	8.32	0	0	1.71	4.02	0	8.54	2.39	11.14	0	0	2.68	2.65	0	0	5.21	5.51	0	0	2.77	5.16	0	0	5
6	1.83e-1	0	3.95	13.37	0	0	3.84	15.05	0	0	3.73	15.94	0	0	3.2	21.79	0	0	1.8	8.79	0	0	2.08	10.77	0	12.1	1.65	6.51	0	0	3.21	1.21	0	0	11.4	10.56	0	0	2.02	3.13	0	0	3
7	6.74e-2	0	6.63	13.38	0	0	6.51	14.65	0	0	6.35	15.96	0	0	5.57	21.98	0	0	1.88	6.05	0	0	2.05	13.19	0	0	2.81	6.6	0	0	3.05	3.44	0	0	12.6	5.45	0	0	3.09	5.2	0	0	3
8	2.48e-2	0	9.29	12.03	0	0	9.1	13.94	0	0	8.85	15.05	0	0	8.11	60.4	0	0	2.41	3.85	0	0	4.6	8.81	0	0	5.1	8.27	0	0	8.05	0.83	0	0	13.2	13.42	0	0	4.43	7.89	0	0	3
9	9.12e-3	0	12.7	12.47	0	0	12.4	12.8	0	0	12	14.27	0	0	11.0	182.3	0	0	1.82	0.71	0	0	3.98	8.56	0	0	2.5	4.64	0	0	3.61	2.82	0	0	10.2	12.97	0	0	5.93	6.96	0	0	3
10	2.04e-3	0	17.7	13.57	0	0	17.2	276.5	0	0	16.6	20.01	0	0	15.0	317.0	0	0	1.93	0.43	0	0	4.16	11.23	0	0	16.1	29.24	0	0	2.96	2.41	0	0	7.79	2.75	0	0	6.83	10.55	0	0	3
11	4.54e-4	0	11.4	8.31	0	0	11.0	8.66	0	0	10.69	8.6	0	0	9.61	13.94	0	0	1.85	1.47	0	0	4.28	11.25	0	0	8.48	10.69	0	0	2.77	2.4	0	0	7.37	2.7	0	0	6.73	5.95	0	0	3
12	2.26e-5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.82	1.82	0	0	4.31	11.25	0	0	7.69	11.67	0	0	2.74	2.4	0	0	7.21	2.69	0	0	6.7	2.75	0	0	2			
13	4.00e-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.8	1.86	0	0	4.31	11.25	0	0	7.65	11.73	0	0	2.73	2.4	0	0	7.22	2.68	0	0	6.7	2.56	0	0	2			
14	5.40e-7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.8	1.87	0	0	4.31	11.25	0	0	7.65	11.74	0	0	2.73	2.4	0	0	7.21	2.68	0	0	6.7	2.53	0	0	1			
15	1.00e-7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.78	1.87	0	0	4.21	9.4	0	0	7.48	10.79	0	0	2.67	2.21	0	0	7.05	2.24	0	0	6.7	2.52	0	0	1			

Gr	E [MeV]	Gd157				Gd158				Gd160				Er166				Er167				Er168				Er170				F19				C				B10				He4
		Ginel	Gel	Gcapt	Gn,2n	Ginel	Gel	Gcapt	Gn,2n	Ginel	Gel	Gcapt	Gn,2n	Ginel	Gel	Gcapt	Gn,2n	Ginel	Gel	Gcapt	Gn,2n	Ginel	Gel	Gcapt	Gn,2n	Ginel	Gel	Gcapt	Gn,2n	Ginel	Gel	Gcapt	Gn,2n	Ginel	Gel	Gcapt	Gn,2n	Ginel	Gel	Gcapt	Gn,2n	
1	19.6	12.52	2.2	69.27	8.92	35.34	1.51	57.37	8.07	42.58	1.15	78.62	18.01	15.73	0.91	41	7.16	9.17	0.9	67.61	30.27	19.36	0.93	31.96	9.13	26.88	0.92	25.17	15.84	4.54	8.71	15.86	7.2	30	5	20	0	30	10	15	0	0
2	6.07	10.26	3.55	16.18	0	7.39	1.32	18.37	0	25.24	1.77	30.38	0	16.82	1.55	32	0	12.29	1.54	49.14	0	13.43	1.36	49.67	0	14.56	1.3	26	0	9.12	8.12	15.42	0	35	5	20	0	30	10	15	0	0
3	2.23	16.28	3.73	8.67	0	18.98	1.57	13.6	0	18.96	1.08	22.68	0	49.08	2.35	15.82	0	10.2	2.45	30.21	0	44.37	2.43	21.89	0	53.41	2.58	11.08	0	7.3	2.12	8.5	0	0	5	20	0	30	10	15	0	0
4	1.35	22.82	6.12	6.13	0	22.9	3.71	5.31	0	18.88	2.44	10.96	0	35.11	0.57	9.53	0	5.97	0.78	12.93	0	37.67	0.54	12.98	0	51.08	0.68	8.9	0	22.03	1.65	5.14	0	0	5	20	0	35	10	15	0	0
5	4.98e-1	28.53	5.23	5.2	0	36.88	4.19	4.94	0	69.41	3.26	10.99	0	3.54	6.33	11.59	0	4.22	6.65	9.33	0	7.28	5.96	14.97	0	19.18	5.62	7.47	0	15.88	1.02	6.13	0	0	5	20	0	0	10	15	0	0
6	1.83e-1	30.01	3.7	3.97	0	60.91	2.93	3.82	0	91.34	2.74	9.18	0	50	5.47	13.43	0	6.61	5.57	8.23	0	50	4.82	19.44	0	41.37	4.07	4.99	0	0	1.08	2	0	0	3	20	0	0	10	10	0	0
7	6.74e-2	21.25	2.47	3.12	0	0	2.98	3.71	0	0	2.77	6.91	0	0	3.9	3.24	0	0	3.82	3.92	0	0	3.09	9.74	0	0	7.65	6.22	0	0	0.97	2.89	0	0	3	20	0	0	10	10	0	0
8	2.48e-2	0	3.29	3.11	0	0	3.88	6.24	0	0	3.51	9.15	0	0	2.6	6.26	0	0	2.92	4.01	0	0	2.32	3.1	0	0	5.7	4.7	0	0	1.68	2.55	0	0	3	20	0	0	5	8	0	0
9	9.12e-3	0	4.1	5.27	0	0	0.1	0.73	0	0	1.83	9.09	0	0	3.32	3.15	0	0	2.92	4.47	0	0	3.8	3.21	0	0	4.17	2.26	0	0	2.05	4.39	0	0	3	20	0	0	5	8	0	0
10	2.04e-3	0	5.58	6.18	0	0	0.33	0.77	0	0	3.26	9.17	0	0	4.74	3.13	0	0	5.12	2.81	0	0	5.34	6.15	0	0	4.82	3.38	0	0	2.05	6.05	0	0	3	20	0	0	5	5	0	0
11	4.54e-4	0	2.27	1.49	0	0	1.18	0.85	0	0	8.6	7.76	0	0	7.71	3.97	0	0	6.69	1.88	0	0	8.6	5.05	0	0	7.7	7.21	0	0	2.05	6.09	0	0	3	20	0	0	5	5	0	0
12	2.26e-5	0	7.66	2.29	0	0	4.76	3.36	0	0	11.06	17.51	0	0	4.39	6.59	0	0	5.88	1.13	0	0	8.67	3.22	0	0	17.9	4.91	0	0	2.05	6.1	0	0	2	20	0	0	3	5	0	0
13	4.00e-6	0	1.15	2.96	0	0	1.86	7.46	0	0	11.64	17.1	0	0	3.85	9.14	0	0	29.7	1.04	0	0	8.8	3.07	0	0	17.8	3.92	0	0	2.05	6.11	0	0	2	20	0	0	3	3	0	0
14	5.40e-7	0	0.87	1.49	0	0	1.91	8.49	0	0	11.75	17.05	0	0	3.78	9.36	0	0	9.57	1.02	0	0	8.83	3.05	0	0	17.8	3.7	0	0	2.05	6.11	0	0	1	2	0	0	2	1	0	0
15	1.00e-7	0	1.15	0.2	0	0	1.91	8.64	0	0	11.72	17.05	0	0	3.69	8.63	0	0	57.6	1.2	0	0	8.65	2.8	0	0	17.5	3.41	0	0	2	5.55	0	0	1	2	0	0	2	1	0	0

Covariance Data: ANL covariance Matrix

1-Group Simplified Variance (%) Matrix

	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$
U235	1	5	10	5	15	50
U238	2	5	15	5	15	50
Np237	2	10	50	5	10	100
Pu238	2	10	15	10	10	100
Pu239	1	5	10	5	15	50
Pu240	2	5	15	10	20	100
Pu241	1	10	15	10	10	100
Pu242	2	10	15	10	20	100
Am241	2	10	50	10	10	100
Am242m	3	20	50	10	40	100
Am243	3	10	50	10	10	100
Cm242	5	40	50	10	40	100
Cm243	5	40	50	10	40	100
Cm244	5	40	50	10	40	100
Cm245	5	40	50	10	40	100
Cm246	5	40	50	10	40	100
C			30	5	20	
He4				5	20	
Si28			30	5	20	100
Zr90			20	20	20	100

ANL Variance Matrix (Fissile Isotopes)

		Th232					U233					U234					U235					U236					U238				
Gr	E [MeV]	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$
1	19.6	2	5	15	10	20	100	2	20	20	10	10	100	3	20	15	10	20	100	1	5	10	5	15	50	3	20	15	10	20	100
2	6.07	2	5	15	10	20	100	2	20	20	10	10	100	3	20	15	10	20	100	1	5	10	5	15	50	2	20	15	10	20	0
3	2.23	2	5	15	10	20	0	2	10	20	10	10	0	3	20	15	10	20	0	1	5	10	5	15	0	2	20	10	5	5	0
4	1.35	2	5	15	10	20	0	2	10	20	10	10	0	3	20	15	10	20	0	1	5	15	5	15	0	2	20	10	5	5	0
5	4.98e-1	2	5	20	10	15	0	2	10	30	10	10	0	3	20	20	10	20	0	1	5	15	5	15	0	2	20	10	5	5	0
6	1.83e-1	2	5	0	10	15	0	2	10	0	10	10	0	3	20	20	10	20	0	1	5	15	5	15	0	2	20	10	5	5	0
7	6.74e-2	2	5	0	10	15	0	2	10	0	10	10	0	3	20	25	10	20	0	1	5	20	5	10	0	2	20	25	10	20	0
8	2.48e-2	2	5	0	10	10	0	2	10	0	10	20	0	3	20	0	10	20	0	1	5	25	5	10	0	2	20	0	5	5	0
9	9.12e-3	2	10	0	10	10	0	2	10	0	10	20	0	3	20	0	10	20	0	1	5	25	5	5	0	2	20	0	5	3	0
10	2.04e-3	2	10	0	10	10	0	2	10	0	10	20	0	3	20	0	10	20	0	0.5	3	30	5	5	0	2	20	0	5	3	0
11	4.54e-4	2	10	0	10	10	0	2	10	0	10	10	0	3	20	0	10	10	0	0.5	3	0	5	5	0	2	20	0	5	3	0
12	2.26e-5	2	10	0	10	5	0	2	10	0	10	5	0	3	20	0	10	10	0	0.5	3	0	5	5	0	2	20	0	5	3	0
13	4.00e-6	2	10	0	10	5	0	2	5	0	10	5	0	3	20	0	10	10	0	0.5	3	0	5	3	0	2	20	0	5	3	0
14	5.40e-7	2	50	0	5	1	0	2	2	0	10	3	0	3	20	0	2	10	0	0.5	1	0	5	1	0	2	20	0	1	1	0
15	1.00e-7	2	50	0	5	1	0	2	2	0	10	3	0	3	20	0	2	5	0	0.3	1	0	5	1	0	2	20	0	1	1	0
		Np237					Pu238					Pu239					Pu240					Pu241					Pu242				
Gr	E [MeV]	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$
1	19.6	3	10	50	5	10	100	3	10	15	10	10	100	1	5	10	5	15	50	3	5	15	10	20	100	1	20	15	10	10	100
2	6.07	2	10	50	5	10	0	2	10	15	10	10	0	1	5	10	5	15	50	2	5	15	10	20	0	1	20	15	10	20	0
3	2.23	2	10	50	5	10	0	2	10	15	10	10	0	1	5	10	5	15	0	2	5	15	10	20	0	1	10	15	10	20	0
4	1.35	2	10	50	5	10	0	2	10	15	10	10	0	1	5	15	5	15	0	2	5	15	10	20	0	1	10	15	10	20	0
5	4.98e-1	2	10	50	5	10	0	2	10	20	10	10	0	1	5	15	5	15	0	2	5	20	10	20	0	1	10	20	10	20	0
6	1.83e-1	2	10	50	5	10	0	2	10	20	10	10	0	1	5	15	5	15	0	2	5	20	10	20	0	1	10	20	10	15	0
7	6.74e-2	2	10	60	5	10	0	2	30	25	10	10	0	1	5	20	5	10	0	2	5	25	10	20	0	1	10	25	10	10	0
8	2.48e-2	2	10	0	5	10	0	2	30	0	10	20	0	1	5	25	5	10	0	2	5	0	10	10	0	1	10	0	10	20	0
9	9.12e-3	2	10	0	5	10	0	2	30	0	10	20	0	1	5	30	5	5	0	2	10	0	10	10	0	1	10	0	10	20	0
10	2.04e-3	2	20	0	5	10	0	2	30	0	10	20	0	0.5	3	0	5	5	0	2	10	0	10	10	0	1	10	0	10	20	0
11	4.54e-4	2	20	0	5	10	0	2	30	0	10	20	0	0.5	3	0	5	5	0	2	10	0	10	10	0	1	10	0	10	20	0
12	2.26e-5	2	20	0	5	10	0	2	20	0	10	10	0	0.5	3	0	5	5	0	2	10	0	10	10	0	1	10	0	10	5	0
13	4.00e-6	2	20	0	5	10	0	2	20	0	10	10	0	0.5	3	0	5	4	0	2	10	0	10	7	0	1	5	0	10	5	0
14	5.40e-7	2	20	0	5	4	0	2	20	0	7	10	0	0.5	2	0	5	3	0	2	50	0	5	3	0	0.5	2	0	7	3	0
15	1.00e-7	2	20	0	5	4	0	2	20	0	7	10	0	0.3	1	0	5	2	0	2	50	0	5	2	0	0.5	2	0	7	3	0
		Am241					Am242m					Am243					Cm242 / Cm243 / Cm244					Cm245									
Gr	E [MeV]	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$	v	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$
1	19.6	3	10	50	10	10	100	5	20	50	10	40	100	5	10	50	10	10	100	5	40	50	10	40	100	5	40	50	10	40	100
2	6.07	2	10	50	10	10	0	3	20	50	10	40	100	3	10	50	10	10	0	5	40	50	10	40	0	5	40	50	10	40	100
3	2.23	2	10	50	10	10	0	3	20	50	10	40	0	3	10	50	10	10	0	5	40	50	10	40	0	5	40	50	10	40	0
4	1.35	2	10	50	10	10	0	3	20	50	10	40	0	3	10	50	10	10	0	5	40	50	10	40	0	5	40	50	10	40	0
5	4.98e-1	2	10	50	10	10	0	3	20	50	10	40	0	3	10	50	10	10	0	5	40	50	10	40	0	5	40	50	10	40	0
6	1.83e-1	2	10	50	10	10	0	3	20	50	10	40	0	3	10	50	10	10	0	5	40	50	10	40	0	5	40	50	10	40	0
7	6.74e-2	2	10	55	10	10	0	3	20	55	10	40	0	3	10	55	10	10	0	5	40	55	10	40	0	5	40	55	10	40	0
8	2.48e-2	2	20	0	10	10	0	3	10	0	10	40	0	3	10	0	10	10	0	5	40	0	10	40	0	5	40	0	10	40	0
9	9.12e-3	2	20	0	10	10	0	3	10	0	10	40	0	3	10	0	10	10	0	5	40	0	10	40	0	5	40	0	10	40	0
10	2.04e-3	2	20	0	10	10	0	3	10	0	10	40	0	3	10	0	10	10	0	5	40	0	10	40	0	5	40	0	10	40	0
11	4.54e-4	2	20	0	10	10	0	3	10	0	10	40	0	3	10	0	10	20	0	5	40	0	10	40	0	5	40	0	10	40	0
12	2.26e-5	2	20	0	10	10	0	3	10	0	10	40	0	3	40	0	10	20	0	5	40	0	10	40	0	5	40	0	10	40	0
13	4.00e-6	2	20	0	10	10	0	3	5	0	10	40	0	3	40	0	10	20	0	5	40	0	10	40	0	5	40	0	10	40	0
14	5.40e-7	2	20	0	10	1																									

ANL Variance Matrix (Structural Isotopes)

Pb				Bi				Fe56				Fe57				Ni58				Cr52				Zr				N15				Si					
Gr	E [MeV]	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$																																
1	19.6	40	20	20	100	40	20	20	100	20	30	45	100	20	30	45	100	20	20	15	100	40	10	20	100	20	20	20	100	30	5	30	100	30	5	20	100
2	6.07	40	20	20	0	40	20	20	0	15	20	30	0	15	20	30	0	20	20	10	0	10	10	20	0	20	20	0	35	5	30	0	30	5	20	0	
3	2.23	40	20	20	0	40	20	20	0	10	10	15	0	10	10	15	0	20	20	10	0	10	10	20	0	20	20	0	0	5	30	0	35	5	20	0	
4	1.35	45	20	20	0	45	20	20	0	20	10	10	0	10	10	10	0	25	10	10	0	15	10	15	0	25	20	0	0	5	30	0	0	5	20	0	
5	4.98e-1	0	20	20	0	0	20	20	0	0	10	8	0	10	10	8	0	0	10	10	0	0	10	10	0	0	20	20	0	0	5	30	0	0	5	20	0
6	1.83e-1	0	20	20	0	0	20	20	0	0	10	8	0	20	10	8	0	0	10	10	0	0	10	10	0	0	20	20	0	0	5	30	0	0	5	20	0
7	6.74e-2	0	20	20	0	0	20	20	0	0	8	8	0	20	8	8	0	0	10	10	0	0	10	10	0	0	20	20	0	0	5	30	0	0	5	20	0
8	2.48e-2	0	20	20	0	0	20	20	0	0	6	8	0	25	6	8	0	0	10	10	0	0	10	10	0	0	20	20	0	0	5	30	0	0	5	20	0
9	9.12e-3	0	20	20	0	0	20	20	0	0	4	8	0	0	4	8	0	0	10	10	0	0	10	10	0	0	20	20	0	0	5	30	0	0	5	20	0
10	2.04e-3	0	20	20	0	0	20	20	0	0	4	8	0	0	4	8	0	0	4	10	0	0	10	10	0	0	20	20	0	0	5	30	0	0	5	20	0
11	4.54e-4	0	20	20	0	0	20	20	0	0	4	8	0	0	4	8	0	0	4	10	0	0	10	10	0	0	20	20	0	0	5	30	0	0	5	20	0
12	2.26e-5	0	20	20	0	0	20	20	0	0	4	8	0	0	4	8	0	0	4	10	0	0	10	10	0	0	20	20	0	0	5	30	0	0	5	20	0
13	4.00e-6	0	20	20	0	0	20	20	0	0	4	8	0	0	4	8	0	0	4	10	0	0	10	10	0	0	20	20	0	0	5	30	0	0	5	20	0
14	5.40e-7	0	20	20	0	0	20	20	0	0	4	8	0	0	4	8	0	0	4	5	0	0	4	8	0	0	20	20	0	0	5	30	0	0	5	20	0
15	1.00e-7	0	20	20	0	0	20	20	0	0	4	8	0	0	4	8	0	0	4	5	0	0	4	8	0	0	20	20	0	0	5	30	0	0	5	20	0

C				O				Na				B10				H (bonded)				Al				Gd				Er				Be9					
Gr	E [MeV]	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$																																
1	19.6	30	5	20	0	35	5	20	0	30	5	10	100	30	10	15	0	0	2	20	0	15	8	10	100	30	5	15	0	30	5	15	0	30	5	10	0
2	6.07	35	5	20	0	0	5	20	0	30	5	10	0	30	10	15	0	0	2	20	0	15	8	10	0	30	5	10	0	30	5	10	0	35	5	10	0
3	2.23	0	5	20	0	0	5	20	0	30	5	10	0	30	10	15	0	0	2	20	0	15	8	10	0	30	5	8	0	30	5	8	0	35	5	10	0
4	1.35	0	5	20	0	0	5	20	0	30	5	10	0	35	10	15	0	0	2	20	0	20	6	7	0	30	5	8	0	30	5	8	0	30	5	10	0
5	4.98e-1	0	5	20	0	0	5	20	0	35	5	10	0	0	10	15	0	0	2	20	0	0	6	7	0	30	5	8	0	30	5	8	0	30	5	10	0
6	1.83e-1	0	3	20	0	0	3	20	0	0	5	8	0	0	10	10	0	0	1	20	0	0	6	7	0	30	5	8	0	30	5	8	0	30	5	8	0
7	6.74e-2	0	3	20	0	0	3	20	0	0	5	8	0	0	10	10	0	0	1	20	0	0	5	7	0	35	5	8	0	30	5	8	0	30	5	8	0
8	2.48e-2	0	3	20	0	0	3	20	0	0	5	8	0	0	5	8	0	0	1	20	0	0	5	6	0	0	5	8	0	0	5	8	0	0	5	8	0
9	9.12e-3	0	3	20	0	0	3	20	0	0	5	8	0	0	5	8	0	0	1	20	0	0	5	6	0	0	5	8	0	0	5	8	0	0	5	8	0
10	2.04e-3	0	3	20	0	0	3	20	0	0	5	7	0	0	5	5	0	0	1	20	0	0	5	6	0	0	4	8	0	0	4	8	0	0	5	7	0
11	4.54e-4	0	3	20	0	0	3	20	0	0	5	7	0	0	5	5	0	0	1	10	0	0	4	5	0	0	4	8	0	0	4	8	0	0	5	7	0
12	2.26e-5	0	2	20	0	0	2	20	0	0	3	7	0	0	3	5	0	0	1	10	0	0	4	5	0	0	4	8	0	0	4	8	0	0	3	7	0
13	4.00e-6	0	2	20	0	0	2	20	0	0	2	5	0	0	2	1	0	0	0.5	2	0	0	3	5	0	0	1	8	0	0	1	8	0	0	2	5	0
14	5.40e-7	0	1	2	0	0	1	2	0	0	2	5	0	0	2	5	0	0	1	2	0	0	1	2	0	0	1	8	0	0	1	8	0	0	2	5	0
15	1.00e-7	0	1	2	0	0	1	2	0	0	2	5	0	0	2	1	0	0	0.5	2	0	0	3	5	0	0	1	8	0	0	1	8	0	0	2	5	0

Li6				Li7				F19				He4					
Gr	E [MeV]	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$												
1	19.6	30	5	10	0	30	5	10	100	30	5	10	100	0	5	20	0
2	6.07	30	5	10	0	30	5	10	0	0	5	20	0	0	5	20	0
3	2.23	35	5	10	0	30	5	10	0	30	5	10	0	0	5	20	0
4	1.35	0	5	10	0	35	5	10	0	30	5	10	0	0	5	20	0
5	4.98e-1	0	5	10	0	0	5	10	0	30	5	10	0	0	5	20	0
6	1.83e-1	0	5	8													

The approach and theoretical background

Integral experiments play an essential role in the reduction of design uncertainties related to reactor neutronics calculations. For this purpose, it is recommended that the proposed experiment shows neutronics features similar to those of the calculated reactor.

- ➡ A quantitative and synthetic measure to judge the relevance of selected experiments to the ABR systems can be based on the “representativity” concept. The approach uses a sensitivity methodology associated to selected integral parameters and based on the Generalized Perturbation Theory (GPT).
- ➡ To carry out a representativity study, sensitivity coefficients have to be first calculated for the selected integral parameters.

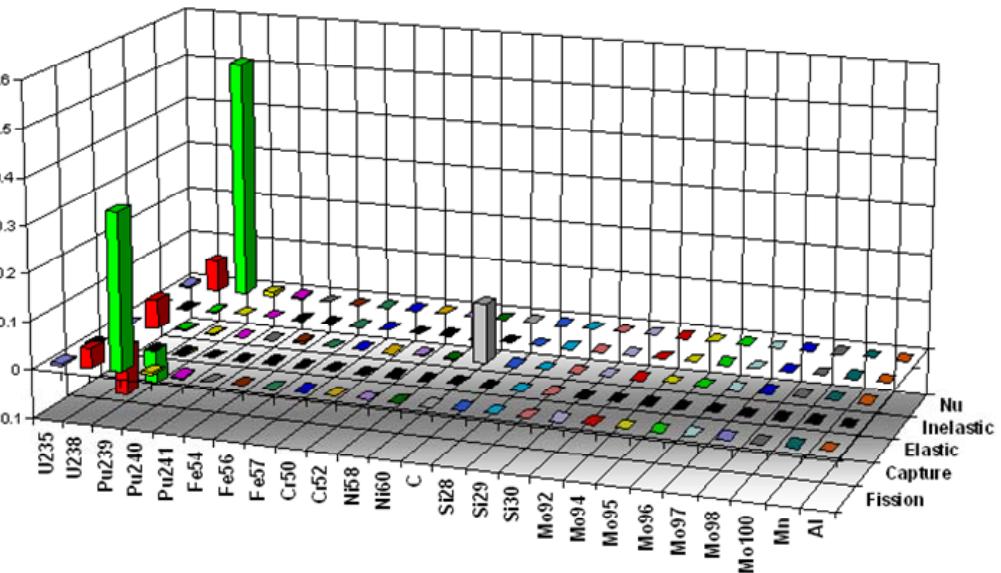
The approach and theoretical background: Meaning of sensitivity coefficient

$$S = \frac{dP}{P} / \frac{d\sigma}{\sigma}$$

$\sigma = \sigma_x$ with $x = \text{isotope, energy group, cross-section type.}$

k_{eff} ZPR3 Assembly 53: Sensitivity Coefficients (%) by Isotope

Isotope	σ_{fiss}	σ_{capt}	σ_{el}	σ_{inel}	v	Total
U235	0.724	-0.092	0.019	-0.002	1.024	1.673
U238	7.458	-17.224	10.559	-0.433	11.288	11.648
Pu239	57.671	-12.087	0.671	0.180	88.567	135.002
Pu240	0.994	-0.821	0.052	0.008	1.434	1.666
Pu241	0.587	-0.068	0.004	0.001	0.913	1.437
Fe54	-	-0.012	0.094	-0.002	-	0.081
Fe56	-	-0.112	0.776	-0.022	-	0.642
Fe57	-	-0.007	0.024	0.001	-	0.018
Cr50	-	-0.005	0.035	-	-	0.030
Cr52	-	-0.017	0.162	-0.012	-	0.133
Ni58	-	-0.101	0.472	-0.009	-	0.362
Ni60	-	-0.037	0.140	-0.005	-	0.099
C	-	-0.003	21.448	-0.082	-	21.363
Si28	-	-0.001	0.025	-	-	0.024
Si29	-	-	0.001	-	-	0.001
Si30	-	-0.001	0.001	-	-	-
Mo92	-	-0.016	0.015	-0.001	-	-0.002
Mo94	-	-0.017	0.009	-	-	-0.008
Mo95	-	-0.140	0.013	0.001	-	-0.127
Mo96	-	-0.054	0.016	-	-	-0.037
Mo97	-	-0.084	0.008	0.001	-	-0.074
Mo98	-	-0.079	0.024	0.001	-	-0.055
Mo100	-	-0.024	0.009	-	-	-0.015
Mn	-	-0.096	0.147	0.002	-	0.053
Al	-	-0.003	0.028	-	-	0.025
Total	67.434	-31.100	34.753	-0.373	103.226	173.940



$$S_x / S_{\text{tot}}$$

Covariance Data: BOLNA covariance Matrix

BOLNA Standard Deviations in Relevant Energy Groups

	ν	σ_f	σ_{inel}	σ_{el}	σ_{capt}	$\sigma_{n,2n}$
U235	0.61	0.5	6.79	4.54	22.13	20.35
U238	2	5.26	20.58	5.35	1.41	5.32
Np237	0.6	5.79	22.35	4.12	5.79	9.51
Pu238	7	17.11	28.65	6	12.31	58.36
Pu239	0.19	0.68	22.18	10.3	11.55	8.53
Pu240	4.81	3.91	9.78	5.53	14.29	54.09
Pu241	0.29	13.54	20.92	5.16	20.51	39.68
Pu242	4.01	18.63	29.28	3.68	24.05	51.7
Am241	1	8.29	29.63	5.5	5.29	10.03
Am242m	0.7	16.57	23.84	11.1	29	31.77
Am243	1.2	9.62	17.87	7.49	8.92	26.63
Cm242	5.6	23.39	11.03	5.27	18.18	53.54
Cm243	1.31	37.25	7.42	10.71	29.72	52.75
Cm244	5.6	36.53	22.67	9.33	22.54	40.91
Cm245	3.01	37.22	13.01	7.45	19.32	22.43
Cm246	5	40	50	10	40	100
Fe56			7.23	5.89	4.02	7.05
Na23			8.87	3.31	6.81	11.07
C			35	5	20	-
O16			100	1.68	81.81	100

Integral Parameters

Calculated Parameter Absolute Values (DIF3D – Diffusion).

	k_{eff}	$\frac{\langle \sigma_{\text{fiss,U}^{238}} \Phi \rangle}{\langle \sigma_{\text{fiss,Pu}^{239}} \Phi \rangle}$	$\frac{\langle \sigma_{\text{capt,U}^{238}} \Phi \rangle}{\langle \sigma_{\text{fiss,Pu}^{239}} \Phi \rangle}$	Void Coefficient [pcm]
ZPR3 Assembly 53	0.968750	0.039	0.166	
ZPR3 Assembly 54	0.888737	0.041	0.162	
ZPPR-2	0.988176	0.024	0.151	-74
ZPPR-9	0.989594	0.022	0.150	1413
ZPPR-15A	0.987255	0.019	0.138	1508
ZPR6 Assembly 6A	0.997041			-1855
ZPR6 Assembly 7	0.992170	0.024	0.151	200
CIRANO – ZONA2A	0.993716	0.041	0.123	-1642
CIRANO – ZONA2A3	0.985850	0.041	0.123	-1678
CIRANO – ZONA2B	0.981893	0.042	0.122	-1776
ABR Metal	0.972326	0.022	0.133	1402
ABR Oxide	0.988232	0.021	0.168	2443

Sensitivity coefficients for the parameters under study

Case of the multiplication factor k_{eff} :

$$S_j = \frac{\partial k}{\partial \sigma_j} \cdot \frac{\sigma_j}{Ak} = -\frac{k}{I_f} \left\langle \Phi^*, \left(\frac{\partial A}{\partial \sigma_j} - \frac{1}{k} \frac{\partial F}{\partial \sigma_j} \right) \Phi \right\rangle$$

with Φ and Φ^* solutions of:

$$M\Phi = \left(A - \frac{1}{k} F \right) \Phi = 0 \quad M^* \Phi^* = \left(A^* - \frac{1}{k} F^* \right) \Phi^* = 0$$

and $I_f = \langle \Phi^*, F\Phi \rangle$, F being the fission production part of the M [=A-(1/k)F] operator.

Case of a reactivity coefficient (like the void reactivity coefficient):

$$\Delta\rho = \left(1 - \frac{1}{k_p} \right) - \left(1 - \frac{1}{k} \right) = \frac{1}{k} - \frac{1}{k_p}$$

Where k_p corresponds to a variation of the Boltzmann operator:

$$M \rightarrow M_p (= M + \delta M), \quad \Phi \rightarrow \Phi_p (= \Phi + \delta\Phi_p)$$

$$\Phi^* \rightarrow \Phi_p^* (= \Phi^* + \delta\Phi_p^*), \quad k \rightarrow k_p (= k + \delta k_p)$$

with Φ_p and Φ_p^* solutions of:

$$M_p \Phi_p = \left(A_p - \frac{1}{k_p} F_p \right) \Phi_p = 0 \quad M_p^* \Phi_p^* = \left(A_p^* - \frac{1}{k_p} F_p^* \right) \Phi_p^* = 0$$

⇒ The sensitivity coefficients (at first order) for $\Delta\rho$ to variations of the σ_j are given as:

$$S_j = \frac{\partial(\Delta\rho)}{\partial \sigma_j} \cdot \frac{\sigma_j}{\Delta\rho} = \frac{1}{\Delta\rho} \left\{ \frac{1}{I_f^p} \langle \Phi_p^*, \sigma_{p,j} \Phi_p \rangle - \frac{1}{I_f} \langle \Phi^*, \sigma_j \Phi \rangle \right\} \quad \text{with} \quad I_f^p = \langle \Phi_p^*, F_p \Phi_p \rangle$$