Fast Reactor Sensitivity Studies at ANL

Nuclear Physics Working Group Meeting

June 24, 2010

Port Jefferson, NY

W. S. Yang, G. Aliberti, R. D. McKnight

Nuclear Engineering Division

Argonne National Laboratory

Objective and Background

- The main objective is to reduce the uncertainty of reactor performance predictions due to cross section uncertainties by the combined use of covariance data and integral experiments in statistical uncertainty evaluation and/or cross section adjustment
- In FY 2008, a collaborative project among INL, ANL, BNL, and LANL was initiated
  - Selection of a set of experiments (INL, ANL)
  - Analysis of experiments using the best methodologies available (INL, ANL)
  - Production of science based covariance data (BNL, LANL)
  - Sensitivity analysis of selected configurations including reference design configurations (ANL, INL)
  - Uncertainty evaluation and target accuracy assessment (ANL, INL)
  - Statistical uncertainty evaluation and cross section adjustment using calculation/experiment discrepancies (INL, ANL)

Performance Parameter Uncertainty due to Nuclear Data Uncertainty

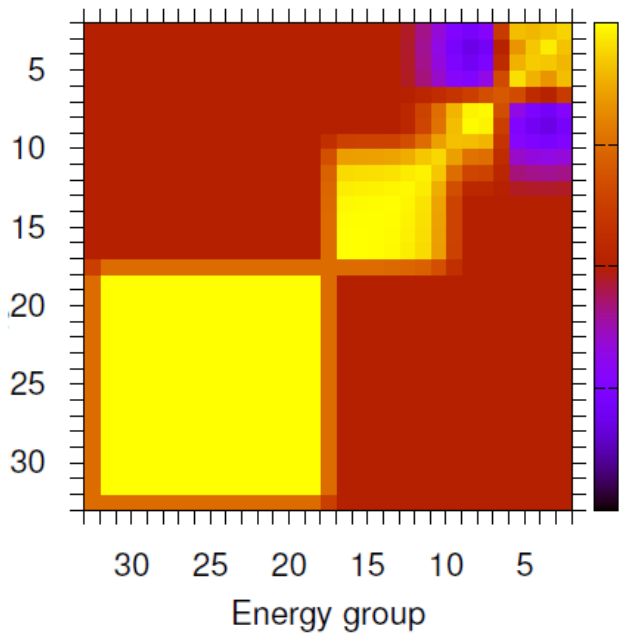
- Sensitivity coefficient vector of a performance parameter $\boldsymbol{R}$ with respect to all input parameters $\boldsymbol{\alpha}$ (e.g., cross sections)

$$\mathbf{s} = \left(\frac{\alpha_1}{R}\frac{\partial R}{\partial \alpha_1}, \frac{\alpha_2}{R}\frac{\partial R}{\partial \alpha_2}, \cdots, \frac{\alpha_N}{R}\frac{\partial R}{\partial \alpha_N} \right)^T$$

- Response parameter uncertainty

$$\sigma_R^2 = \mathbf{s}^T \mathbf{M} \mathbf{s} = \sum_{i=1}^{N}\sum_{j=1}^{N} s_i s_j m_{ij}$$

  - $\mathbf{M} = [m_{ij}]$: relative covariance matrix of input parameters
  - The latest covariance data is AFCI-1.3 prepared by BNL and LANL



Correlation and Representativity Analysis

- Denoting the sensitivity vectors of a performance parameter in two different systems by $\mathbf{s}_1$ and $\mathbf{s}_2$, the correlation of two systems with respect to the parameter of interest can be represented as

$$\rho_{12} = \frac{\mathbf{s}_1^T \mathbf{M} \mathbf{s}_2}{[(\mathbf{s}_1^T \mathbf{M} \mathbf{s}_1)(\mathbf{s}_2^T \mathbf{M} \mathbf{s}_2)]^{1/2}}$$

  - This correlation factor quantifies the similarity between two systems with respect to the performance parameter
- Let the system 1 be an integral experiment E and the system 2 be the target system . Then, using the integral experiment E, the uncertainty σ_R of a response parameter R of the target system can be reduced to

$$\sigma'_R = \sigma_R (1 - r_{RE}^2)^{1/2}$$

  - r_{RE} is the correlation of the parameters of the target and experimental systems and is often called the representativity factor.

Uncertainty Reduction Methods

- Consider multiple response parameters and experiments
  - Subscripts 1 and 2 respectively denote the experiments and target system
- No Experimental Information

$$\mathbf{V}_2 = \mathbf{G}_2 \mathbf{M} \mathbf{G}_2^T$$

- Bias Factor Method

$$\mathbf{V}_2 = (\mathbf{G}_2 - \mathbf{G}_1)\mathbf{M}(\mathbf{G}_2 - \mathbf{G}_1)^T$$

- Cross Section Adjustment Method

$$\mathbf{M}' = \mathbf{M} - \mathbf{M}\mathbf{G}_1^T(\mathbf{V}_1 + \mathbf{G}_1\mathbf{M}\mathbf{G}_1^T)^{-1}\mathbf{G}_1\mathbf{M}$$

$$\mathbf{V}_2 = \mathbf{G}_2 \mathbf{M}' \mathbf{G}_2^T$$

- Combined Method (bias factor method after cross section adjustment)

$$\mathbf{V}_2 = (\mathbf{G}_2 - \mathbf{G}_1)\mathbf{M}'(\mathbf{G}_2 - \mathbf{G}_1)^T$$

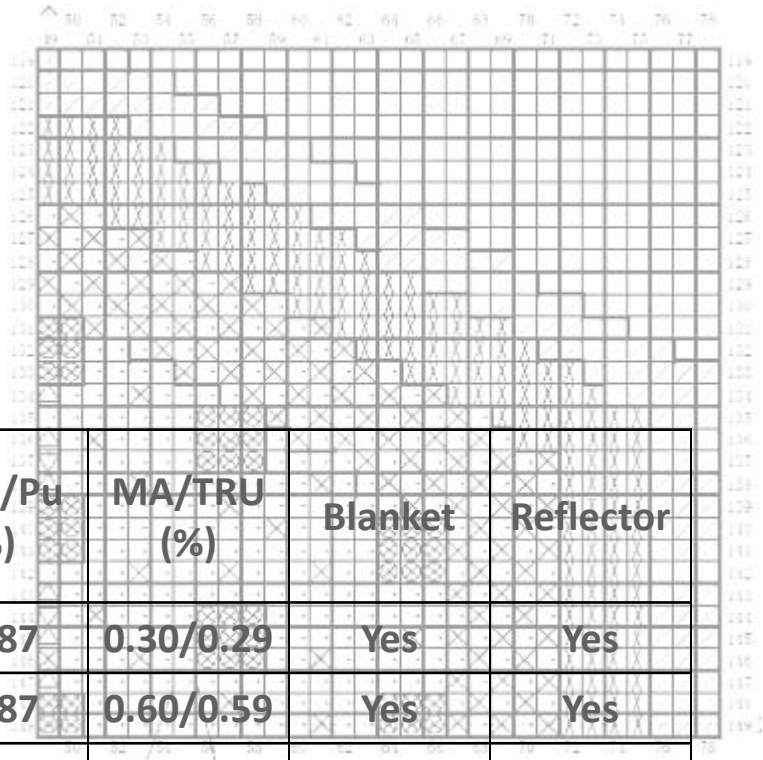
Summary of Sensitivity Studies Completed (1/5)

- Eight potential target systems
  - Multiplication factor, power peaking factor, Doppler coefficient, coolant void worth, burnup reactivity swing

System	Power (MWt)	Core volome (m^3)	Coolant	Fuel type	TRU/HM (%)	MA/TRU (%)	Cycle length (EFPD)	BR or TRU CR
ABTR	250	1.0	Sodium	Metal	18.8	~0	110	0.65
ABR metal	1000	3.5	Sodium	Metal	22.1	8.6	330	0.73
ABR oxide	1000	4.7	Sodium	MOX	28.0	11.1	330	0.66
SFR	840	2.6	Sodium	Metal	60.5	17.5	155	0.47
EFR	3600	11.8	Sodium	MOX	23.7	5.1	1700	1.07
GFR	2400	24.0	Helium	Carbide	21.7	23.0	415	1.03
LFR	900	7.4	Lead	Metal	23.3	10.3	310	0.83
ADMAB	380	2.5	LBE	Nitride	100.0	68.0	365	0.00

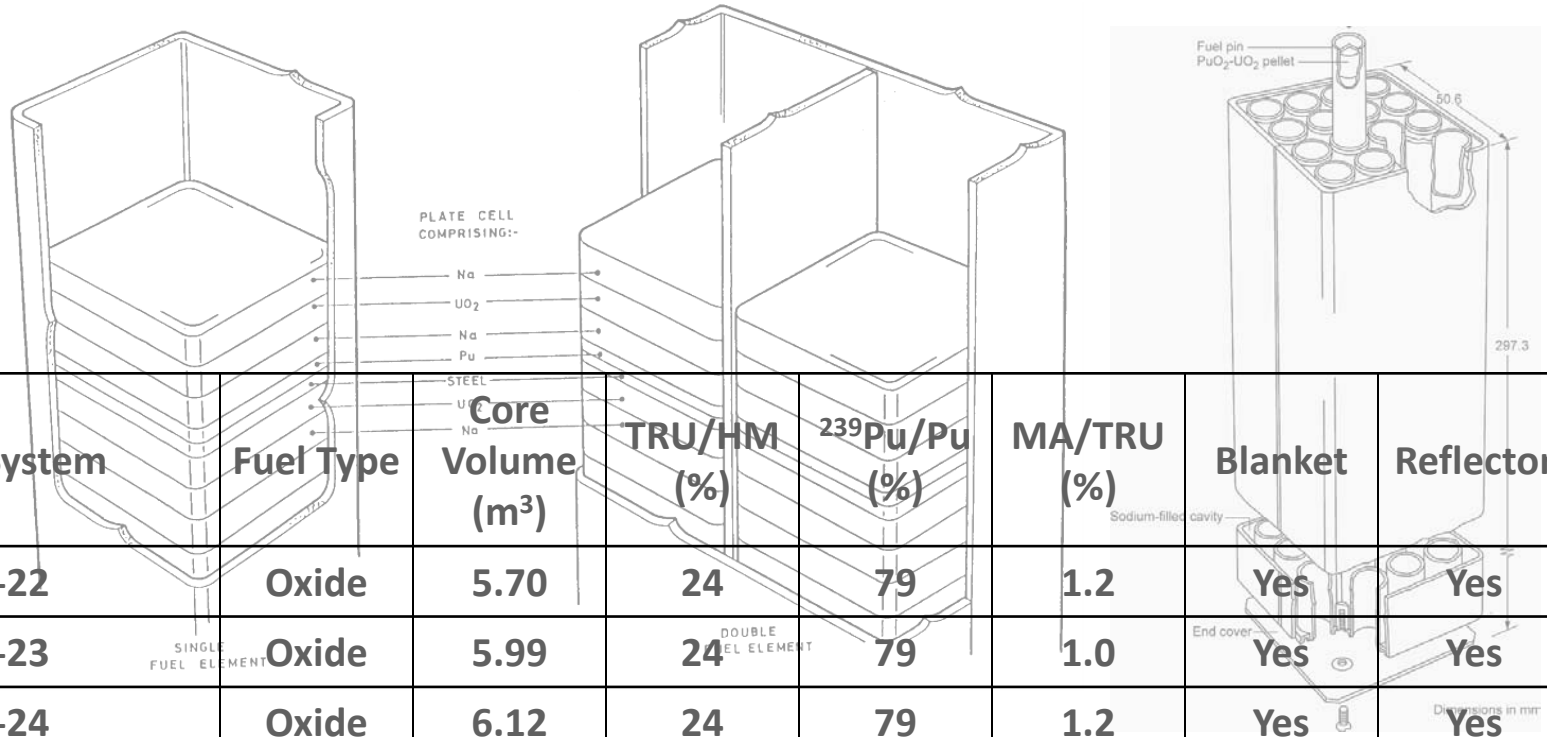
Summary of Sensitivity Studies Completed (2/5)

- Seventeen fast critical experiments
  - Multiplication factor, two spectral indices at core center (U-238 fission to Pu-239 fission and U-238 capture to Pu-239 fission), coolant void worth



System	Fuel Type	Core Volume (m^3)	TRU/HM (%)	^{239}Pu/Pu (%)	MA/TRU (%)	Blanket	Reflector
ZPPR-2	Oxide	2.41	15/22	87/87	0.30/0.29	Yes	Yes
ZPPR-9	Oxide	4.60	11/16	87/87	0.60/0.59	Yes	Yes
ZPPR-10A	Oxide	4.92	12/16	87/87	0.70/0.70	Yes	Yes
ZPPR-15A	Metal	2.50	12/18	87/87	0.99/0.98	Yes	Yes
ZPR6-6A	Oxide	4.00	-	-	-	Yes	No
ZPR6-7	Oxide	3.12	15/15	87/87	0.29/0.23	Yes	No
ZPR3-53	Carbide	0.23	40	93	0	Yes	No
ZPR3-54	Carbide	0.20	40	93	0	No	Yes

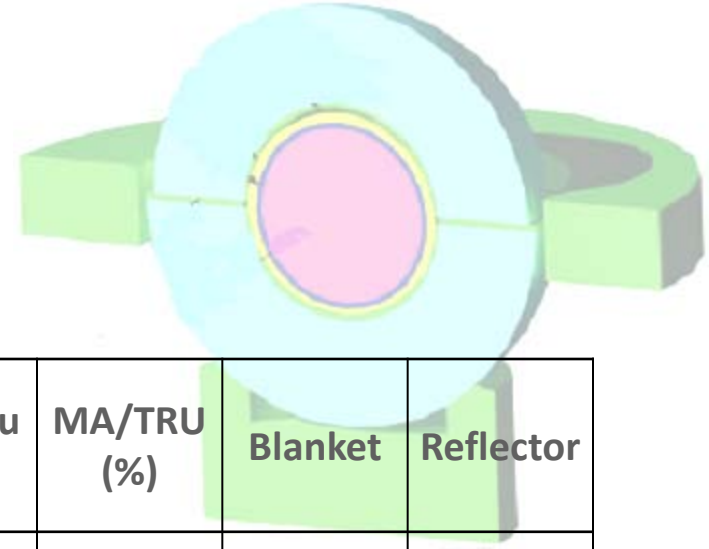
Summary of Sensitivity Studies Completed (3/5)



System	Fuel Type	Core Volume (m^3)	TRU/HM (%)	$^{239}Pu/Pu$ (%)	MA/TRU (%)	Blanket	Reflector
ZEBRA-22	Oxide	5.70	24	79	1.2	Yes	Yes
ZEBRA-23	Oxide	5.99	24	79	1.0	Yes	Yes
ZEBRA-24	Oxide	6.12	24	79	1.2	Yes	Yes
ZEBRA-25	Oxide	6.44	24	79	1.0	Yes	Yes
CIRANO/ZONA2A	Oxide	4.77	27/25	79/92	2.7/0.3	Yes	No
CIRANO/ZONA2A3	Oxide	4.25	27	79	2.7	Yes	Yes
CIRANO/ZONA2B	Oxide	3.88	27	79	2.7	No	Yes
COSMO	Oxide	1.98	27	79	2.7	No	Yes
MUSE-4	Oxide	4.77	27	79	3.1	No	Yes

Summary of Sensitivity Studies Completed (4/5)

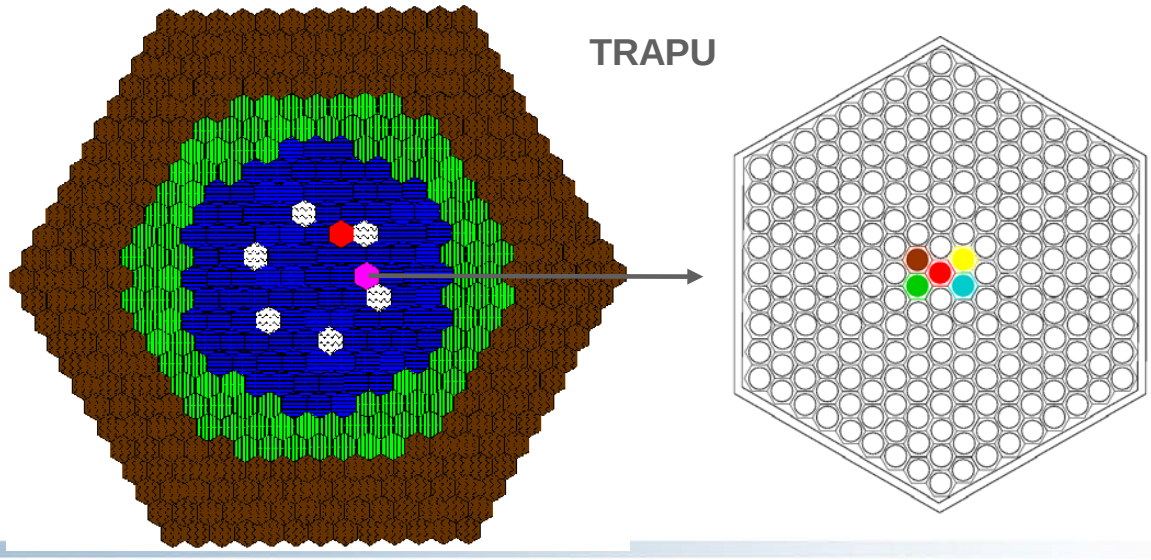
- Nine LANL critical assemblies
  - Multiplication factor, two spectral indices at core center (U-238 fission to Pu-239 fission and U-238 capture to Pu-239 fission)

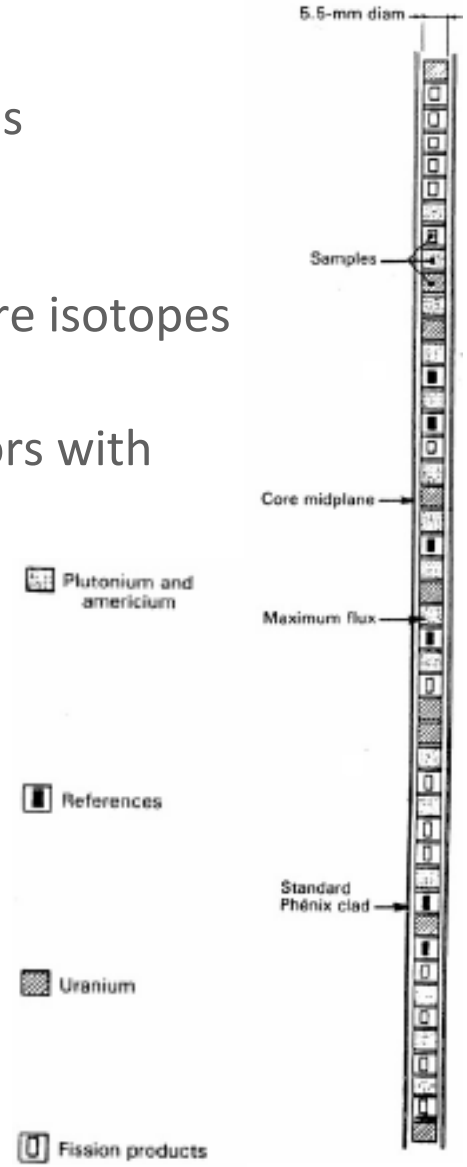


System	Fuel Type	Core Volume (cm^3)	TRU/HM (%)	$^{239}Pu/Pu$ (%)	MA/TRU (%)	Blanket	Reflector
FLATTOP Pu	Metal	390	100	95	-	No	Yes
FLATTOP U-235	Metal	958	-	-	-	No	Yes
FLATTOP U-233	Metal	538	-	-	-	No	Yes
JEZEBEL Pu-239	Metal	1090	100	95	-	No	No
JEZEBEL Pu-240	Metal	1237	100	76	-	No	No
JEZEBEL U-233	Metal	897	-	-	-	No	No
GODIVA	Metal	2797	-	-	-	No	No
Np Sphere	Metal	299	-	88	83	No	No
BIGTEN	Metal	64394	-	-	-	No	Yes

Summary of Sensitivity Studies Completed (5/5)

- Three irradiation experiments
  - PROFIL-1: a pin containing 46 samples of pure isotopes
    - Pu-240 in Pu-239 samples
    - Am-242m, Pu-238, and Cm242 in the Am241 samples
  - PROFIL-2: two pins, each containing 42 samples of pure isotopes
    - Pu-238 in Np-237 samples, Pu-239 in Pu-238 samples
  - TRAPU: fuel pins of different plutonium isotopic vectors with high Pu-240, Pu-241 and Pu-242 fractions
    - Np-237 and Am-243 in TRAPU pin





Sensitivity Coefficient Calculation Methods

- Composition-dependent cross sections were generated in a 33-group energy structure using the MC^2-2 code and the ENDF/B-VII.0 nuclear data
- Sensitivity coefficients were calculated using the generalized perturbation theory methods
  - The generalized perturbation theory code VARI3D was generally used. Flux, adjoint, and generalized adjoint flux were calculated with the finite difference diffusion theory option of the DIF3D code.
  - For small critical assemblies of LANL where transport effects are significant, the sensitivity coefficients were calculated using the ERANOS code system with the discrete ordinate transport solver BISTRO
    - Sensitivity coefficients of the potential target systems (except for two ABR cores) were calculated with this method for the WPEC Subgroup 26 activity
  - For time-dependent response parameters (i.e., nuclide densities and burnup reactivity swing), the sensitivity coefficients were calculated using the depletion perturbation code DPT. Depletion calculations were performed using the REBUS-3 fuel cycle analysis code.

Verification Tests of DPT

- Verification tests were performed by comparing the DPT sensitivity coefficients with those obtained from direct perturbation calculations
- Sensitivity coefficients obtained with DPT agree well with the values determined from direct perturbation calculations (even with different methods and data)

Sensitivity Coefficients of ^{240}Pu Final Density in ^{239}Pu Sample of PROFIL-1 Irradiation Experiment

Cross section	Direct perturbation calculation		DPT
^{239}Pu σ_c	REBUS-3	0.829	0.813
^{238}U ν	ERANOS	-0.669	-0.660

Sensitivity Coefficients of GFR

Integral parameter	Cross section	Direct perturb	DPT
BOC k_{eff}	^{238}U σ_c	-0.191	-0.191
	^{239}Pu σ_f	0.393	0.392
	^{239}Pu σ_c	-0.039	-0.039
	^{241}Pu σ_f	0.100	0.101
	^{241}Am σ_c	-0.052	-0.051
burnup reactivity swing ($\Delta\rho$)	^{238}U σ_c	6.345	6.342
	^{239}Pu σ_f	-4.651	-4.586
	^{239}Pu σ_c	-1.224	-1.280
	^{241}Pu σ_f	-2.981	-2.972
	^{241}Am σ_c	1.777	1.724

Transport Effects on Sensitivity Coefficients

- It was generally considered that the transport effects on sensitivity coefficients would be insignificant even if the effects on integral parameter values are non-negligible
- It was however found that the transport effects are non-negligible for the small critical assemblies of LANL

Sensitivity Coefficients of FLATTOP Multiplication Factor

Isotope	Method	σ_f	σ_c	σ_e	σ_{in}	ν	$\sigma_{n,xn}$
^{235}U	diffusion	0.0147	-0.0008	0.0014	0.0007	0.0188	-
	transport	0.0075	-0.0006	0.0008	0.0004	0.0105	-
^{238}U	diffusion	0.1139	-0.0565	0.2316	0.1313	0.1400	0.0021
	transport	0.0575	-0.0408	0.1124	0.0670	0.0791	0.0011
^{239}Pu	diffusion	0.6306	-0.0129	0.0270	0.0278	0.8159	0.0003
	transport	0.6287	-0.0130	0.0227	0.0133	0.8809	0.0003
^{240}Pu	diffusion	0.0180	-0.0009	0.0015	0.0014	0.0227	-
	transport	0.0191	-0.0010	0.0013	0.0006	0.0267	-
^{241}Pu	diffusion	0.0020	-0.0001	0.0001	0.0001	0.0026	-
	transport	0.0019	-0.0001	0.0001	-	0.0027	-

Uncertainties (%) Estimated with AFCI-1.3 Data (1/2)

System	Multiplication Factor	^{238}U Fission / ^{239}Pu Fission	^{238}U Capture / ^{239}Pu Fission	Coolant Void Worth (pcm)
ZPPR-2	0.86	5.72	1.69	197. (-146)
ZPPR-9	1.19	7.10	1.72	12.1 (171)
ZPPR-10A	1.11	6.69	1.69	11.5 (149)
ZPPR-15A	0.96	5.89	1.77	8.94 (135)
ZPR6-6A	1.57	-	-	11.9 (-221)
ZPR6-7	0.97	5.65	1.67	98.3 (197)
ZPR3-53	0.74	2.56	1.73	-
ZPR3-54	1.61	2.78	1.79	-
ZEBRA-22	0.91	4.73	1.74	16.6 (-110)
ZEBRA-23	0.86	4.67	1.73	14.9 (-136)
ZEBRA-24	1.00	5.06	1.82	-
ZEBRA-25	0.95	5.02	1.82	-
CIRANO/ZONA2A	0.70	4.92	1.87	6.70 (-157)
CIRANO/ZONA2A3	0.81	4.91	1.90	8.08 (-166)
CIRANO/ZONA2B	0.94	4.88	1.93	8.84 (-164)
COSMO	0.93	4.93	1.93	9.12 (183)
MUSE-4	0.96	4.98	1.85	10.4 (151)

Uncertainties (%) Estimated with AFCI-1.3 Data (2/2)

System	Multiplication Factor	^{238}U Fission / ^{239}Pu Fission	^{238}U Capture / ^{239}Pu Fission
FLATTOP Pu	0.55	2.07	2.32
FLATTOP U-235	1.07	0.73	1.48
FLATTOP U-233	0.62	2.61	2.98
JEZEBEL Pu-239	0.62	2.61	2.98
JEZEBEL Pu-240	0.48	2.43	2.73
JEZEBEL U-233	0.87	2.04	2.57
GODIVA	0.89	1.87	2.15
Np Sphere	1.85	0.82	1.53
BIGTEN	2.61	12.4	2.69

Main Contributors to Coolant Void Worth Uncertainties

	R-2	R-9	10A	15A	6/6A	6/7	Ze22	Ze23	Z2A	Z2A3	Z2B	Cos	Mu4	ABRm	ABRo
$^{235}U\ \sigma_c$					5.89										
$^{235}U\ \sigma_f$					3.91										
$^{238}U\ \sigma_c$	66.3	4.23	4.11	3.69	3.07	34.5	3.33	3.19							1.56
$^{238}U\ \sigma_e$	60.1			2.51	3.18	33.4	8.00	7.02	2.99	2.20					
$^{238}U\ \sigma_{in}$	111.	7.91	7.19	2.83	5.83	62.1	11.5	10.3	2.79	2.75	2.28	2.35	2.78	3.75	2.16
$^{238}Pu\ \sigma_f$														3.37	1.33
$^{239}Pu\ \sigma_f$	58.9	3.01	3.02	2.13		27.0									
$^{239}Pu\ \nu$	54.0			2.28					1.49						
$^{56}Fe\ \sigma_e$				2.89					2.01	4.74	5.98	6.22	7.14	8.02	2.00
$^{52}Cr\ \sigma_e$										2.38	3.18	3.19	3.64	2.90	
$^{23}Na\ \sigma_e$				2.55	4.55		4.53	4.46	3.41	3.64	3.72	3.74	3.91		1.38
$^{23}Na\ \sigma_{in}$	78.4	5.54	5.35	3.92		35.9			1.48					6.54	4.10
Total	197.	12.1	11.5	8.94	11.9	98.3	16.6	15.0	6.70	8.08	8.84	9.12	10.4	13.1	6.17

Correlation Matrix of Multiplication Factor (1/3)

	R-2	R-9	10A	15A	6/6A	6/7	3/53	3/54	Ze22	Ze23	Ze24	Ze25	Z2A	Z2A3
R-2	1.00	0.96	0.97	0.96	0.35	0.99	0.82	0.38	0.96	0.98	0.94	0.95	0.96	0.87
R-9		1.00	1.00	0.96	0.35	0.99	0.74	0.17	0.96	0.96	0.96	0.96	0.88	0.74
10A			1.00	0.97	0.35	0.99	0.75	0.20	0.97	0.97	0.96	0.96	0.89	0.76
15A				1.00	0.34	0.97	0.71	0.24	0.98	0.97	0.97	0.97	0.91	0.81
6/6A					1.00	0.35	0.23	0.13	0.38	0.38	0.37	0.37	0.34	0.32
6/7						1.00	0.80	0.28	0.97	0.98	0.95	0.96	0.92	0.81
3/53							1.00	0.43	0.73	0.77	0.69	0.72	0.79	0.68
3/54								1.00	0.30	0.33	0.26	0.29	0.47	0.68
Ze22									1.00	1.00	0.99	1.00	0.94	0.85
Ze23										1.00	0.98	0.99	0.96	0.86
Ze24											1.00	1.00	0.91	0.81
Ze25												1.00	0.93	0.83
Z2A													1.00	0.94
Z2A3														1.00

Correlation Matrix of Multiplication Factor (2/3)

	Z2B	Cos	Mu4	F/Pu	F/235	F/233	J/239	J/240	J/233	God	NpS	Big	ABRm	ABRo
R-2	0.75	0.77	0.76	-0.26	-0.07	-0.01	0.10	0.13	-0.01	-0.02	0.00	0.68	0.80	0.77
R-9	0.61	0.63	0.61	-0.43	-0.10	-0.01	0.05	0.08	-0.01	-0.03	0.00	0.81	0.83	0.78
10A	0.63	0.64	0.63	-0.41	-0.09	-0.01	0.06	0.08	-0.01	-0.03	0.00	0.79	0.83	0.79
15A	0.69	0.71	0.70	-0.33	-0.08	-0.01	0.07	0.11	-0.01	-0.03	0.00	0.73	0.85	0.79
6/6A	0.28	0.28	0.28	-0.13	0.81	0.00	0.00	0.00	0.00	0.79	0.11	0.58	0.29	0.26
6/7	0.68	0.70	0.68	-0.34	-0.08	-0.01	0.08	0.11	-0.01	-0.03	0.00	0.73	0.81	0.78
3/53	0.57	0.58	0.57	0.00	-0.03	0.00	0.21	0.24	0.00	-0.01	0.00	0.41	0.51	0.54
3/54	0.78	0.77	0.78	0.18	0.01	0.00	0.15	0.15	0.00	0.01	0.00	-0.05	0.20	0.18
Ze22	0.73	0.75	0.73	-0.33	-0.06	-0.01	0.10	0.13	-0.01	0.00	0.00	0.77	0.85	0.78
Ze23	0.75	0.76	0.75	-0.31	-0.05	-0.01	0.10	0.14	-0.01	0.00	0.00	0.75	0.84	0.78
Ze24	0.70	0.72	0.70	-0.37	-0.07	-0.01	0.08	0.12	-0.01	-0.01	0.00	0.81	0.86	0.78
Ze25	0.72	0.73	0.72	-0.35	-0.06	-0.01	0.09	0.12	-0.01	0.00	0.00	0.79	0.85	0.78
Z2A	0.85	0.86	0.85	-0.14	-0.05	0.00	0.14	0.19	0.00	-0.01	0.00	0.60	0.76	0.71
Z2A3	0.98	0.98	0.98	-0.10	-0.04	0.00	0.13	0.18	0.00	-0.01	0.00	0.48	0.70	0.63

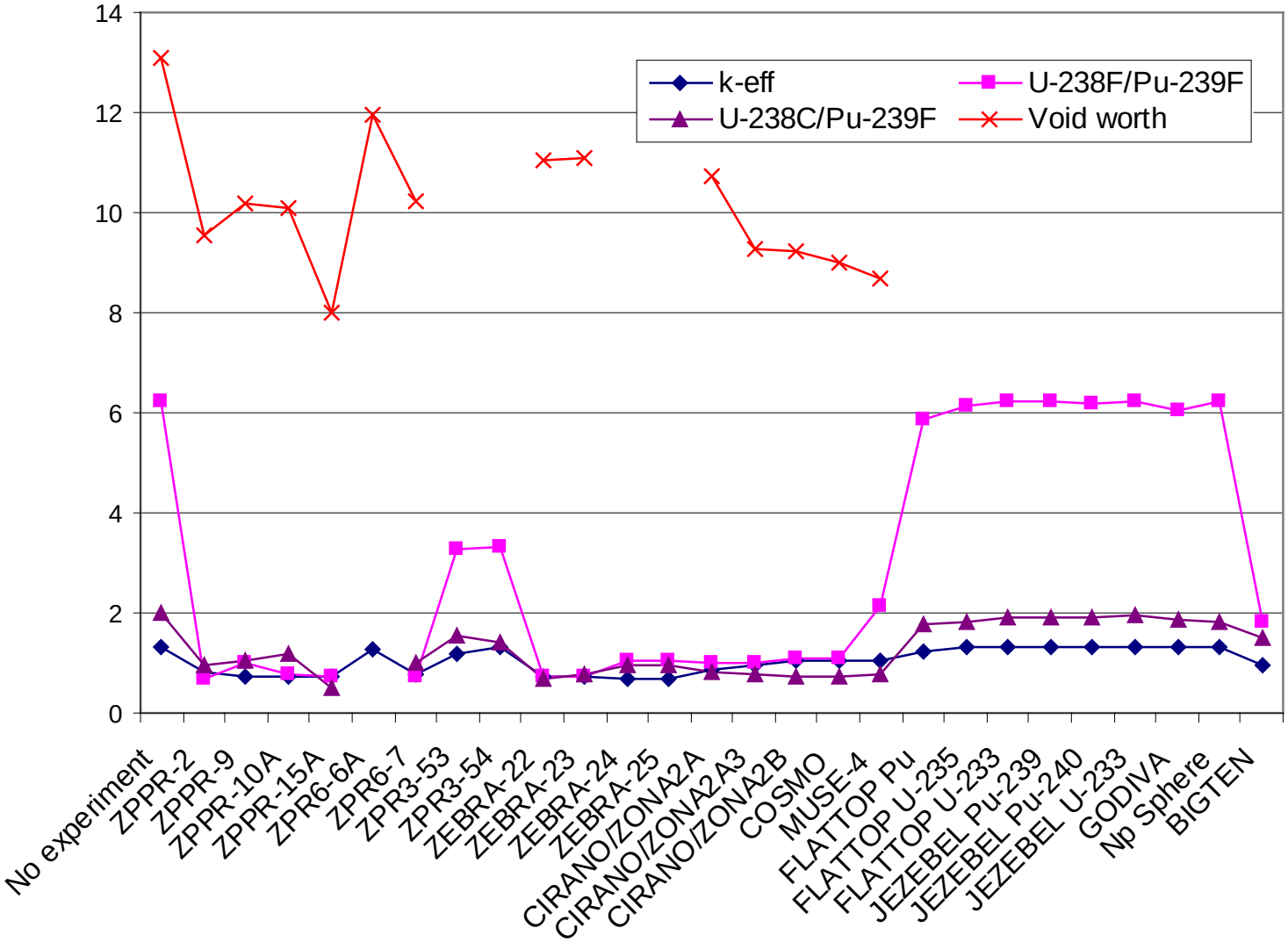
Correlation Matrix of Multiplication Factor (3/3)

	Z2B	Cos	Mu4	F/Pu	F/235	F/233	J/239	J/240	J/233	God	NpS	Big	ABRm	ABRo
Z2B	1.00	1.00	1.00	-0.06	-0.04	0.00	0.12	0.16	0.00	-0.01	0.00	0.37	0.61	0.54
Cos		1.00	1.00	-0.07	-0.04	0.00	0.12	0.16	0.00	-0.01	0.00	0.39	0.64	0.57
Mu4			1.00	-0.07	-0.04	0.00	0.12	0.15	0.00	-0.01	0.00	0.38	0.63	0.56
F/Pu				1.00	0.13	0.01	0.54	0.55	0.01	0.06	0.01	-0.58	-0.41	-0.35
F/235					1.00	0.01	0.00	0.00	0.01	0.98	0.13	0.21	-0.10	-0.08
F/233						1.00	0.00	0.00	0.98	0.01	0.00	-0.01	-0.01	-0.01
J/239							1.00	0.95	0.00	0.00	0.00	0.00	0.02	0.03
J/240								1.00	0.00	0.00	0.00	0.00	0.05	0.07
J/233									1.00	0.02	0.00	-0.01	-0.01	-0.01
God										1.00	0.13	0.25	-0.04	-0.03
NpS											1.00	0.04	0.01	0.01
Big												1.00	0.71	0.62
ABRm													1.00	0.97
ABRo														1.00

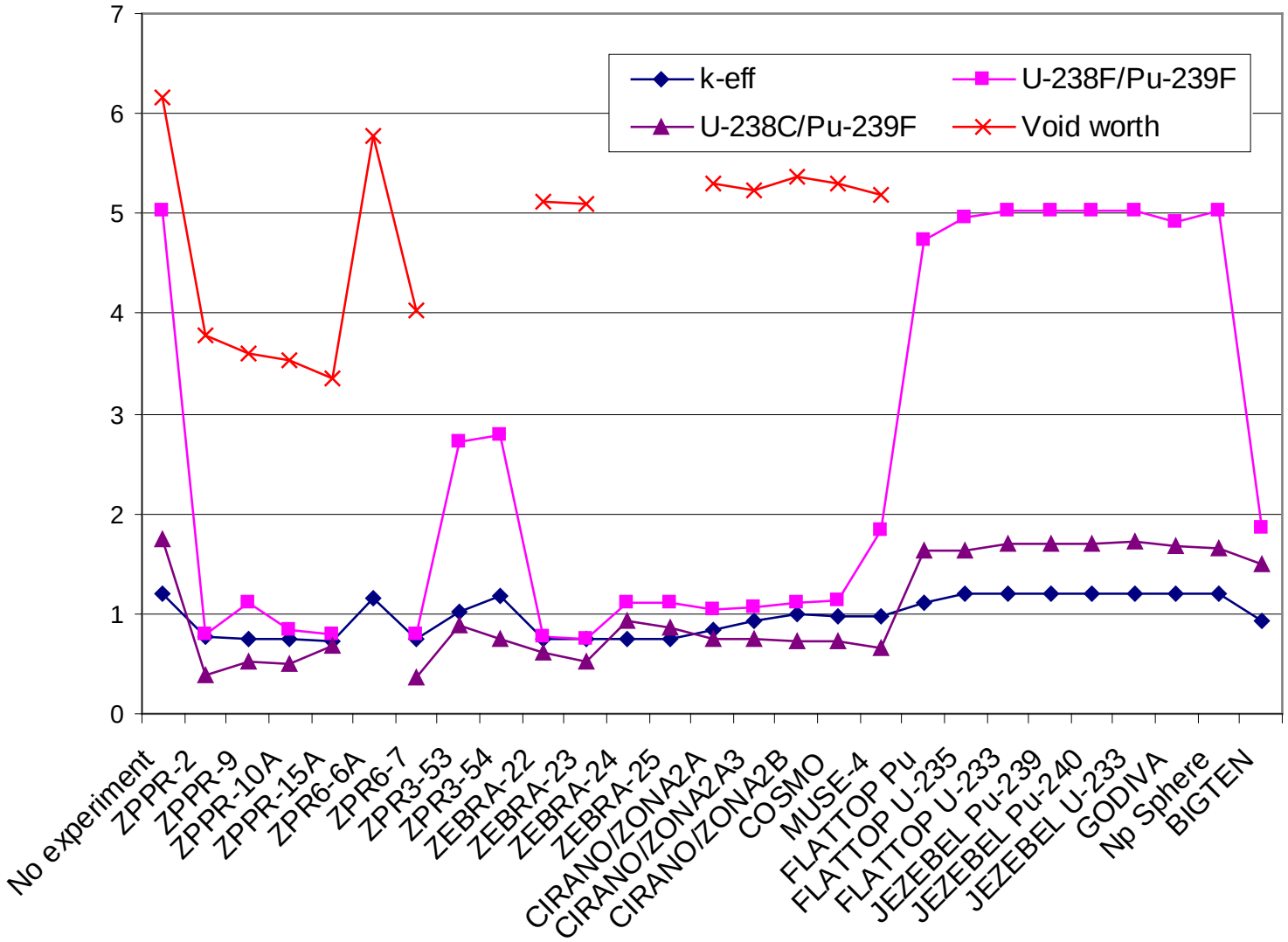
Correlation Matrix of Sodium Void Worth

	R-2	R-9	10A	15A	6/6A	6/7	Ze22	Ze23	Z2A	Z2A3	Z2B	Cos	Mu4	ABRm	ABRo
R-2	1.00	0.95	0.96	0.80	-0.62	0.99	-0.88	-0.89	-0.84	-0.75	-0.65	-0.66	-0.68	0.68	0.79
R-9		1.00	1.00	0.81	-0.52	0.97	-0.83	-0.84	-0.68	-0.54	-0.43	-0.44	-0.46	0.63	0.81
10A			1.00	0.82	-0.52	0.97	-0.83	-0.84	-0.69	-0.55	-0.45	-0.45	-0.48	0.64	0.82
15A				1.00	-0.35	0.77	-0.64	-0.63	-0.57	-0.56	-0.52	-0.54	-0.56	0.79	0.84
6/6A					1.00	-0.62	0.69	0.70	0.69	0.64	0.58	0.57	0.58	-0.40	-0.35
6/7						1.00	-0.90	-0.92	-0.81	-0.68	-0.57	-0.57	-0.59	0.62	0.76
Ze22							1.00	1.00	0.90	0.73	0.61	0.61	0.61	-0.53	-0.56
Ze23								1.00	0.91	0.74	0.62	0.62	0.62	-0.53	-0.57
Z2A									1.00	0.91	0.82	0.82	0.82	-0.57	-0.52
Z2A3										1.00	0.98	0.98	0.98	-0.70	-0.53
Z2B											1.00	1.00	1.00	-0.71	-0.50
Cos												1.00	1.00	-0.73	-0.51
Mu4													1.00	-0.75	-0.54
ABRm														1.00	0.89
ABRo															1.00

Reduced Uncertainties of ABR Metal Core Integral Parameters After Applying Single Experiment



Reduced Uncertainties of ABR Oxide Core Integral Parameters After Applying Single Experiment



Summary and Future Work

- Sensitivity and uncertainty analysis was performed for the experiments selected in FY 2008 for the uncertainty reduction project
  - Static sensitivity analysis for total 89 integral parameters of 26 experiments
  - Time-dependent sensitivity analysis for 14 measurements in the PROFIL-1, PROFIL-2 and TRAPU irradiation experiments
- Sensitivity and uncertainty analysis will be continued for an extended set of experiments (ZPR6 Assembly 7 with high Pu-240 content, SNEAK 6 and 7) and integral measurements (reactivity coefficients, reaction rate traverses, etc.)
- The formal data adjustment procedure based on the Bayesian method will be applied to quantify the uncertainties associated with predicted values of reactor performance parameters with full application of integral experiment values
  - The cross section adjustment code GMADJ of ANL will be modified to be applicable on modern computer environments and the methods of GMADJ will be improved as needed
  - This activity will also support the participation in the cross section adjustment exercise of the OECD/NEA WPEC Subgroup 33
  - Evaluation of the correlation matrix of integral measurements