Covariances of prompt fission neutron spectra

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Objectives

Investigate:

1. Validation of fission spectrum covariance preparation and processing (analytical and MC)
2. Effects of different spectra on criticality (k_{eff}) (ENDF/B-VII, Watt, Kornilov model)
3. Consistency of the uncertainties in k_{eff} calculated from fission spectrum covariances

Data Formats for Cross Section Covariances in Evaluated Data Files

- ***MF*=31:** covariance of average number of neutrons per fission (ν - MT=452, 455, 456)
- ***MF*=32:** Shape and area of individual resonances
- ***MF*=33:** covariance of neutron cross section
- ***MF*=34:** covariance of angular distribution of secondary neutron (currently MT=2/P_1 only)
- ***MF*=35:** covariance of energy distribution of secondary particles (currently MT=18 only)

No processing available:

- ***MF*=30**: Covariances obtained from parameter covariances and sensitivities
- ***MF*=40**: Covariances for production of radioactive nuclei

Processing available (NJOY-ERROR and ERRORJ)

Prompt fission neutron spectrum models

Maxwell distribution

$$\chi(E) = \frac{2\cdot\sqrt{E}}{\sqrt{\pi\cdot T^3}} e^{-E/T}$$

U-235

Watt distribution

$$\chi(E) = \frac{2}{\sqrt{\pi a^3 b}} \sinh \sqrt{bE} \exp -\left(\frac{ab}{4} + \frac{E}{a}\right)$$

a=0.988 MeV $\pm$1.2%
b=2.249/MeV $\pm$5.9%

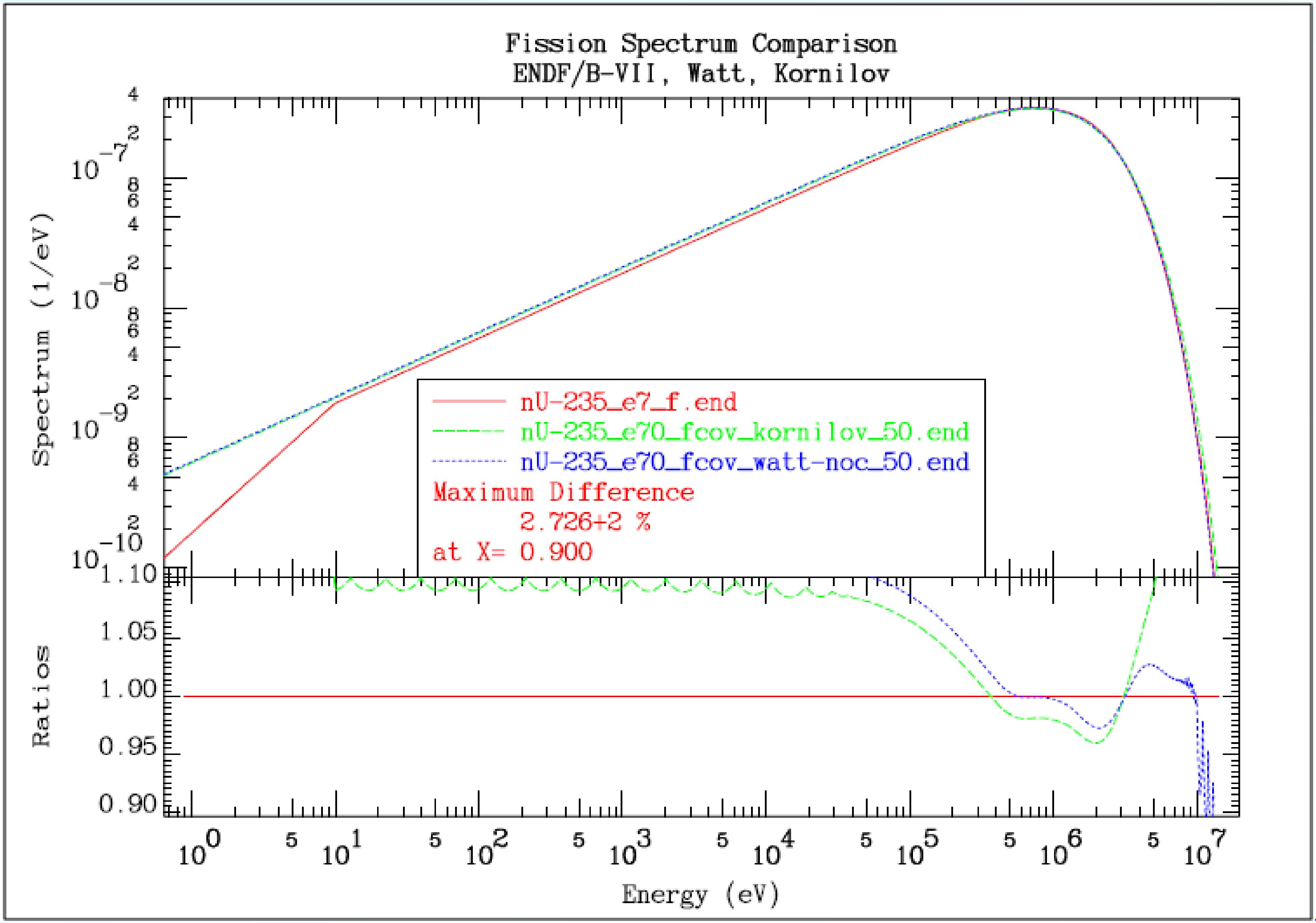
Kornilov spectrum

$$\chi(E,r,\alpha) = \frac{W_L(E,T_L,\alpha\varepsilon_L) + W_H(E,T_H,\alpha\varepsilon_H)}{2}$$

$$W(E,T,\varepsilon) = \frac{1}{\sqrt{\pi T\varepsilon}} \sinh\left[\frac{2\sqrt{\varepsilon E}}{T}\right] \exp\left[-\frac{E+\varepsilon}{T}\right]$$

$$r = \frac{T_L}{T_H} = 1.248 \pm 0.031$$

$$\alpha = 0.936 \pm 0.027$$



Fission Spectrum Comparison
ENDF/B-VII, Watt, Kornilov
Spectrum (1/eV)
nU-235_e7_f.end
nU-235_e70_fcov_kornilov_50.end
nU-235_e70_fcov_watt-noc_50.end
Maximum Difference
2.726+2 %
at X= 0.900
Ratios
Energy (eV)

Prompt fission neutron spectrum properties

- Fission spectrum is normalised to 1
  sum of bin probabilities equals 1: $\sum_g \chi_g = 1$

- Covariances are given as absolute covariances of the bin probabilities (not average probability distributions)

- To assure that the perturbed spectrum remains normalised the covariance matrix should comply with the «zero sum» rule: sum of abolute matrix elements in each line and column must be zero: $\sum_g \overline{\delta\chi_g \delta\chi_{g'}} = 0$

Normalisation applied to the sensitivity coefficients

- If the matrix does not sattisfy the “zero sum” rule, the ENDF-6 manual suggests the correction formula:

$$\tilde{V}_{i,j} = V_{i,j} - \chi_i \sum_k V_{k,j} - \chi_j \sum_k V_{k,i} + \chi_i \chi_j \sum_{k,k'} V_{k,k'}$$

or in matrix notation: $\tilde{V} = S_\chi^{\ T} \cdot V \cdot S_\chi$ $\quad S^{\chi}_{g,g'} = \delta_{g,g'} - \chi_{g'}$

- Instead of “correcting” the matrices, we can apply the «correction» to the sensitivities :

$$(\Delta R)^2 = S_R^T \cdot \tilde{V} \cdot S_R = S_R^T \cdot \left(S_\chi^{\ T} \cdot V \cdot S_\chi\right) \cdot S_R$$

$$= \left(S_\chi \cdot S_R\right)^T \cdot V \cdot S_\chi \cdot S_R = S_{RN}^T \cdot V \cdot S_{RN}$$

Normalised sensitivities

Constructing fission spectra covariance matrices– **analytic method**

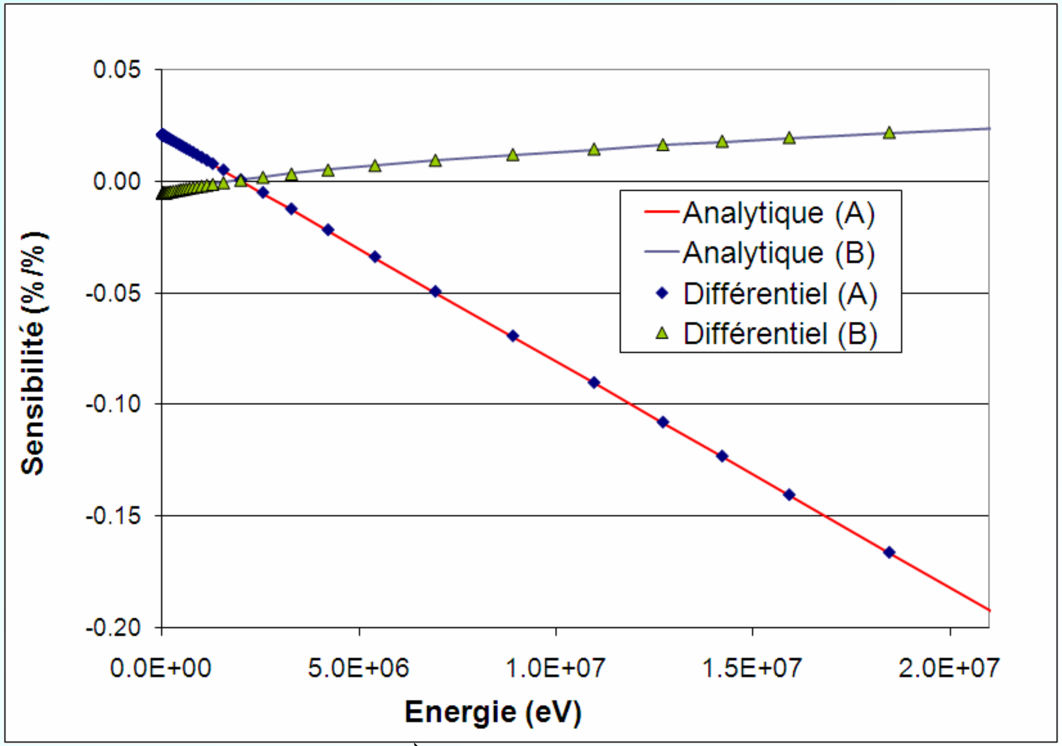
$$M_{\chi} = S_{ab}^{T} \cdot M_{ab} \cdot S_{ab}$$

- Covariance matrix of the parameters *a* and *b* of the watt spectrum:

δa/a=1.2%

δb/b=5.9%

$$R = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$



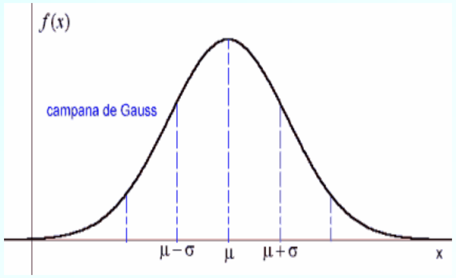
$$(\Delta R)^2 = S_R^T \cdot \tilde{V} \cdot S_R = S_R^T \cdot \left(S_{\chi}^{\ T} \cdot S_{ab}^T \cdot V_{ab} \cdot S_{ab} \cdot S_{\chi}\right) \cdot S_R$$

$$= \left(S_{ab} \cdot S_{\chi} \cdot S_R\right)^T \cdot V_{ab} \cdot \left(S_{ab} \cdot S_{\chi} \cdot S_R\right) = S_{RN}^T \cdot V_{ab} \cdot S_{RN}$$

File-30

Constructing fission spectra covariance matrices – **Monte Carlo method**

- Gaussian probability density distribution of the parameters *a* and *b* (non-correlated):



$$P(a,b) = P(a)\cdot P(b) = \frac{1}{2\pi\cdot\delta a\cdot\delta b}\cdot e^{-\frac{1}{2}\left[\left(\frac{a-\bar{a}}{\delta a}\right)^2+\left(\frac{b-\bar{b}}{\delta b}\right)^2\right]}$$

\- Mean:

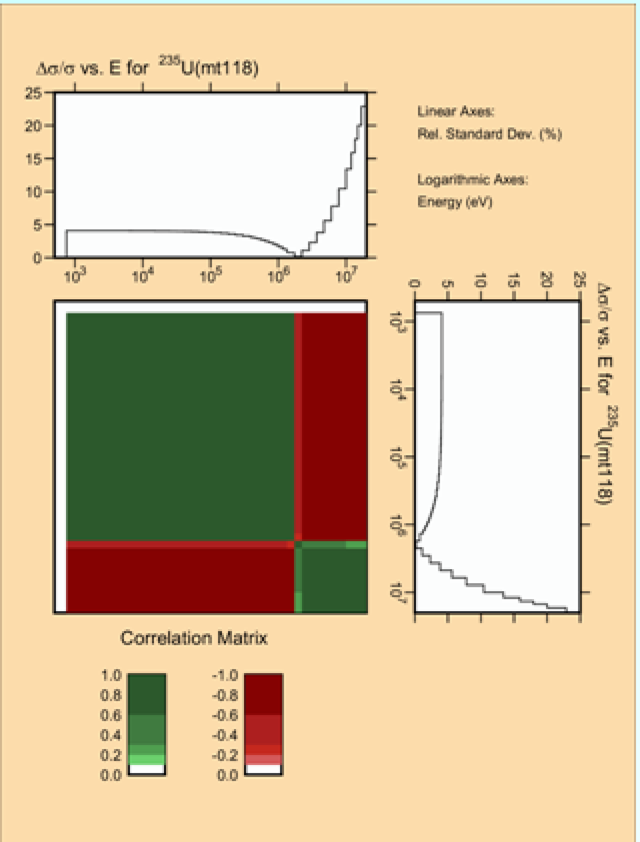
$$\overline{\chi}_g = \sum_i P(a_i,b_i)\chi_g(a_i,b_i) = \frac{1}{n}\sum_{i=1}^{n}\chi_{g,i}$$

\- Covariance:

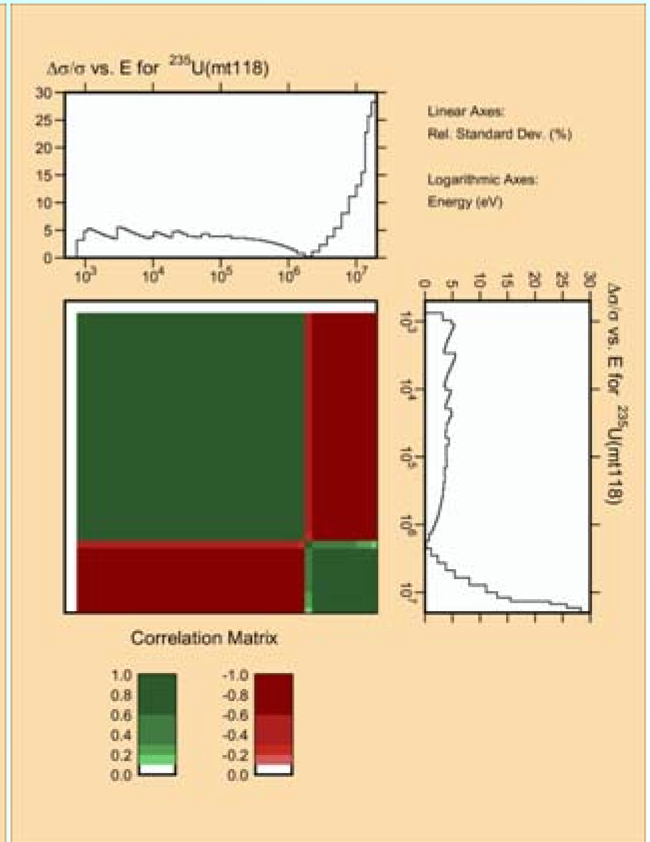
$$\overline{\delta\chi_g\cdot\delta\chi_{g'}} = \sum_{i=1}^{n} P(a_i,b_i)\left(\chi_g(a_i,b_i)-\overline{\chi_g}\right)\left(\chi_{g'}(a_i,b_i)-\overline{\chi_{g'}}\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(\chi_{g,i}-\overline{\chi_g}\right)\left(\chi_{g',i}-\overline{\chi_{g'}}\right)$$

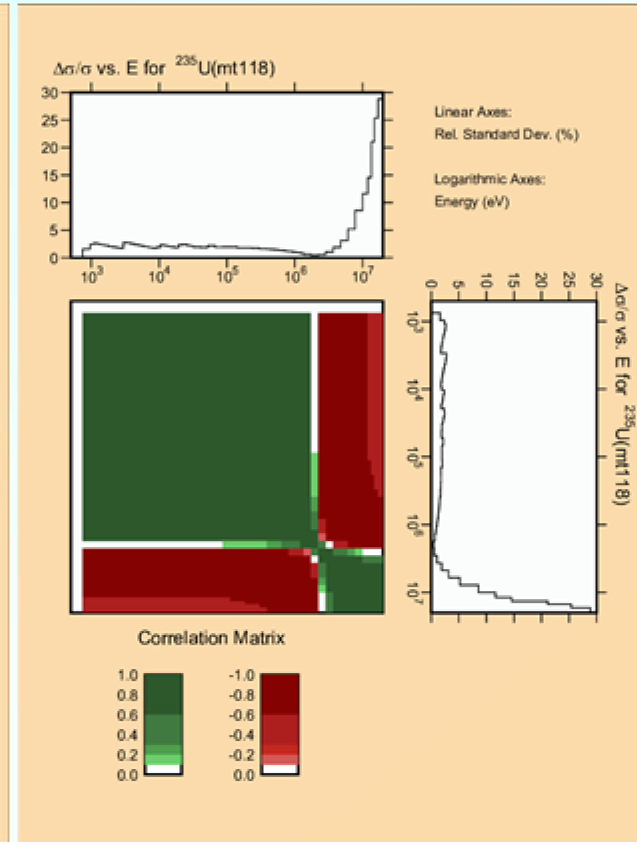
Covariance matrices of U-235 fission spectrum



Analytical-WATT



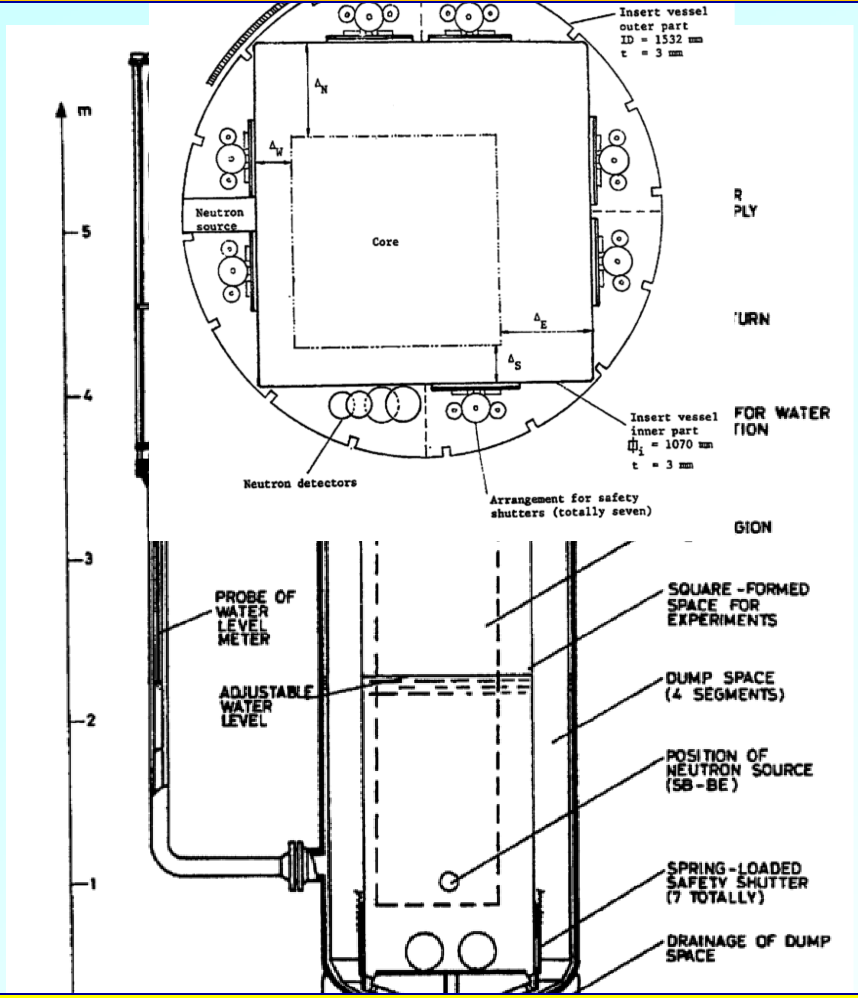
MC-WATT

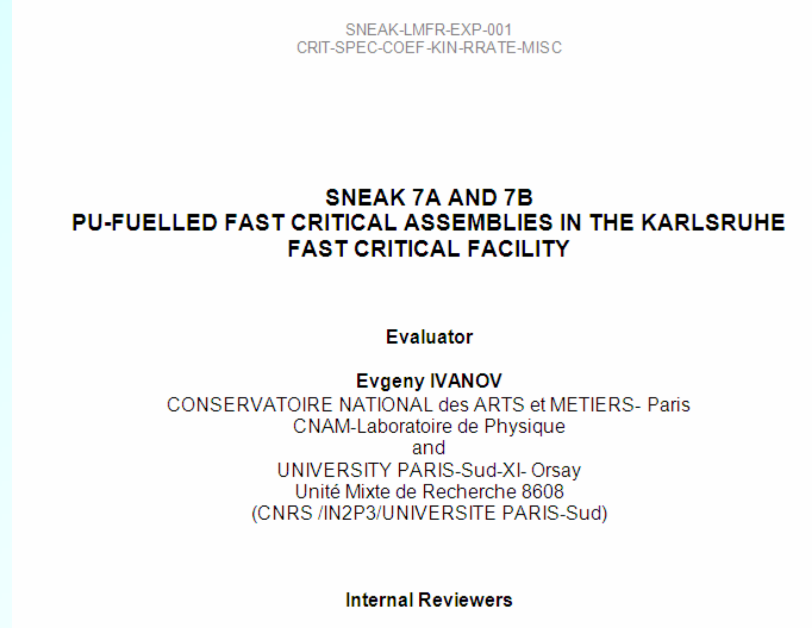


MC-Kornilov

IRPhE-International Reactor Physics Experiments Database (OCDE/AEN)

KRITZ-2 –thermal benchmarks **6 critical configurations with UO2 and MOX fuel.**	**SNEAK-7 – fast benchmarks** **2 configurations MOX**





SNEAK-LMFR-EXP-001
CRIT-SPEC-COEF-KIN-RRATE-MISC

SNEAK 7A AND 7B
PU-FUELLED FAST CRITICAL ASSEMBLIES IN THE KARLSRUHE FAST CRITICAL FACILITY

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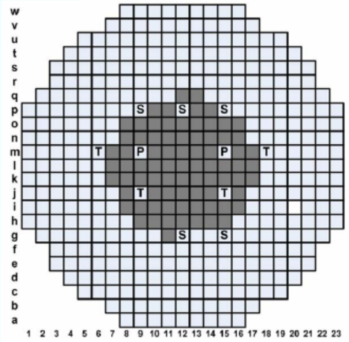
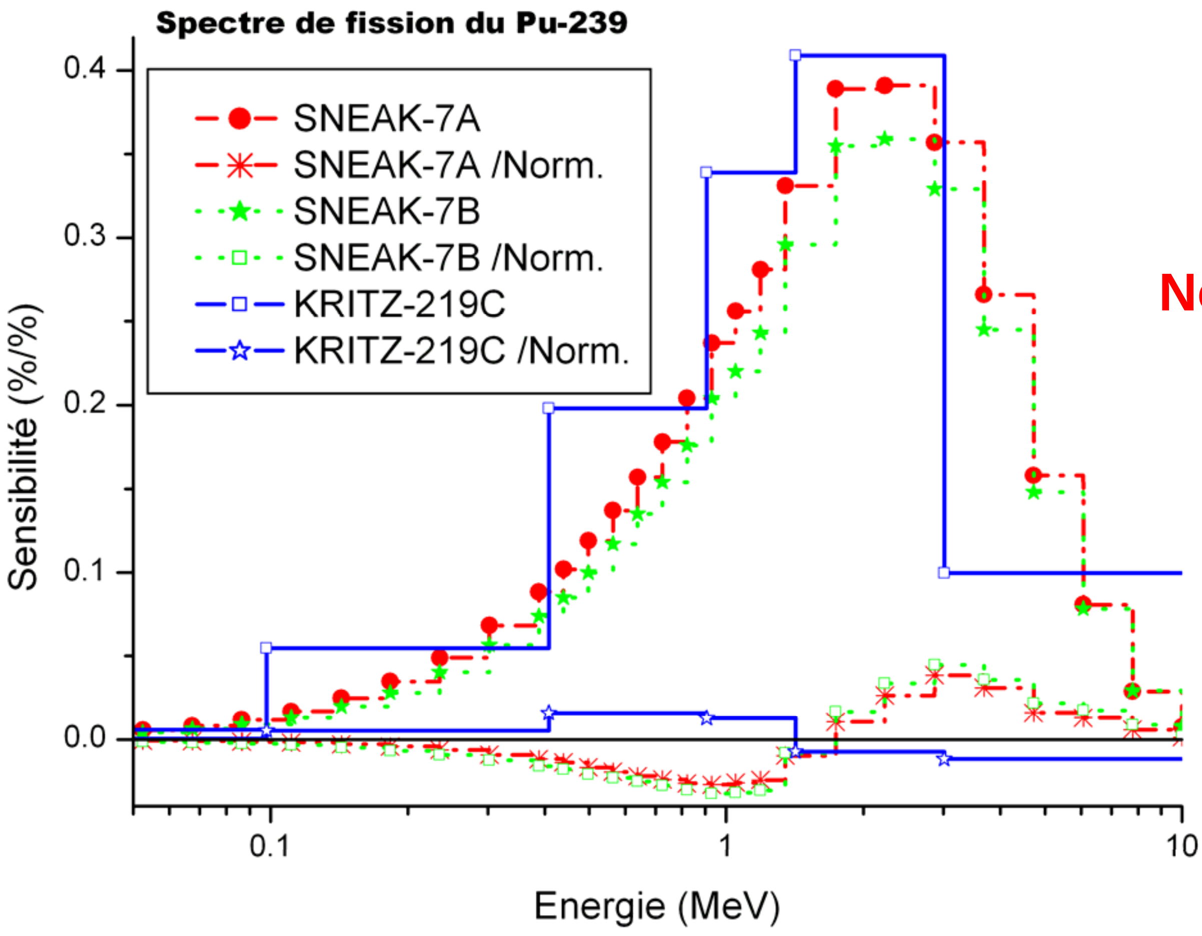


Analysis based on déterministic transport (TWODANT, THREEDANT) and cross section sensitivity and uncertainty code (SUSD3D)

Normalised and non-normalised sensitivities



Non-normalised

Normalised

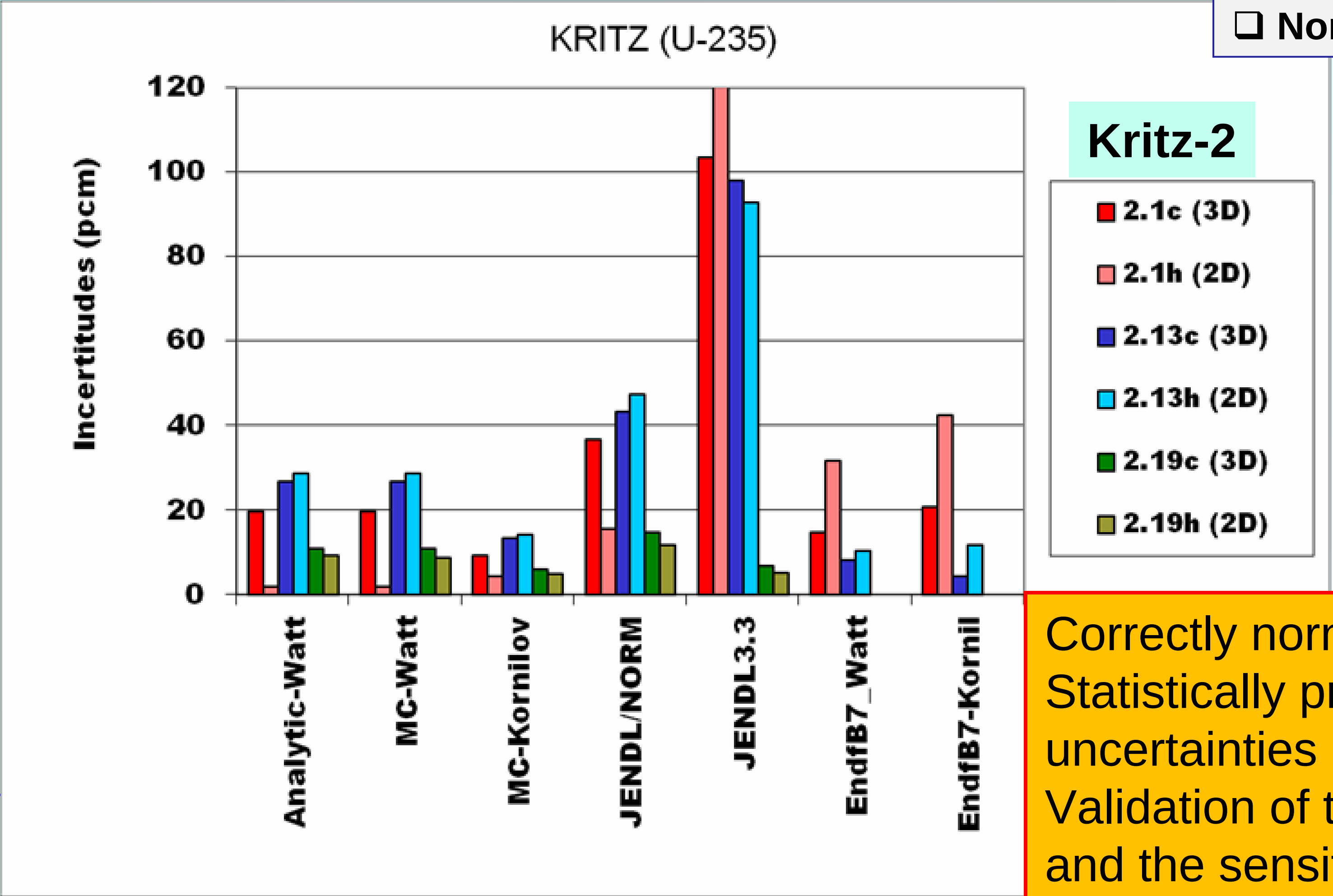
Differences in k-eff (pcm) due to different fission spectra (relative to ENDF/B-VII)

	Δk_{eff} [pcm]							
	KRITZ						**SNEAK**	
	2.1c	**2.1h**	**2.13c**	**2.13h**	**2.19c**	**2.19h**	**7A**	**7B**
Watt	14.9 [a]	31.7	8.2	10.5	-59.0	-77.8	861.7	614.0
Kornilov	20.8	42.3	4.5	11.8	-78.1	-64.1	59.0	171.6

Uncertainty in k-eff (pcm) due to **U-235** fission spectra uncertainty (KRITZ)

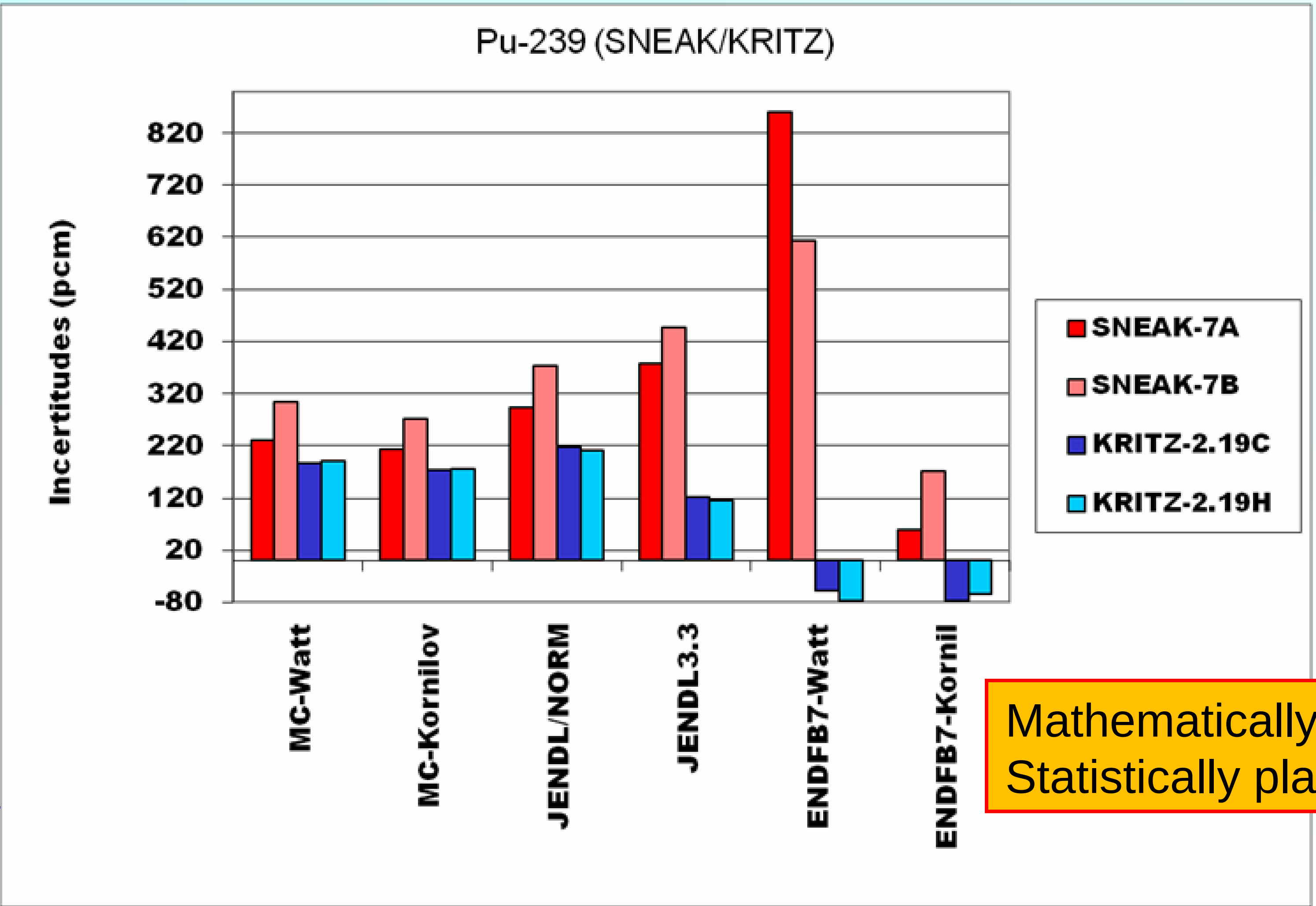
Covariance matrices
- *Analytic-Watt*
- *MC-Watt*
- *MC-Kornilov*
- *JENDL3.3*

Sensitivities
- Normalised
- Non-normalised



Correctly normalised matrices;
Statistically probable uncertainties ;
Validation of the MC method and the sensitivity method.

Uncertainty in k-eff (pcm) due to **Pu-239** fission spectra uncertainty (SNEAK-80g and KRITZ-18g)



Mathematically correct matrices
Statistically plausible

Conclusions

- **MC method** was used to produce covariances for **^{235}U, ^{238}U, ^{239}Pu fission spectra** based on Watt and Kornilov models (E<7MeV).
- Spectra were **validated** against analytical approach (restricted to linear approximation).
- New **"normalised" sensitivity method (**discussed during WPEC-SG26) was implemented in SUSD3D.
- "Normalised" sensitivity method and new covariance matrices were **tested** on sets of thermal and fast critical experiments.
- **Incorrectly normalised matrices** give much higher uncertainties.
- Uncertainty in *k-eff* due to the fission spectra uncertainties are ~10-30 pcm for thermal and **~200 – 300 pcm for fast systems.**
- Differences due to different spectra are consistent with the uncertainties.