Multilevel Breit-Wigner (MLBW) elastic angular distributions

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BROOKHAVEN NATIONAL LABORATORY
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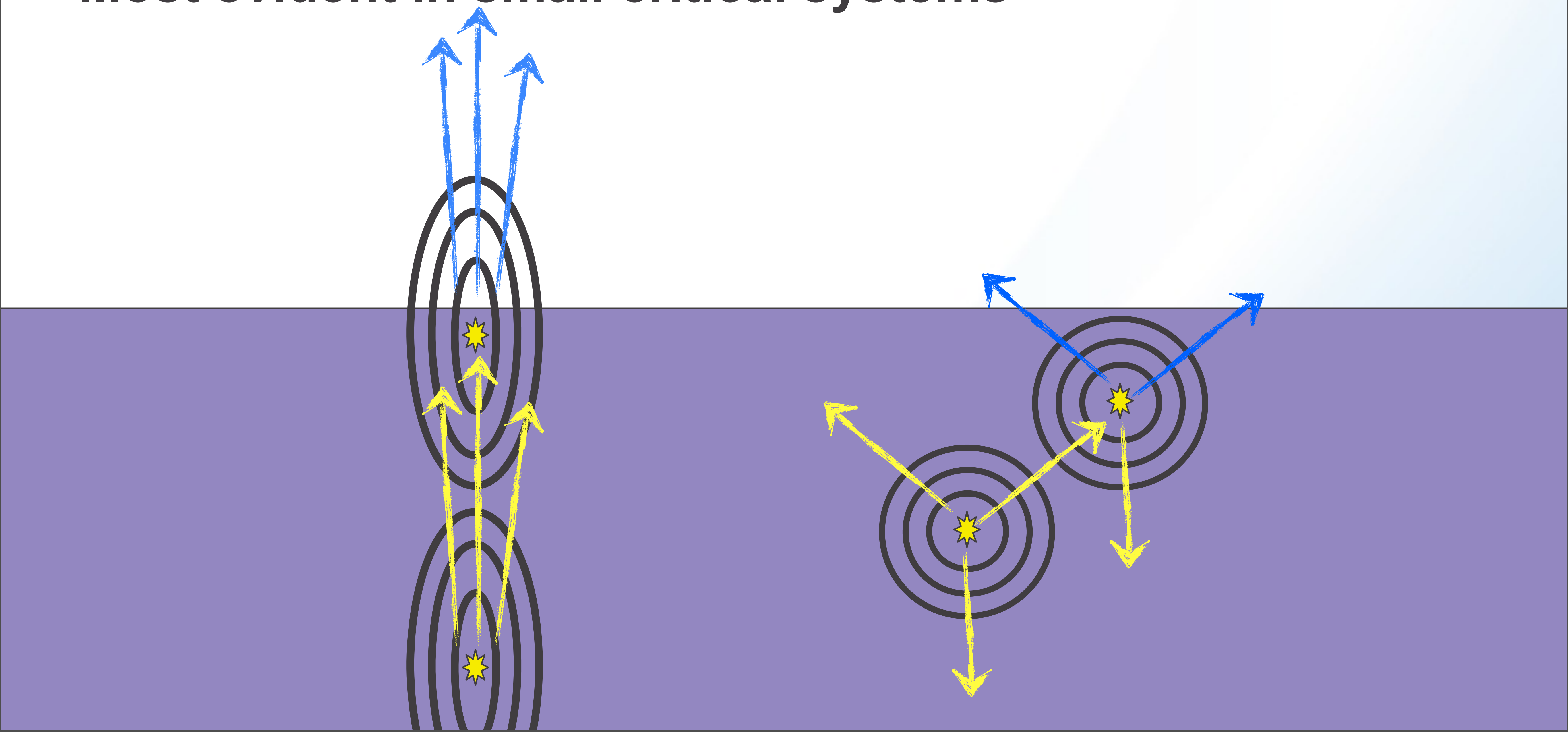
Leakage from a critical assembly depends greatly on angular dists.

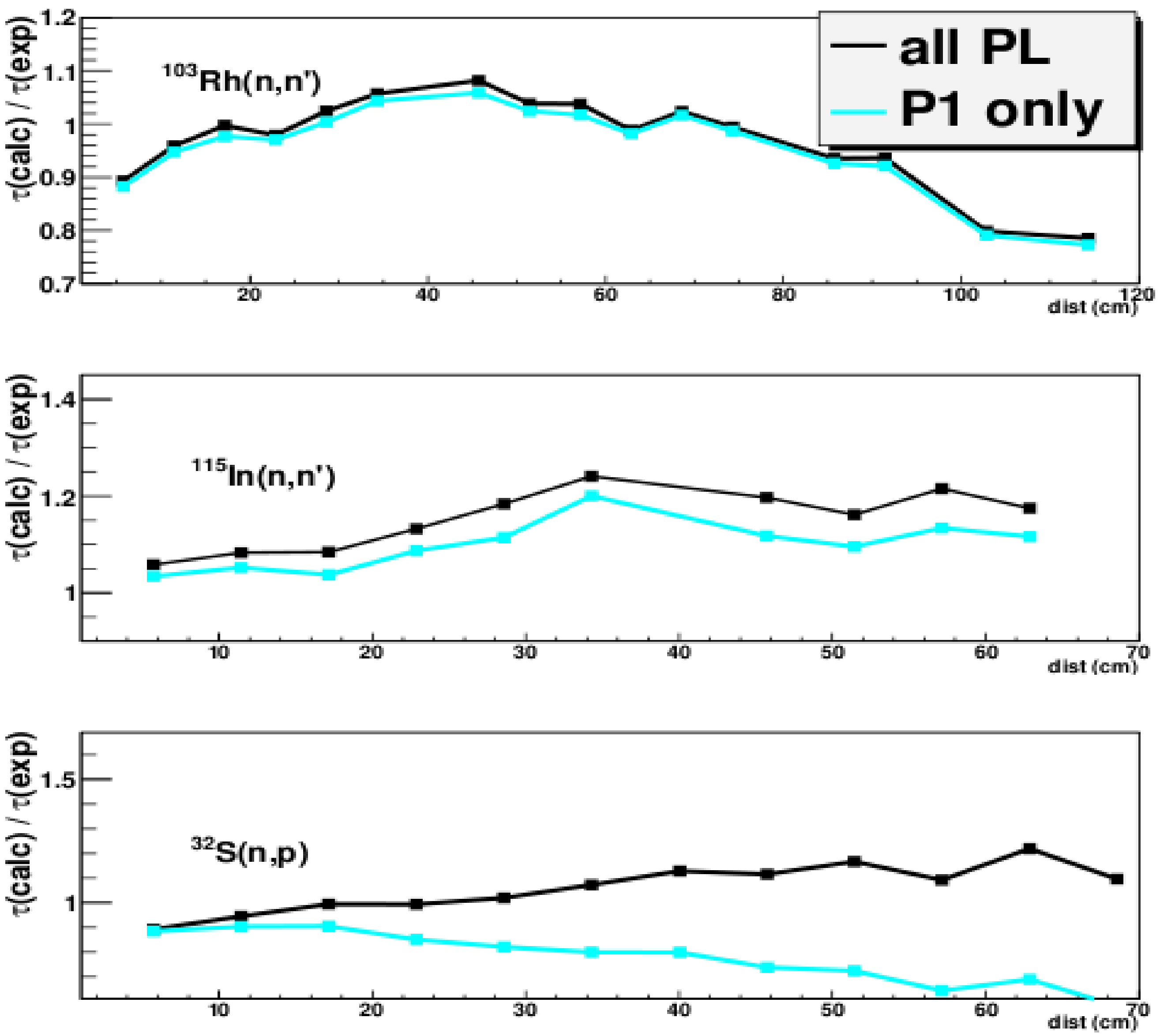
- A forward peaked distribution lets more particles escape from collision events on boundaries than an isotropic one
- Most evident in small critical systems



Shielding benchmark

ASPIS benchmark (iron)

Total Monte Carlo results obtained with the TRIPOLI code shows that **above 20 cm**, the flux attenuation is dominated by higher order polynomial coefficient uncertainties



from G. Noguere

What angular distributions are available in the resonance region?

Format	Number occurrences	Number with angular distributions enabled
scattering radius only	66	0
SLBW	8	0
MLBW	270	0
Reich Moore (LRF=3)	54	46
R Matrix Limited (LRF=7)	1	0
URR only	26	0

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can we salvage these guys?

MLBW Approximation to the R matrix

- R matrix theory gets us the collision matrix (c=channel index)

$$U_{cc'} = e^{-i(\varphi_c+\varphi_{c'})}\left(\delta_{cc'} + i\sum_{\lambda,\mu}\Gamma_{\lambda c}^{1/2}A_{\lambda\mu}\Gamma_{\mu c'}^{1/2}\right)$$

- in terms of the level matrix A:

$$(\mathbf{A}^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} - \sum_c \gamma_{\lambda c}L_c^0\gamma_{\mu c}$$

- Here, E_λ is the resonance energy, $\gamma_{\lambda c}$ is the reduced width and L^0_c is essentially the hard sphere shift and penetrability

MLBW Approximation to the R matrix

- **With just the scattering matrix, we can compute the total cross section**

$$\sigma_c \equiv \sum_{c'} \sigma_{cc'} = 2\pi \lambda\!\!\!^{-}{}_c^2 (1 - \Re U_{cc})$$

- **and the cross section to channel *c'***

$$\sigma_{cc'} = \pi \lambda\!\!\!^{-}{}_c^2 g_c |\delta_{cc'} - U_{cc'}|^2$$

MLBW Approximation to the R matrix

- **MLBW throws away off diagonal parts of the level matrix and renames things into the widths we are familiar with**

$$(\mathbf{A}^{-1})_{\lambda\mu} = (E_\lambda - E - \sum_c L_c^0 \gamma_{\mu c}^2)\delta_{\lambda\mu}$$

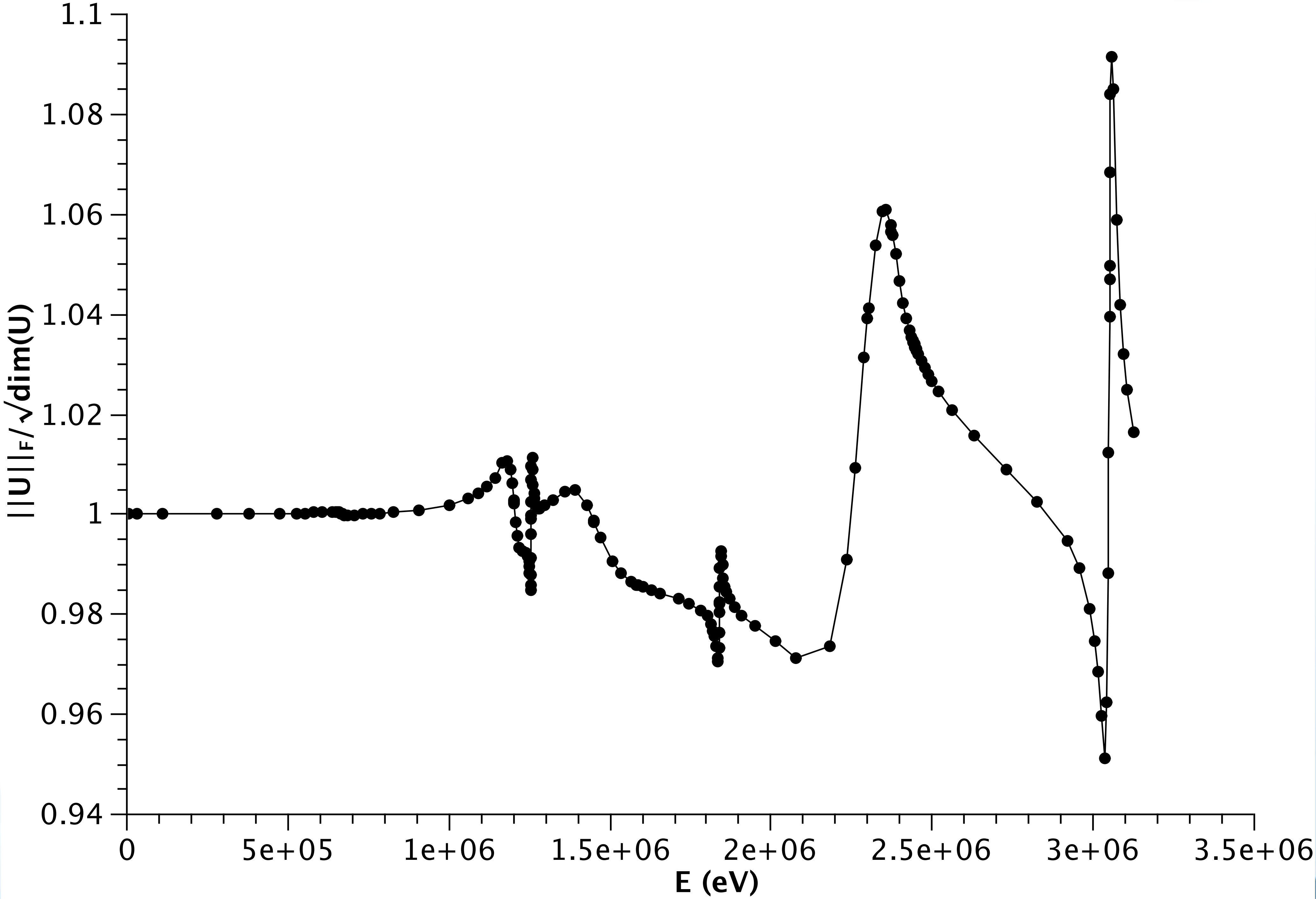
$$\equiv (E_\lambda + \Delta_\lambda - E - i\Gamma_\lambda/2)\delta_{\lambda\mu}$$

- **Result is that collision matrix no longer unitary; we can measure this with Frobenius norm:**

$$||U||_F = \sqrt{\sum_{cc'} U_{cc'}^* U_{c'c}}$$

$$\text{If } U \text{ is unitary, } ||U||_F = \sqrt{\dim(U)}$$

So MLBW is not unitary. Is this a problem?



ENDF's MLBW != R matrix MLBW

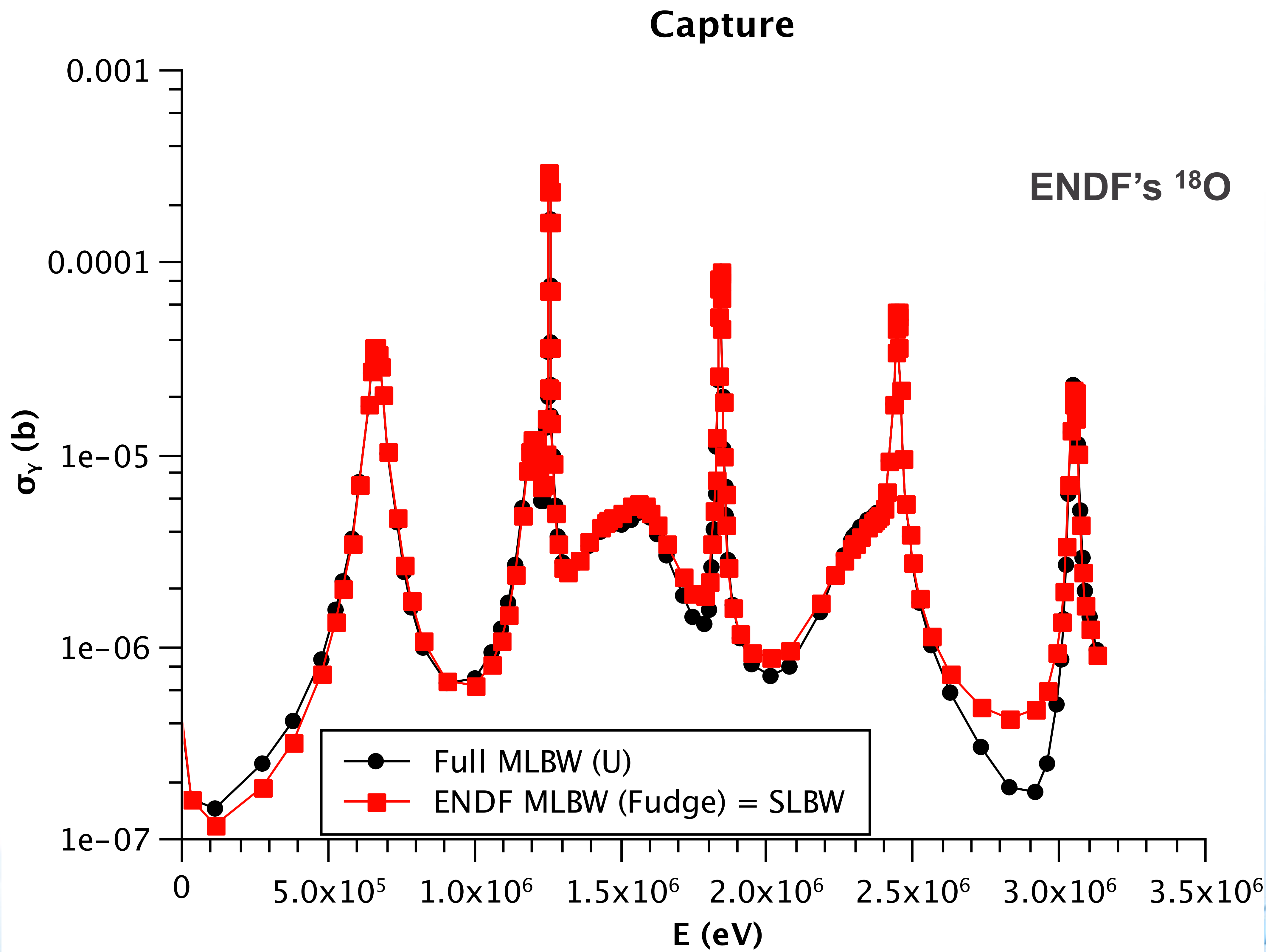
- **ENDF MLBW uses SLBW formulas for non-elastic cross sections**

Bad

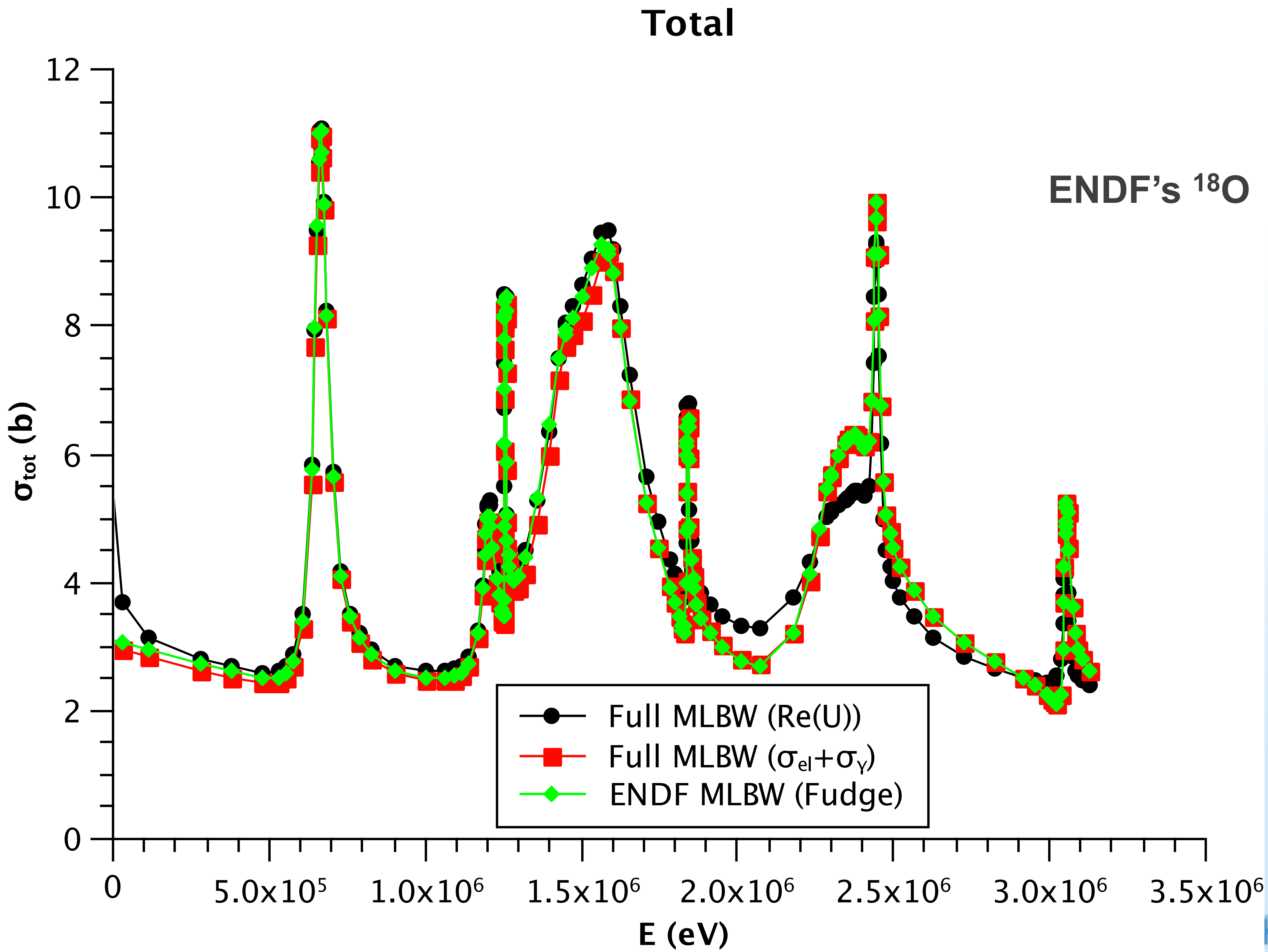
- **ENDF overrides use of potential scattering radius in phase-factor of scattering matrix**

Meh

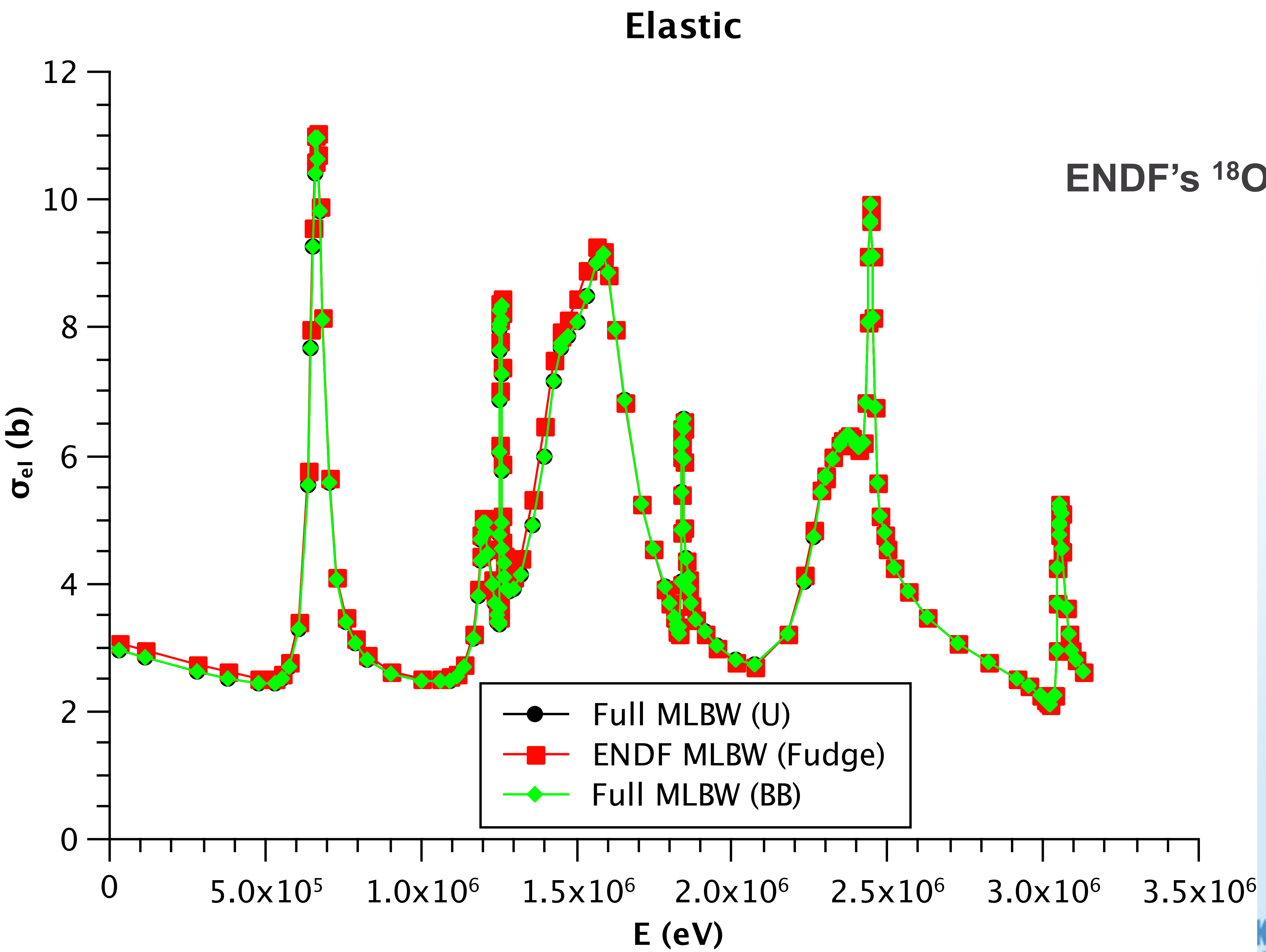
ENDF's MLBW capture is clearly different



This naturally impacts the total too



ENDF MLBW elastic is the same as the R matrix MLBW elastic



Blatt-Beidenharn shows us how to make angular distributions

According to Blatt-Biedenharn formalism, we only need a collision matrix in order to compute an angular distribution:

$$\frac{d\sigma_{\alpha,\alpha'}(E)}{d\Omega} = \frac{1}{k^2(2i+1)(2I+1)} \sum_{s,s'} \sum_{L=0}^{\infty} B_L(\alpha s, \alpha' s'; E) P_L(\mu)$$

where

$$B_L(\alpha s, \alpha' s'; E) =$$

$$\frac{(-)^{s-s'}}{4} \sum_{c_1=\{\alpha\ell_1 s_1 J_1\}} \sum_{c_1'=\{\alpha'\ell_1' s_1' J_1'\}} \sum_{c_2=\{\alpha\ell_2 s_2 J_2\}} \sum_{c_2'=\{\alpha'\ell_2' s_2' J_2'\}} \bar{Z}(\ell_1 J_1 \ell_2 J_2 s L) \bar{Z}(\ell_1' J_1 \ell_2' J_2 s' L)$$

$$\times\, \delta_{ss_1} \delta_{s's_1'} \delta_{J_1 J_1'} \delta_{ss_2} \delta_{s's_2'} \delta_{J_2 J_2'} (\delta_{c_1 c_1'} - U_{c_1 c_1'}(E))^* (\delta_{c_2 c_2'} - U_{c_2 c_2'}(E))$$

We can get MLBW angular distributions!!!

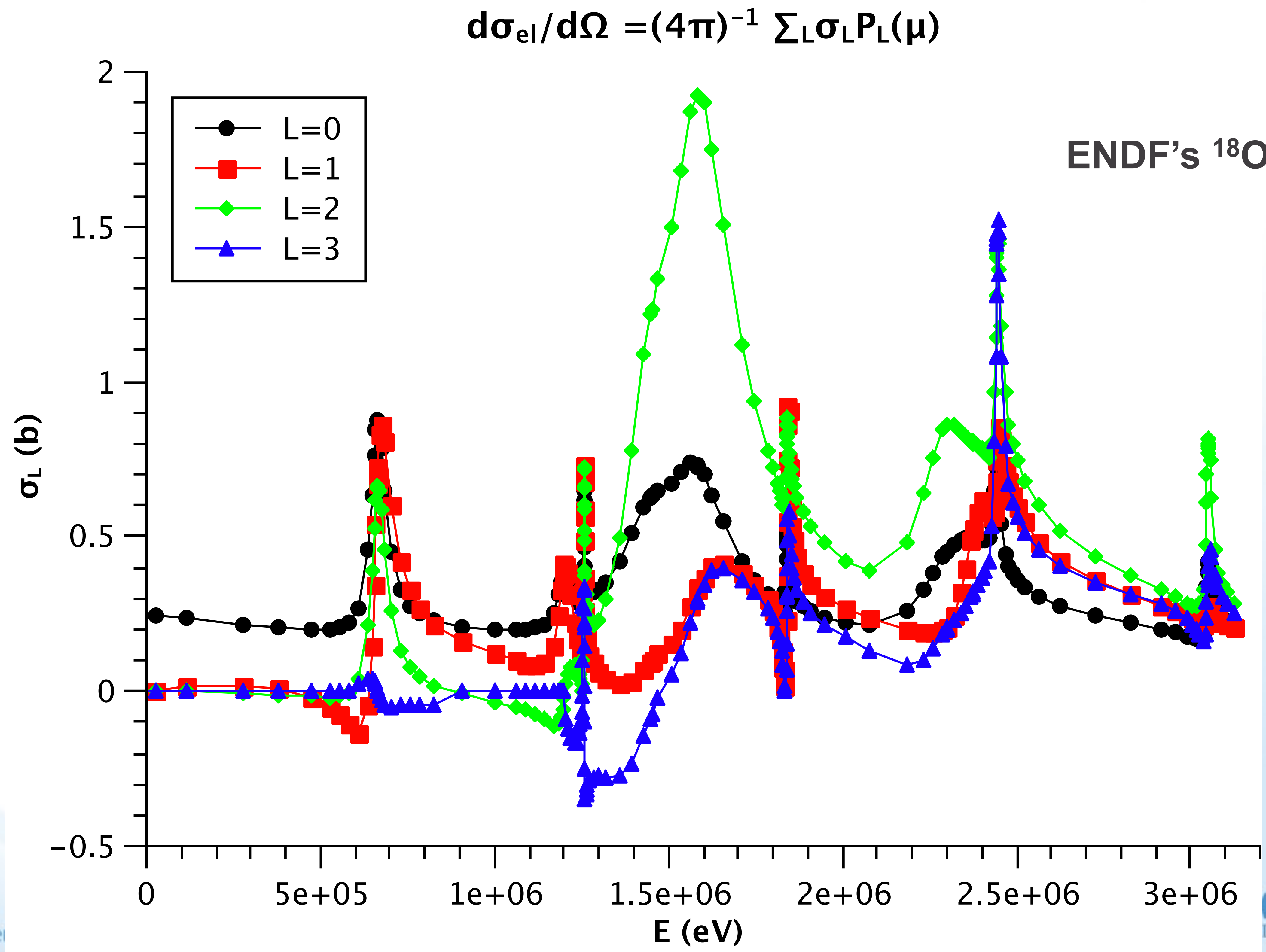
- **For the MLBW format, we have such a thing:**

$$U_{cc'} = e^{-i(\varphi_c + \varphi_{c'})} \left(\delta_{cc'} + \sum_{\lambda} \frac{i\Gamma_{\lambda c}^{1/2} \Gamma_{\lambda c'}^{1/2}}{E_\lambda + \Delta_\lambda - E - i\Gamma_\lambda/2} \right)$$

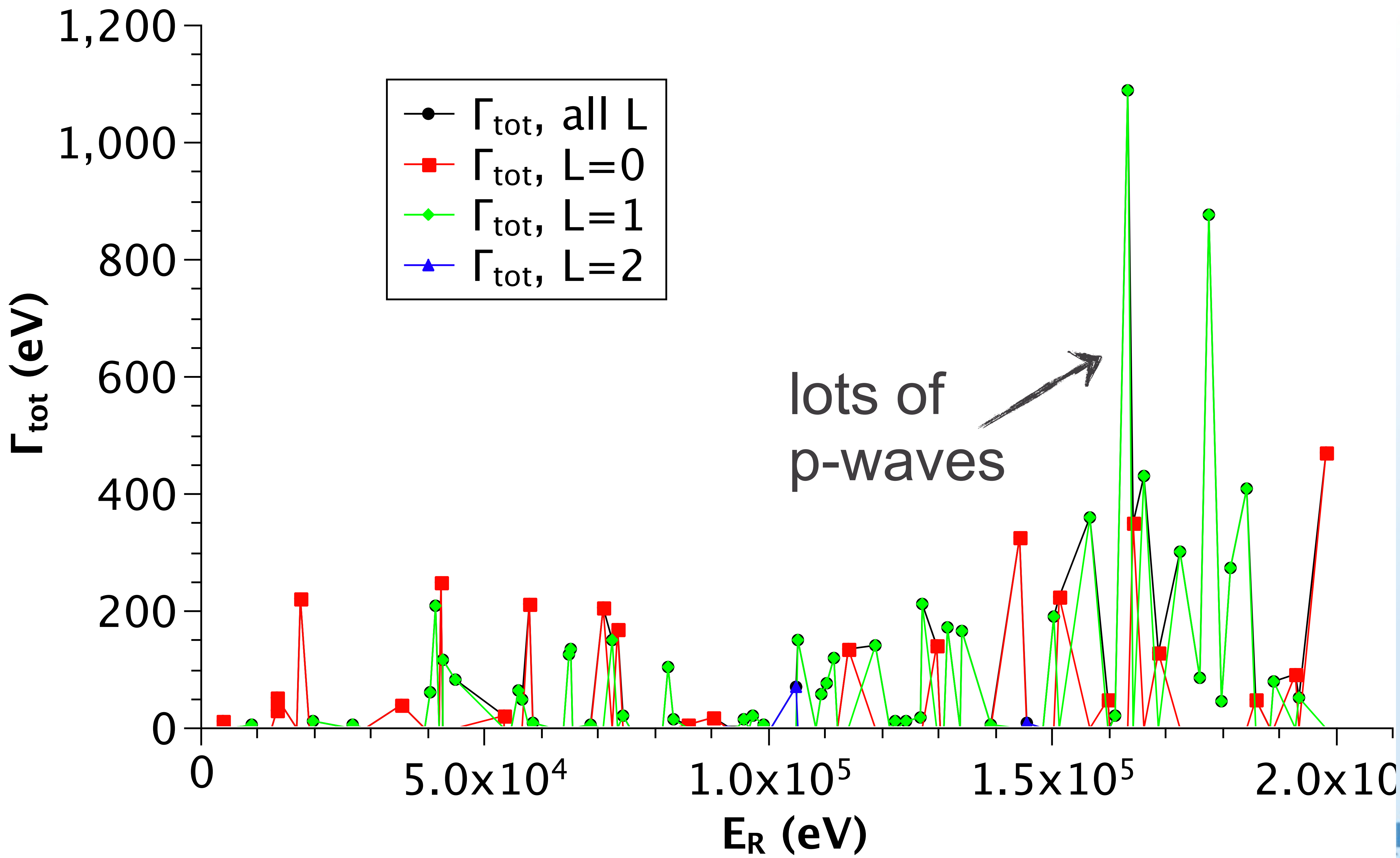
- **The MLBW formalism in ENDF is "broken", but it's elastic channel and the collision matrix are not**

OK then, what does it look like?

The angular distribution directly from ENDF MLBW parameters



Try it on a more popular isotope: ^{90}Zr
distribution of level widths



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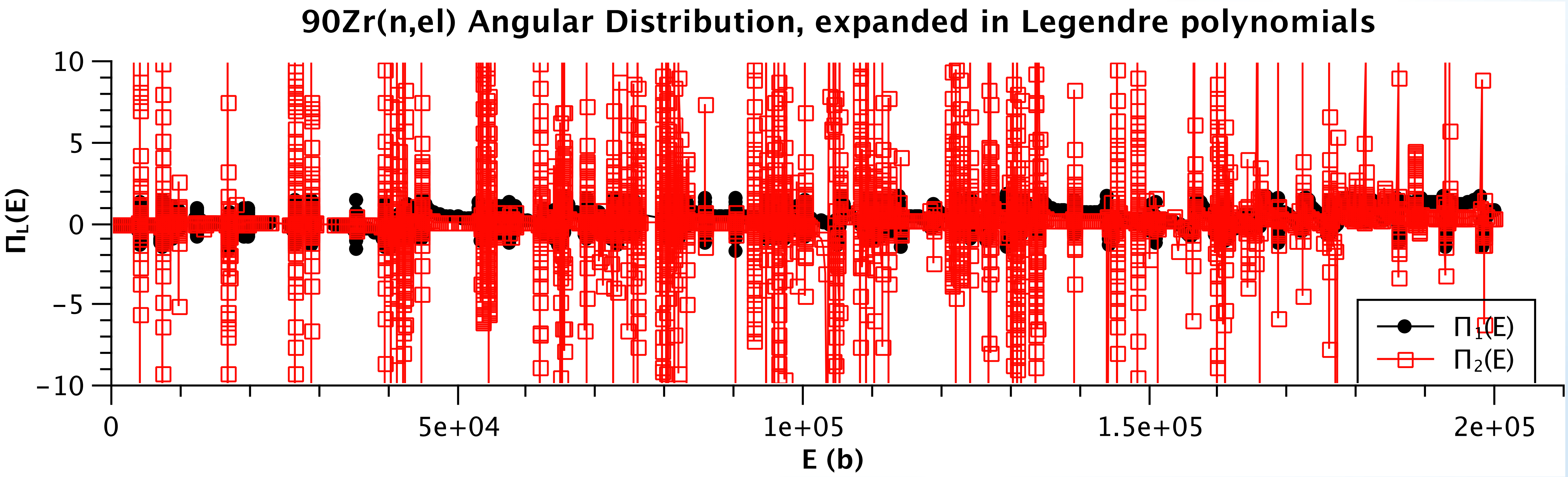
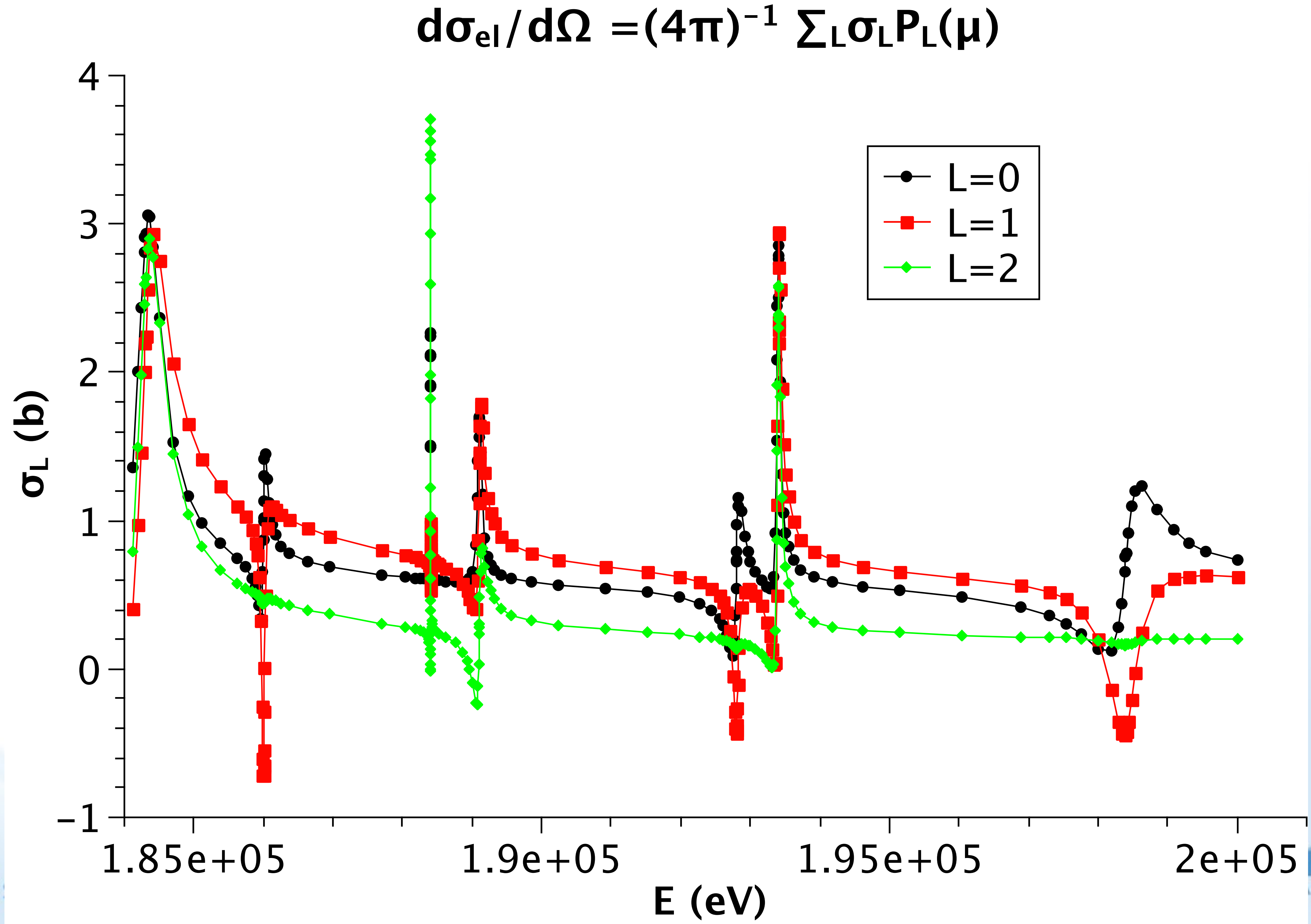
angular distribution



Try it on a more popular isotope: ^{90}Zr

zoom in on angular distribution



What next?

- Finish coding hooks in Fudge
- Get the distributions into an ENDF file (e.g. 90Zr)
- Compare to ^{nat}Zr $\bar{\mu}$
- Try it out in a benchmark