Procedures for Optimum Representation of the Measured B(E2) Values for the First 2+ States

Michael Birch
McMaster University

Outline

- Motivation
  - The Nature of Uncertainties
  - The Physical Significance of the Result
  - The Origin of the Data
- A Method of Representation of Input Data
  - Formulation
  - Examples
  - Issues
- Conclusion

An Example Situation

- Evaluating B(E2) data for the $0^+ \rightarrow 2^+$ transitions of even-even nuclei
  - Two representations of results: lifetime (τ) and B(E2)
  - Diverse data from several methods and laboratories
  - Experiments Separated by many years
- Previous evaluation for comparison
  - 2001RA27 – S. Raman

The Nature of Uncertainties

- Frequently: X σ
- More generally: X $(+\sigma^{+} - \sigma^{-})$
  - Can result from experiment: in the B(E2) evaluation, lifetime measurements sometimes have asymmetric uncertainties.
  - Can also come from calculations where the input uncertainty is large
    - $f(x + \sigma x) \approx f(x) + df/dx(\sigma x) \Rightarrow \sigma f = df/dx(\sigma x)$ if $\sigma x << x$
    - If σx is large need to go back to definition used above $\sigma f^{+} \equiv f(x + \sigma x) - f(x)$
    - Similarly for lower uncertainty $\sigma f^{-} \equiv f(x) - f(x - \sigma x)$
- Ways to handle asymmetry:
  - Symmetrization
  - Generalize the averaging procedure

Symmetrization

- Consider quantity X ($+\sigma^+ - \sigma^-$)
- Simple method
  - $\sigma_{sym} = ½(\sigma^+ + \sigma^-)$
  - New central Value = $½[(X+ \sigma^+) + (X - \sigma^-)] = X + ½(\sigma^+ - \sigma^-)$
- Advanced method: G. Audi – NuBase (2003Au02)
  - Define probability distribution as asymmetric normal distribution with modal value x=X and standard deviation σ^+ for x>X and σ^- for x<X
  - Use variance $\sigma_{sym} = (1 - 2/\pi)(\sigma^+ - \sigma^-)^2 + \sigma^+ \sigma^-$
  - Can find m which divides the distribution into equal area, use this as the new central value

$$\mathrm{erf}\left(\frac{m - X}{\sqrt{2}\ \sigma^+}\right) = \frac{\sigma^+ - \sigma^-}{2\sigma^+}$$ where σ+ > σ-

Symmetrization Consequences

- Both methods shift the central value – that is to say alter the original data
- Consider the following examples given by Audi

Nuclide	Original $T_{½}$	Simple Method		Advanced Method	
		Result	Deviation	Result	Deviation
^{76}Ni	240+550–190 ms	420 ± 370	75.00%	470 ± 390	95.83%
^{222}U	1.0+1.0–0.4 μs	1.3 ± 0.7	30.00%	1.4 ± 0.7	40.00%
^{264}Hs	327+448–120 μs	490 ± 280	49.85%	540 ± 300	65.14%
^{266}Mt	1.01+0.47–0.24 ms	1.1 ± 0.4	8.91%	1.2 ± 0.4	18.81%

Symmetrization Consequences

- Raman symmetrized all his data via the simple method
- The result is internal inconstancies within his tables
  - Example: 124Ce
  - Single lifetime measurement, $\tau = 1270$ 280 ps
  - Converting to B(E2) gives 3.5, but Raman gives 3.7 since the uncertainty would have been asymmetric, 3.50(+99–63)
  - 3.7 as a lifetime is 1200 ps – a difference of 6%
- In general, $\sigma^{+} > \sigma^{-}$
  - all lifetime measurements will be skewed upwards when expressed as B(E2) values

Generalizing the Averaging Procedure

- Weighted averaging can be generalized to handle asymmetric uncertainties

$$w = \frac{2}{(\sigma^{+})^2 + (\sigma^{-})^2}$$

- Removes the problems associated with symmetrization

Physical Significance of the Result

- A measurement represents a physical quantity –there is one correct answer
  - Multiple representations of the same quantity should yield the same average
- B(E2) and τ represent the same underlying characteristic of a nuclear level
- To average a mixed data set one must be converted to the other

Example: ^{20}Ne

Measured Mean Lifetime (ps)	Reference
1.14(24)	1982SP02
0.8(2)	1975HO15
1.15(20)	1971HA26
0.84(20)	1969GR03
1.25(35)	1969AN08
1.27(24)	1969TH01
1.23(12)	1965EV03
0.76(33)	1956DE22

Measured B(E2)	Reference
0.0322(34)	1977SC36
0.037(3)	1972OL02
0.048(7)	1970NA07
0.047(9)	1960AN07
0.041(10)	1959AL91

B(E2)	Reference
0.0308(+82-54)	1982SP02
0.0439(+146-88)	1975HO15
0.0305(+64-45)	1971HA26
0.0418(+131-80)	1969GR03
0.0281(+109-61)	1969AN08
0.0276(+64-44)	1969TH01
0.0285(+31-25)	1965EV03
0.046(+35-14)	1956DE22
0.0322(34)	1977SC36
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1.23(12)	1965EV03
0.76(33)	1956DE22
1.09(+13-10)	1977SC36
0.948(+84-71)	1972OL02
0.731(+125-93)	1970NA07
0.75(+18-12)	1960AN07
0.86(+28-17)	1959AL91

Example: ^{20}Ne

- Weighted average of B(E2) values: 0.0333(16) e^2b^2
- Weighted average of mean lifetimes:
  - 0.966(51) ps ⇔ 0.0363(19) e^2b^2
- 9% difference between mean values
- Larger than uncertainty by a factor of two

The Origin of the Data

- When all data is from a single experiment one attributes any spread in the measurements to random fluctuations
  - Best modelled by the normal distribution
- Hence the fundamental assumption of weighted averaging:

There exists a single normal distribution which is most likely to reproduce the given data set if random points are chosen from it.

The Origin of the Data

- If the data are diverse perhaps there is a probability distribution which *represents* the data better since all fluctuation may not be random

- The Best Representation (or Expected Value) Method

Formulation

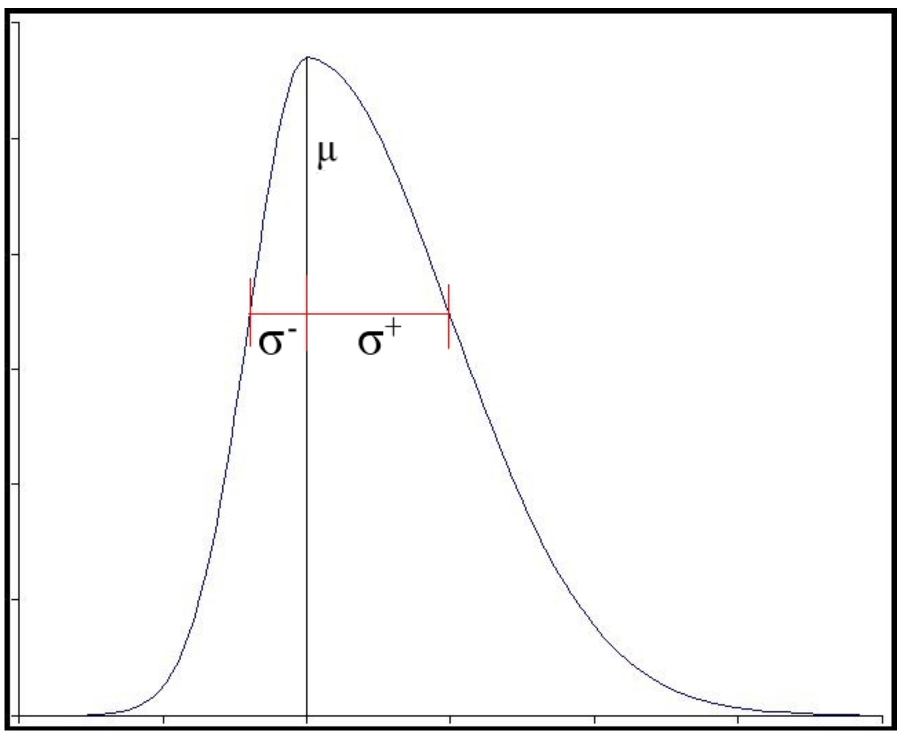
- Basic assumption:

A representative probability distribution for a set of data can be built from modelling each point as a normal distribution

- Call this distribution the *Total Probability Distribution*, G(x)
- Use the asymmetric Gaussian to model the data points, $g(x;\mu, \sigma^+, \sigma^-)$

The Asymmetric Gaussian

$$g(x;\mu,\sigma+,\sigma-) = \sqrt{\frac{2}{\pi(\sigma^{+}+\sigma^{-})^{2}}} \begin{cases} e^{\frac{-(x-\mu)^2}{2(\sigma^{-})^2}} & x \leqslant \mu \\ e^{\frac{-(x-\mu)^2}{2(\sigma^{+})^2}} & x > \mu \end{cases}$$

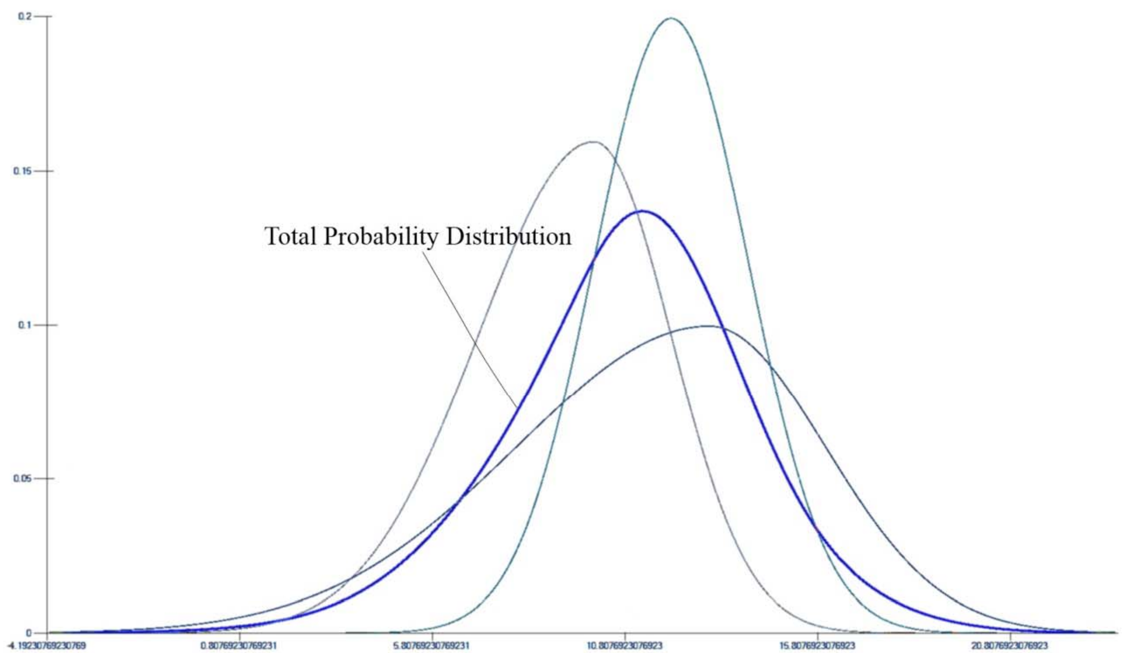


The Total Probability Distribution

For a data set, S, with n points,

$$S = \{\mu_1(+\sigma^+{}_1 - \sigma^-{}_1),\ \mu_2(+\sigma^+{}_2 - \sigma^-{}_2),\ \ldots,\ \mu_n(+\sigma^+{}_n - \sigma^-{}_n)\}$$

$$G(x) = (1/n)\sum_{i=1}^{n} g(x;\ \mu_i,\ \sigma^+{}_i,\ \sigma^-{}_i)$$



Computing the average

- The mean is defined to be the statistical expected value of a discrete variable with probability distribution G(x)

$$\bar{\mu} = \sum_{i=1}^{n} \mu_i \frac{G(\mu_i)}{G_T} \text{, where } G_T = \sum_{i=1}^{n} G(\mu_i)$$

- The internal uncertainty is computed as in weighted averaging, the external uncertainty is calculated as the variance of the distribution

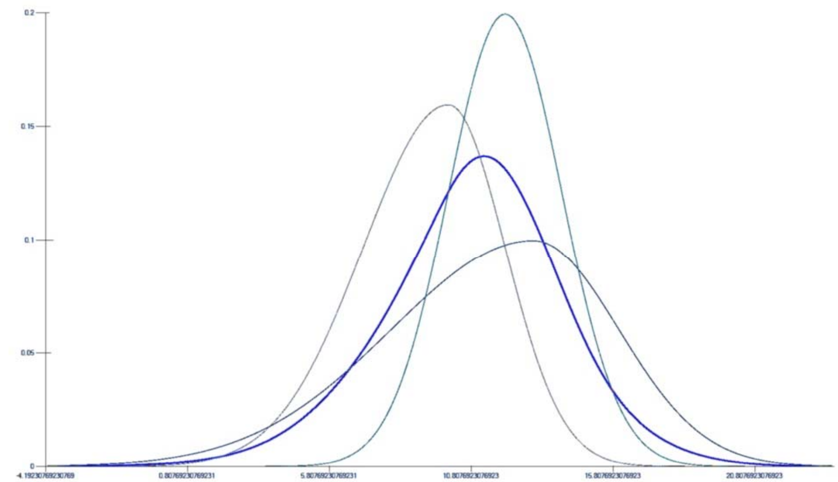
$$\sigma(\bar{\mu}) = \sqrt{\sum_{i=1}^{n} (\bar{\mu} - \mu_i)^2 \frac{G(\mu_i)}{G_T}}$$

Validation of the Method

- Consider a data set with a total probability distribution which resembles a Gaussian
  - The data is well represented by a normal distribution, so the assumption of weighted averaging is satisfied
  - The results returned by both Weighted Averaging and the Best Representation Method should be very similar

Validation of the Method

Data
10 (+2-3)
13 (+3-5)
12 (2)



- Weighted average: 11.5 (+12-17)
- Best Representation: 11.6(+13-20)

- Real Example: ^{26}Si – two data points, in good agreement, give near perfect normal distribution
  - Weighted averaging and Best Representation results are identical

Comments

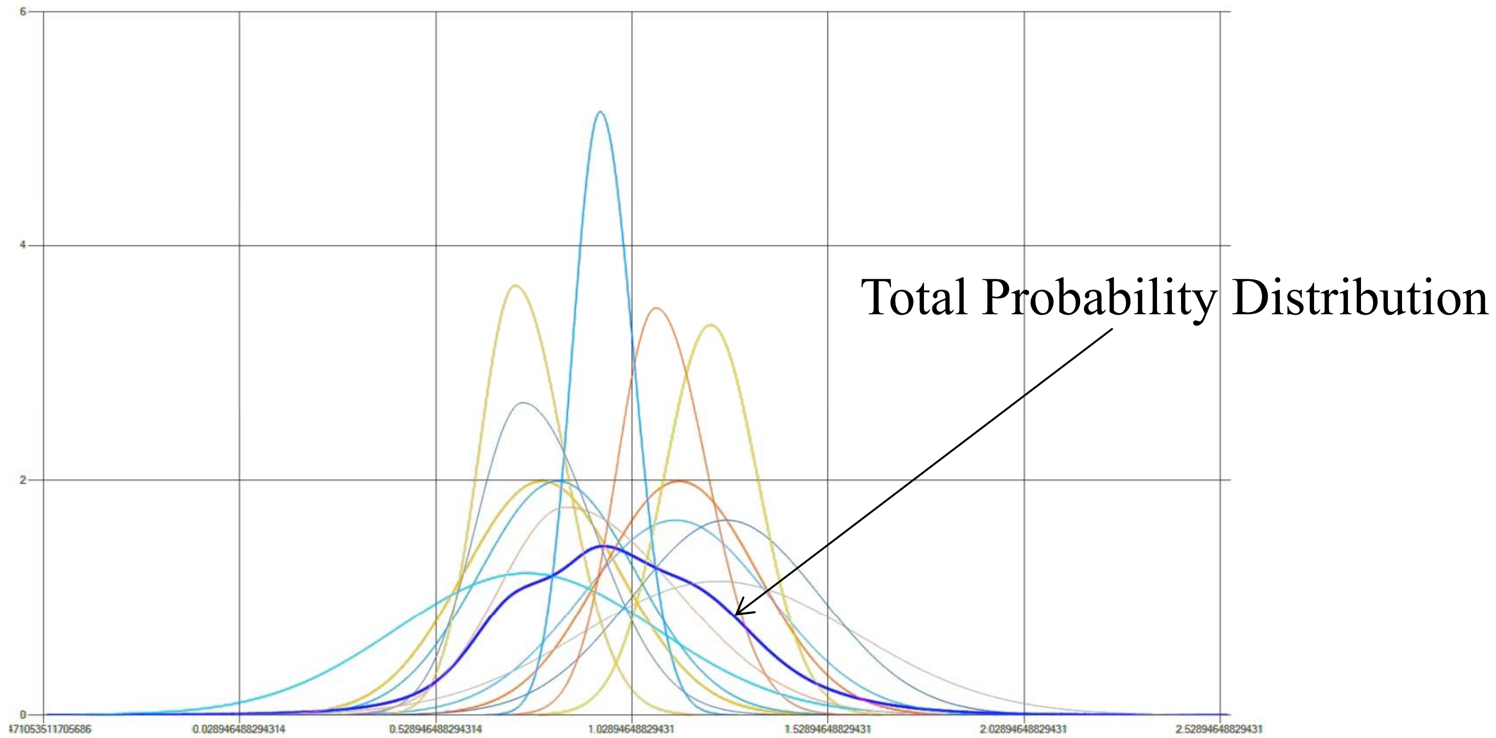
- Results computed via an interactive computer code written for this method (which also returns the weighted average for comparison)
- Method handles asymmetric uncertainties
  - No symmetrisation problem
- No assumptions about the data set as a whole
  - Constructs a probability distribution to give a true representation of the inputs
- Behaves well under data transformations
  - Maintains the physical meaning of the data

^{20}Ne Revisited

- Recall weighted averaging results:
  - Mean of B(E2) values: 0.0333(16) e^2b^2
  - Mean of lifetimes: 0.966(51) ps ⇔ 0.0363(19) e^2b^2
  - 9% difference between results
- Best Representation Method results:
  - Mean of B(E2) values: 0.0349(68) e^2b^2
  - Mean of lifetimes: 0.98(19) ps ⇔ 0.0358(+86-58) e^2b^2
  - 2.6% difference between results, agreement well within uncertainties

^{20}Ne Revisited

- Notice the uncertainties from the Best Representation Method are a factor of 4 higher than Weighted Averaging. Why?
- Look at the probability distribution



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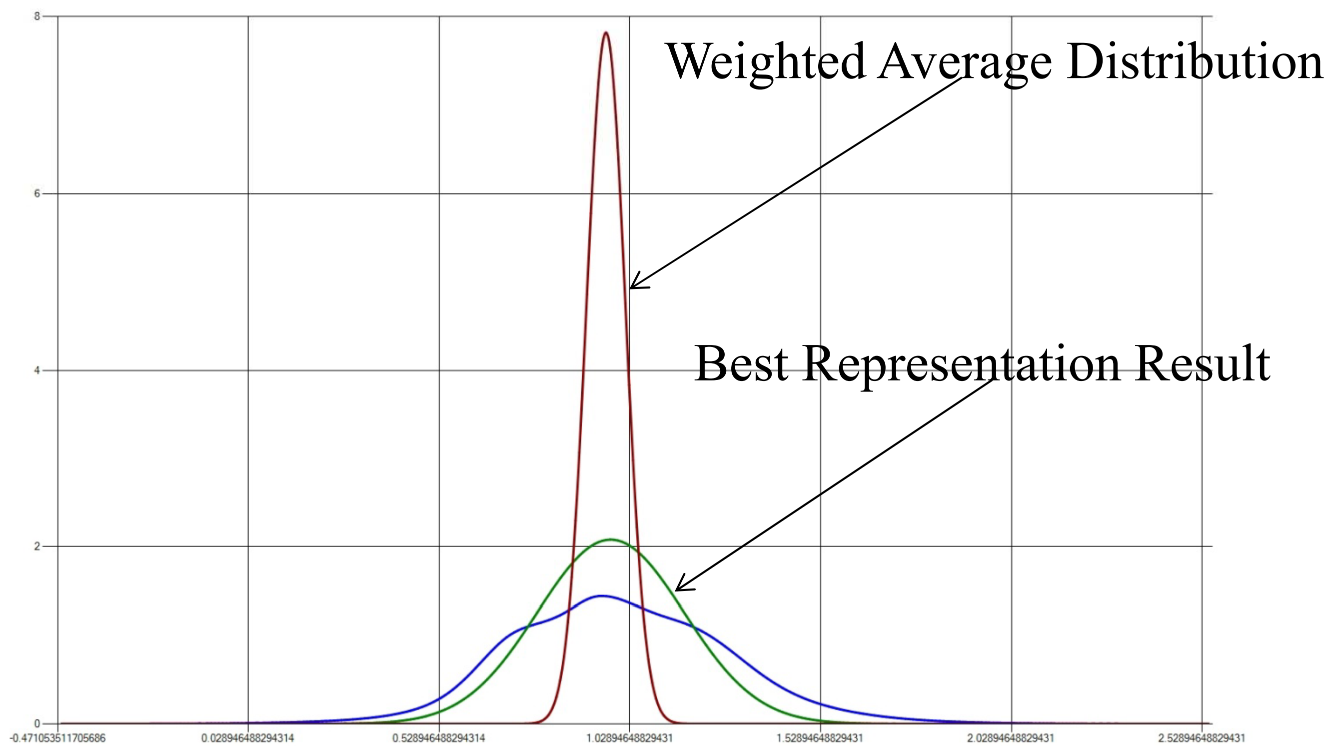

Inflated Uncertainty

- The Best Representation Method returns larger uncertainties than Weighted Averaging in general, however they are not always justified
- They can be artificially inflated by a single data point since the method does in fact represent the data set as a whole
- Consider the following example:

Data
25(3)
26(4)
24(2)
25.5(25)

Weighted Average: 24.8(13)
Best Representation: 25.1(15)

Data
25(3)
26(4)
24(2)
25.5(25)
27(10)

Weighted Average: 24.9(13)
Best Representation: 25.4(20)

Conclusion

- The Best Representation Method builds a probability distribution using the input data to remove the central assumption of Weighted Averaging
- The mean is the expected value of this total probability distribution and the uncertainty is the variance
- The method is aptly named as its results reflect the input data well and are reasonably invariant under conversions in most cases
- It converges with Weighted Averaging in the case where the data is properly described by a normal distribution
- It is most effective when all inputs have comparable uncertainties