Few Group Collapsing of Covariance Matrix Data Based on a Conservation Principle

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Few Group Covariance Data

- The *common* procedure to derive few group covariance data (e.g., use of flat flux weighting function) does not insure that:
  - the uncertainty on any integral parameter obtained using different group structures stays constant.
  - the uncertainty calculated by the finest group structure (reference case) may not be preserved.
- We present the results of a new collapsing algorithm based on a conservation principle.
- The new method allows the conservation of uncertainty on the integral parameter calculated at the fine (reference) energy grid level.

Proposed Collapsing Algorithm (ConsUnce)

- The uncertainty of R_k using the covariance matrix in the fine group grids (i = 1,…,I)

$$\Delta R_k^2 = S_{k,I}^{+} D_I S_{k,I} \quad \text{where} \quad D_I = \begin{pmatrix} d_{11} \cdots\cdots\cdots d_{I1} \\ \\ \\ d_{I1} \cdots\cdots\cdots d_{II} \end{pmatrix}, \quad S_{k,I} = \begin{pmatrix} s_{k,1} \\ \vdots \\ \vdots \\ s_{k,I} \end{pmatrix}$$

- One can find the covariance matrix in broad group grids D_J (j = 1,…,J) such that the fine group uncertainty is conserved:

$$d_{j,j'}^{k} = \frac{\sum_{i\in j} s_{k,i}^{+} \sum_{i'\in j'} d_{i,i'}^{k} s_{k,i'}}{s_{k,j} s_{k,j'}^{+}}, \qquad s_{k,j} = \sum_{i\in j} s_{k,i}$$

- Conservation of Uncertainty (ConsUnce).

Application to Fast Reactor k_{eff}

- The uncertainty on the k_{eff} of different fast reactor systems with different fuel and coolant types, critical or sub-critical (i.e. with potentially very different core neutron spectra) are calculated for the evaluation of the performance of the proposed collapsing algorithm.

- The sensitivity coefficients are calculated using the forward and adjoint neutron fluxes of Advanced Burner Reactor (ABR) fast reactor systems in 230-“fine” group structure, taken as reference.

- The following covariance matrices, based on JENDL 3.3 file, were generated by BNL for U235, U238, Pu239, Fe56, and Na23:
  - 230-”fine” group structure (Reference)
  - 33-group structure generated by both Flat Flux Weighting and ConsUnce.
  - 15-group structure generated by both Flat Flux Weighting and ConsUnce.

- The covariance matrices with ConsUnce are collapsed by weighting ABR-Metal Core sensitivity coefficients.

Uncertainties for k_{eff} [pcm]

Reactor	Matrix Type	230-group	ConsUnce 33-group	Flat Flux 33-group	ConsUnce 15-group	Flat Flux 15-group
ABR Metal	Total	744	744	739	744	709
	U238 σ_{inel}	317	317	315	317	309
	Pu239 σ_f	472	472	468	472	462
	Pu239 σ_{inel}	40.2	40.2	48.1	40.2	60.3
	Fe56 σ_{inel}	284	284	274	284	204
ADMAB	Total	796	748	738	746	604
	Pu239 σ_f	344	339	341	347	340
	Pu239 σ_{inel}	27.5	28.2	30.8	29.1	38.8
	Fe56 σ_{inel}	708	656	643	650	485

ABR - Advanced Burner Reactor for GNEP

ADMAB - Accelerator Driven Minor Actinide Burner

Uncertainties for k_{eff} [pcm] (Cont'd)

Reactor	Matrix Type	230-group	ConsUnce 33-group	Flat Flux 33-group	ConsUnce 15-group	Flat Flux 15-group
SFR	**Total**	**480**	**474**	**461**	**462**	**430**
	U238 σ_{inel}	19.7	19.7	20.0	19.7	20.0
	Pu239 σ_f	245	244	243	245	240
	Pu239 σ_{inel}	14.8	14.8	16.7	15.4	22.6
	Fe56 σ_{inel}	255	231	226	225	172
LFR	**Total**	**542**	**546**	**544**	**548**	**529**
	U238 σ_{inel}	192	195	194	195	188
	Pu239 σ_f	373	378	376	384	375
	Pu239 σ_{inel}	26.9	27.2	31.0	28.7	38.7
	Fe56 σ_{inel}	180	178	173	174	130

SFR - Sodium-cooled Fast Reactor

LFR - Lead-cooled Fast Reactor

Uncertainties for k_{eff} [pcm] (Cont'd)

Reactor	Matrix Type	230-group	ConsUnce 33-group	Flat Flux 33-group	ConsUnce 15-group	Flat Flux 15-group
EFR	**Total**	**752**	**750**	**749**	**747**	**729**
	U238 σ_{inel}	235	234	233	234	230
	Pu239 σ_f	599	597	596	595	582
	Pu239 σ_{inel}	43.4	42.9	52.4	43.5	69.4
	Fe56 σ_{inel}	162	160	154	157	114
GFR	**Total**	**885**	**872**	**882**	**887**	**876**
	U238 σ_{inel}	554	553	548	552	544
	Pu239 σ_f	597	578	594	602	590
	Pu239 σ_{inel}	63.2	61.3	74	56.6	91

EFR - European Fast Reactor
GFR - Gas-cooled Fast Reactor

Energy Breakdown of k_{eff} Uncertainties [pcm]

ABR (Advanced Burner Reactor) Metal Core k_{eff} Uncertainties in 15-group Structure						
Cross Section	U238 σ_{cap}		Pu239 σ_{Inel}		Fe56 σ_{inel}	
Group Upper Bound [eV]	ConsUnce	Flat Flux	ConsUnce	Flat Flux	ConsUnce	Flat Flux
1.96E+07	0	0	10.6	16.8	12.8	25.9
6.07E+06	40.6	30.4	34.5	56.4	101	27.6i
2.23E+06	49.2	71.1	3.58i	14.0i	197	168
1.35E+06	37.5	36.1	7.74	3.99	177	117
4.98E+05	53.0	59.4	11.6	13.4	0	0
1.83E+05	164	159	10.3	12	0	0
6.74E+04	178	175	6.11	6.76	0	0
2.48E+04	179	180	3.12i	3.58i	0	0
:	:	:	:	:	:	:
Total	315	315	40.2	60.3	284	204

E [eV]

(Note: $i = \sqrt{-1}$ appears because of negative correlation.)

Impact of the Type of Sensitivity Coefficients

- Broad-group sensitivity coefficients used so far are collapsed from those in fine-group structure, i.e.:

$$S_{k,j} = \sum_{i \in j} S_{k,i}$$

- For consistency, one should use the sensitivity coefficients directly calculated in the collapsed broad-group structure.
- "Collapsed" and "Direct" sensitivity coefficients produce substantially the same results.

k_{eff} uncertainty [pcm] for ABR-metal core

Energy Groups	33-group		15-group	
Sensitivity Coef.	Collapsed	Direct	Collapsed	Direct
Total	744	736	744	730
Pu239 σ_f	472	465	472	464
Pu239 σ_{cap}	118	119	118	122
Pu239 σ_{elas}	6.80	6.70	6.80	6.59
Pu239 σ_{inel}	40.2	41.4	40.2	39.1
Fe56 σ_{cap}	94.8	95.7	94.8	97.4
Fe56 σ_{elas}	63.0	73.2	63.3	68.1
Fe56 σ_{inel}	284	272	284	255

Application to Reactivity Coefficients

- In principle, for each integral parameter "p", one should calculate the corresponding "broad" group covariance matrix D_J^p by means of ConsUnce algorithm.
- However, the broad group covariance matrix based on k_{eff} sensitivity coefficients D_J^k can be also utilized for the calculation of the uncertainty on reactivity coefficients with a very modest approximation.
- The uncertainty on a reactivity coefficient RC is given by

$$\Delta R_{RC}^2 = S_{RC,J}^+ D_J^{RC} S_{RC,J}$$

$$\cong S_{k,J}^+ D_J^k S_{k,J} - S_{kP,J}^+ D_J^{kP} S_{kP,J} \cong S_{k,J}^+ D_J^k S_{k,J} - S_{kP,J}^+ D_J^k S_{kP,J}$$

Here we made an approximation $D_J^k \cong D_J^{kP}$.

- This approximation implies the similarity of sensitivity coefs, $S_{k,J}$ and $S_{kP,J}$.

Application to Reactivity Coefficients (Cont'd)

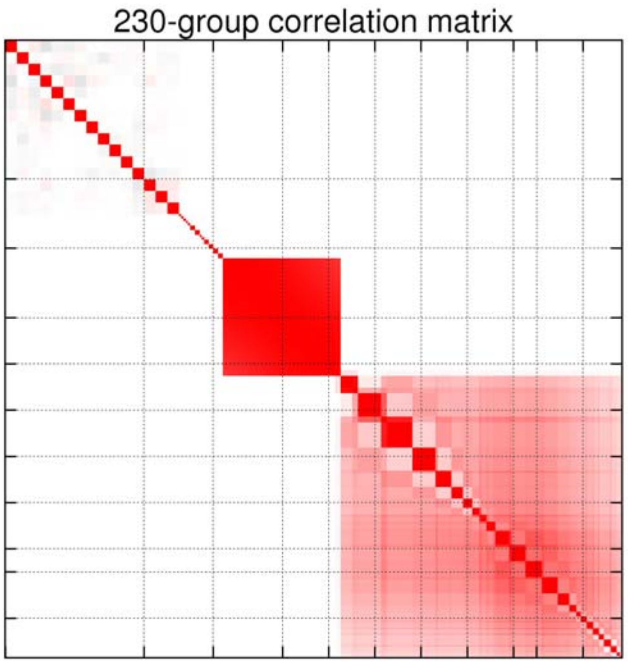
The uncertainty [pcm] in Na-void reactivity for ABR-oxide core

Type of Matrix	230 - group	ConsUnce k_{eff}	ConsUnce Exact	Flat Flux
Total	83.9	82.1	84.4	74.8
U238 σ_{cap}	30.6	30.2	32.0	28.7
Pu239 σ_f	22.9	19.3	18.6	18.9
Pu239 σ_{inel}	3.55	4.89	6.21	7.56
Fe56 σ_{inel}	34.3	33.0	30.6	21.3

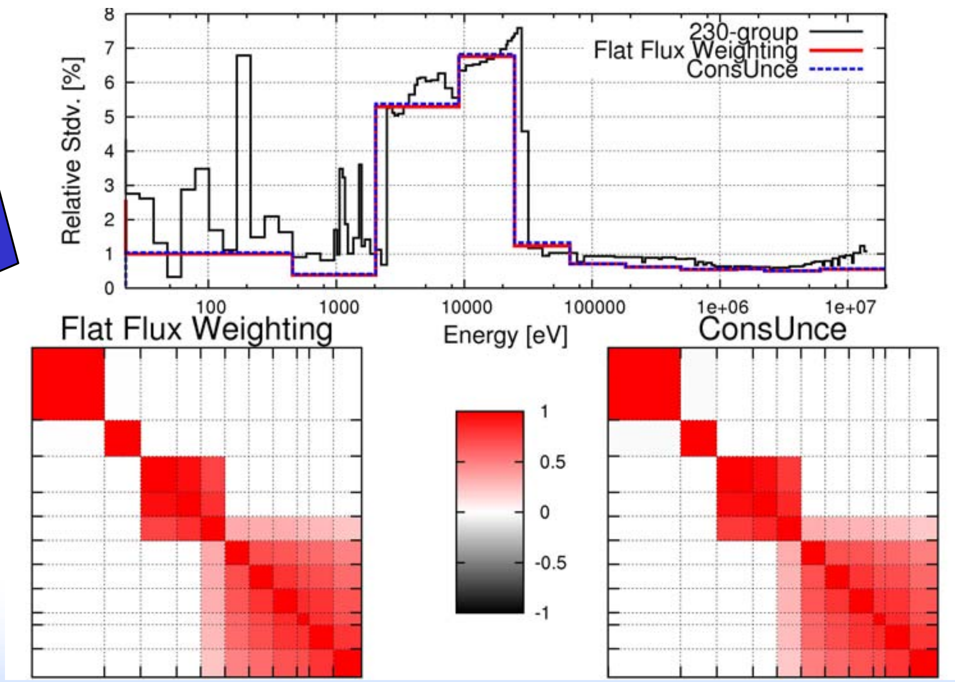
ConsUnce k_{eff} – ConsUnce derived matrices with k_{eff} weighting function.

ConsUnce Exact – ConsUnce derived matrices with Exact weighting function.

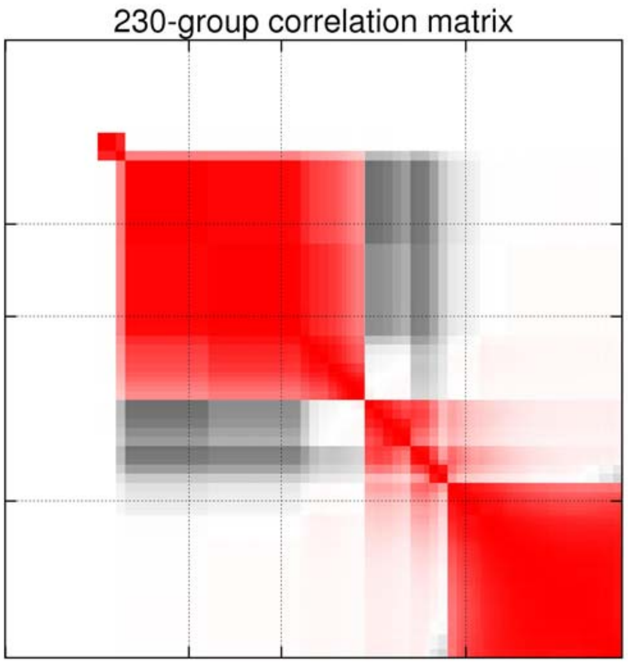
Visual Comparison of Correlations



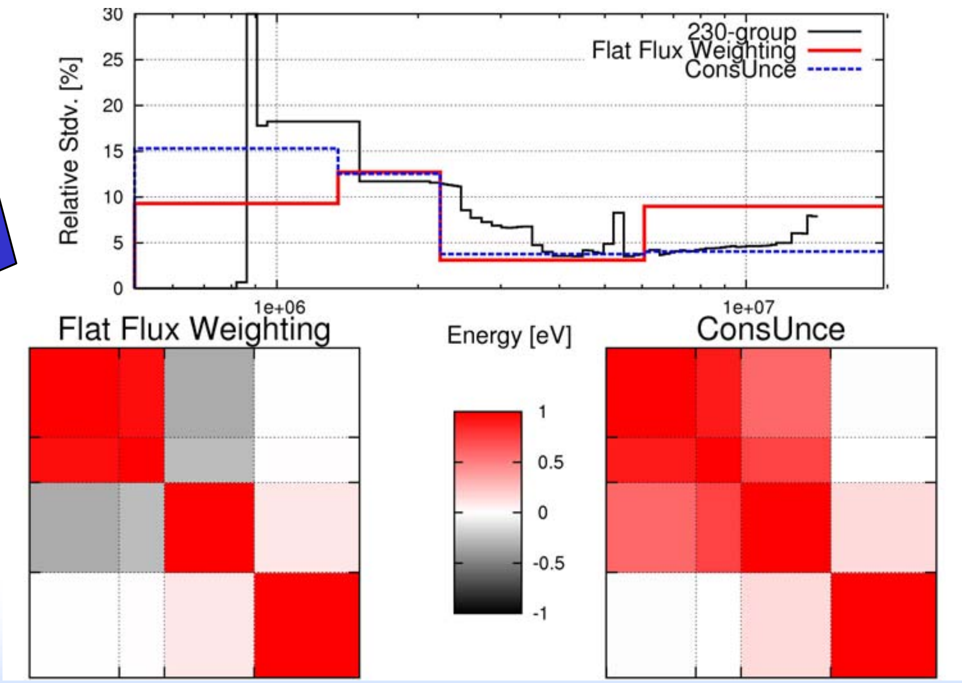
Pu239 (n,f)



Visual Comparison of Correlations (Cont'd)



Fe56 Inelastic



Conclusions

- A new algorithm for a rigorous collapsing of covariance data has been evaluated.
- The method preserves the uncertainty, calculated in a fine-group energy structure, at the broad-group structure.
- Comparisons to uncertainties calculated with covariance data collapsed by the common procedure (flat flux weighting) have shown:
  - No significant effects on very important cross sections of major actinides (Pu-239 fission, U-238 capture).
  - Significant effects on values associated with inelastic cross sections for very broad group energy structure (15-group).
  - The flat flux collapsed data can be used when the sufficient number of energy groups is considered (e.g. 33-group) with some preliminary verifications (e.g. for scattering data).
- When using flat flux weighted covariance data with sufficient number of energy groups (30-50), special caution has to be applied for inelastic cross section in case of data adjustment work.