CSNSM 92.38

New Methods for Extrapolating Masses Far from Stability

C. Borcea[1], and G. Audi

CSNSM, IN2P3-CNRS, Laboratoire René Bernas
Bâtiment 108, F-91405 Orsay Campus, France

Abstract

An "Interactive Graphical" tool has been built for observing the *Surface of Masses* and making close extrapolations. The predictions of 13 models are thoroughly examined with that tool, their predictive power are evaluated, and the coherences of these 13 predictions are studied, allowing thus to set-up a method, based on averaging of these predictions, for medium-range extrapolations.

1 Introduction

One may imagine that when all nuclear masses are displayed as a function of N and Z, one would obtain a "surface" in a 3-dimensional space. At closer look, this surface is rather coarse due to the odd-even staggering of masses in N and Z. However, if one divides the surface into four "sheets" corresponding to the combinations of parity, each of these sheets has a very smooth character. This smoothness is interrupted in some places by "accidents" that almost always can be associated with a relatively violent change in one or more physical parameters (e.g. shell closures, shape transitions like onset of deformation, prolate-oblate or triaxial transitions, etc.). This continuity property of the four sheets is a physically fundamental one and to reproduce it together with the right placement of "accidents" is the main goal of any theoretical mass formula. Moreover, this property permits extrapolation or interpolation to unknown masses. Unfortunately, visualizing and handling these sheets in a tridimensional space together with the constraints concerning their relative distance is not an easy task. A way out will be to look at the "derivatives" of the *Surface of Masses*. The derivatives, while still preserving the continuity property [63Bar], will magnify the amplitude of local "accidents". In addition, a variety of presentations will be possible for these derivatives, allowing to take care of all existing constraints imposed by the topology of the mass sheets. As examples one may quote: neutron and proton separation energies, α- and β-decay energies.

[1]IN2P3-Visitor, on leave from the Institute for Atomic Physics, Bucharest.

The pioneering work of Wapstra [85Bos] in estimating the masses of some unknown nuclei relied mostly on observing the regularities of such derivatives. These few predictions, labelled "systematics", were used primarily as a means to include reactions or decays with known energies, but not connected to nuclides with known experimental mass values [85Wap]. They were also used to define a domain of nuclei, in the (N,Z) plane, as narrow as possible around the known masses, with a smooth contour (interpolations in N, in Z and in N-Z). Therefore these predictions can mostly be considered as interpolations.

2 Close Extrapolations

It may be tempting to extend this procedure to make extrapolations. They are strongly demanded for the planning of experiments, especially considering the increasing number of facilities capable of delivering secondary radioactive beams. The addressed zone of interest will be situated in the immediate vicinity of the present limits of measured masses (close extrapolations).

However it is not easy to obtain reliable extrapolations of the *Surface of Masses*. A way to improve this reliability, is to observe the continuity property in several representations of the various existing derivatives. Therefore an interactive graphic computer program was developed to display simultaneously any four derivatives (separation energies S_n, S_{2n}, S_p, S_{2p}, decay energies Q_α, Q_β, $Q_{2\beta}$, pairing energies Δ_{nn}, Δ_{np}, Δ_{pp}) plotted as a function of any of the parameters: Z, N, A, N-Z or 2Z-N. Continous lines connect (optionally) in each plot nuclei having any of the iso-properties: Z, N, A, N-Z or 2Z-N. One such example is presented in fig. 1. Furthermore, the result of any mass formula can be superimposed on the same figures. Interactively, any point in any diagram can be displaced and all the subsequent changes in the chosen diagram (parentages included) as well as in the others will follow. Also new points may be created in any of the four chosen representations and will show up everywhere.

This **"Interactive Graphical"** tool enforces and extends thus the method of Wapstra and now allows us to make reliable close extrapolations starting from the known masses. It will do of course even better for interpolations. However, one should note that it has a very limited (if any) predictive capability for "accidents", i.e. it cannot substitute for a physical mass formula which can account for shape transitions or shell (subshell) closures. It is nevertheless useful because an experimentally detected "accident" with respect to the smooth behavior will indicate some specific physical properties stimulating further studies.

The usefulness of the **I-G** tool is manyfold, going much beyond the initial purpose of making predictions of unknown masses.

In the evaluation of data on masses, when irregularities are observed, it helps in tracing down the mass responsible for the given irregularity, and also in locating conflicting data.

Moreover, this presentation of derivatives can provide a challenging test ground for various mass formulae and offer useful hints for further improvements in the models. For example, fig.2 shows that the predictions of Möller and Nix [88Mol] have some oscillatory trends not present in the actual mass surface. Also, the Wigner correction terms for N=Z which are essential for predicting masses of proton rich nuclei are considered only

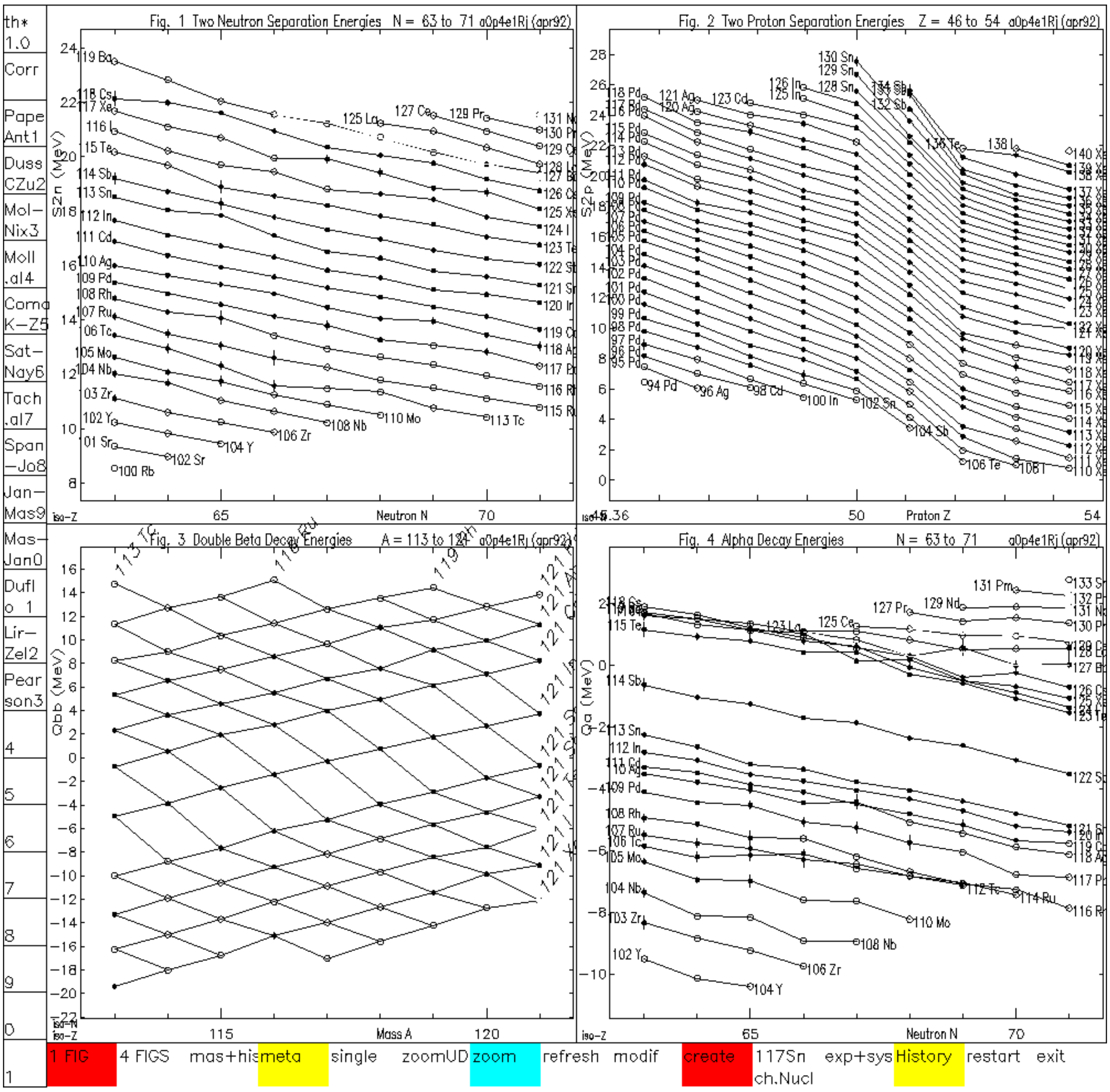

Figure 1: A screen image of the "Interactive Graphical" display of four derivatives of the *Surface of Masses* around ^{116}Sn. The four quadrants display respectively the S_{2n}(N), S_{2p}(Z), Q_{α}(N) and $Q_{2\beta}$(A); the continous lines connect the nuclei having the same Z, N, Z and (Z and N) respectively. The boxes at left and bottom serve for various interactive commands. The Z=50 shell closure is clearly seen in quadrants 2, 3 and 4. Solid points and error bars represent experimental values, open circles Wapstra type predictions from interpolations (the "systematics").

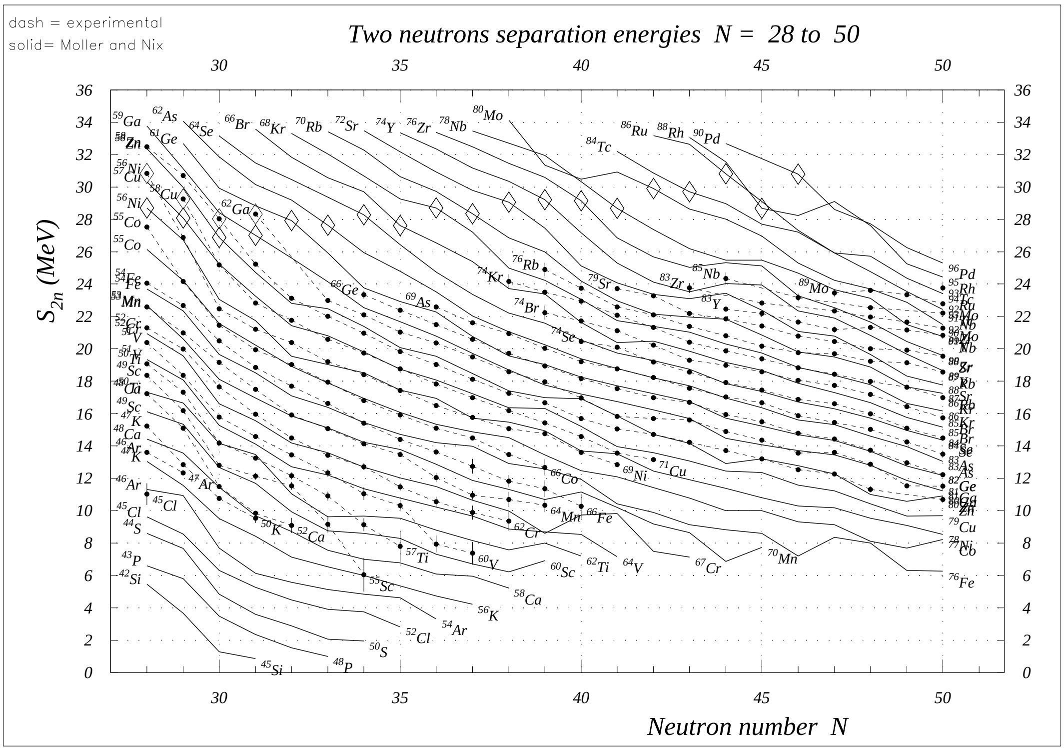

Figure 2: Predictions of the theory of Möller and Nix [88Mol] (solid lines) are compared to the experimental masses (points, error bars and dashed lines) in an S_{2n} representation for N=28-50. Diamonds locate the so-called "Wigner" nuclei (N=Z). Oscillations in the theoretical curves indicate lack of smoothness of the formula. Constraints to limit the variation of some parameters from one nucleus to another could certainly improve the predicted *Surface of Masses*. The Wigner effect, although present in the formula, is damped out completely for odd-Z [81Aud] due to the extra $\frac{1}{A}$ term first introduced in 1976 by Myers [76Mye]. Extension of the predictions up to the drip lines is highly desirable here, as for all "global" models, in order to make "far" extrapolations possible in the future.

in a few mass predictions and often, when taken into account, they do not result in a structure with the same amplitude as that displayed by the experimental masses (figures 2-9). The presence of large amplitude oscillations beyond the limit of known masses may render the predictions of a given mass formula questionable from the point of view of extrapolations(fig.3), even if, like in the present case, it is a "global" model and it does reproduce quite well the masses of nuclei close to the bottom of the valley of stability. Though not illustrated, let us mention a few more observations: only few formulae [88Jan] reproduce the subshell closure at N=56, the well known onset of deformation at N=90, the shape transition at N=108 and the drop in energy at N=152; some irregularities can be observed in [88Dus] in the α-chains connected to ^{125,126}Sn and around A=185; there is an exceedingly strong overbinding just before shell closures in Spanier and Johansson's liquid-drop formula [88Spa]; the predictions of Tachibana et al. [88Tac] are the ones that reproduce the best the smoothness property of the *Surface of Masses*, only few irregularities deserve attention, at N=136 and N=156 for example or at N=109, where

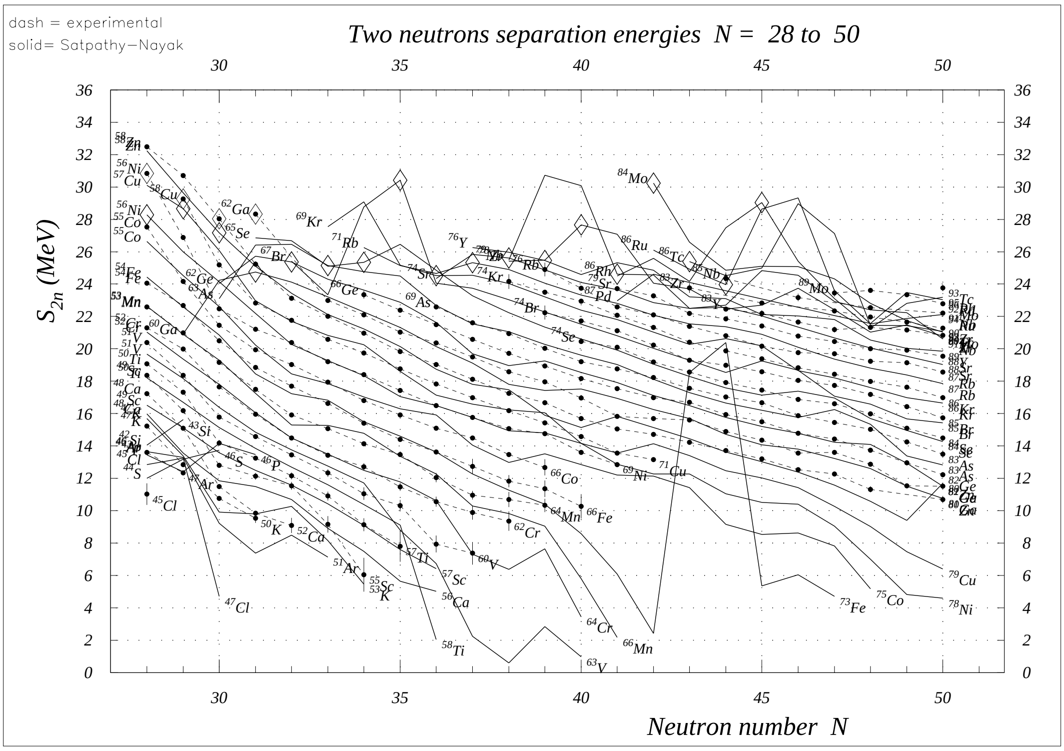

Figure 3: Same as in fig. 2 for another prediction [88Sat]. At the bottom of the valley of stability the model is quite close to the experimental values, and gives consequently a good fit (see Tables 1 and 2). Only when departing from the known masses do very strong oscillations occur, rendering its predictions questionable when medium-range or far extrapolations are considered, even though it is a "global" model.

isotones below Z=73 are underbound whereas those above Z=73 are overbound; strong underbinding of nuclei around A=215 and around ^{195}Fr are evident in both Möller and Nix [88Mol] and Möller et al. [88MMST]; Duflo's calculation [92Duf] manifests irregularities for n-rich nuclei below N=50, for the light rare-earth, the light actinides and for the N-Z=45 nuclei; Liran and Zeldes [76Lir] display a quite smooth *Surface of Masses* and good continuity through shell closures; the ETFSI model (extended Thomas-Fermi plus Strutinsky integral) [92Pea] also reproduces well the general smooth behavior of the *Surface of Masses*, but not so much the "Wigner" effect at N=Z (figure 9).

Similarly to the improvements induced by the analysis of P. Haustein [84Hau] on the mass models, we hope that the type of analysis developed here, based on the derivatives of the *Surface of Masses*, will help improving furthermore the existing models. Such improvements may reflect positively on the predictive power of future versions of the models.

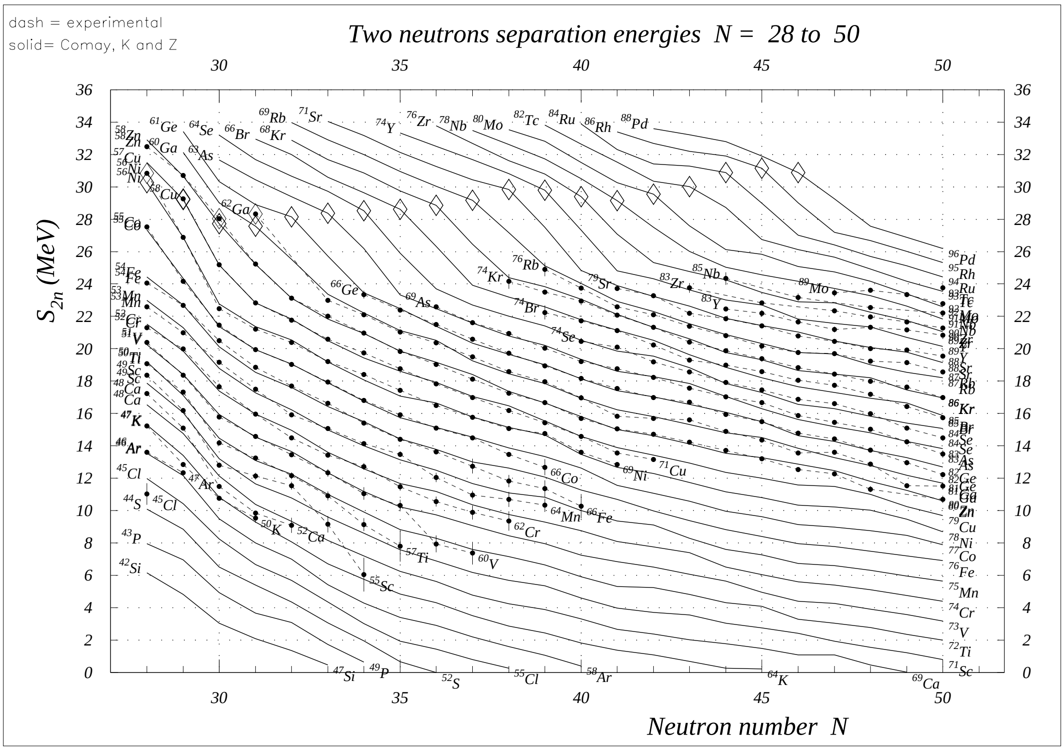

Figure 4: Same as in fig. 2 for the predictions of Comay et al. [88Com].

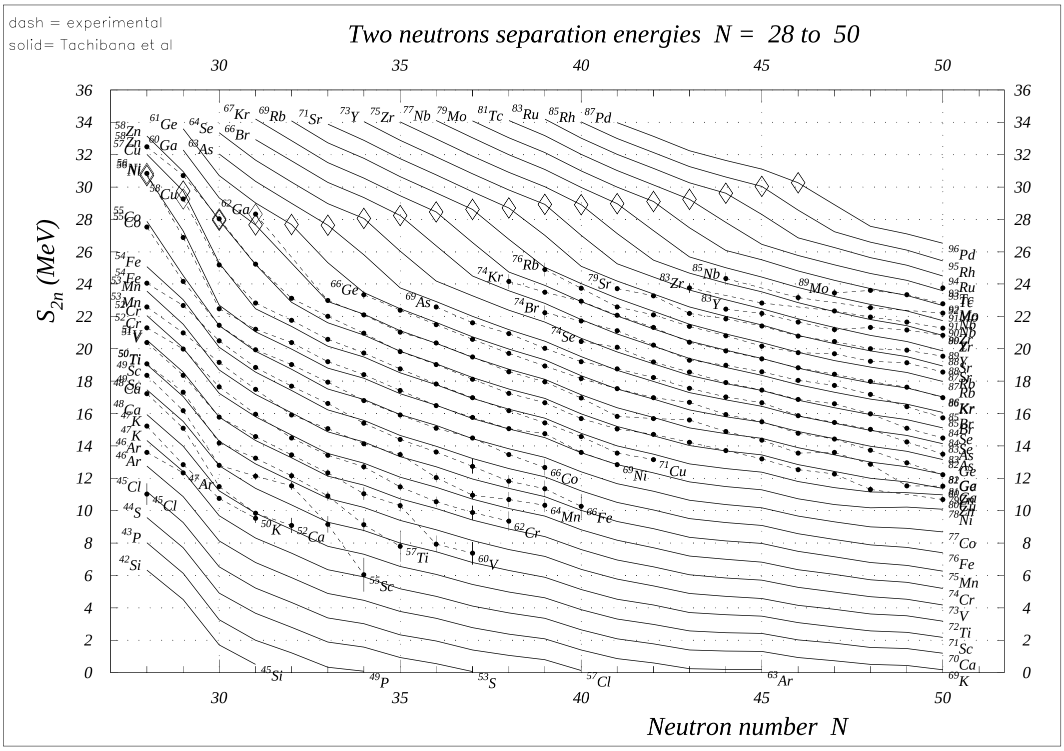

Figure 5: Same as in fig. 2 for the predictions of Tachibana et al. [88Tac].

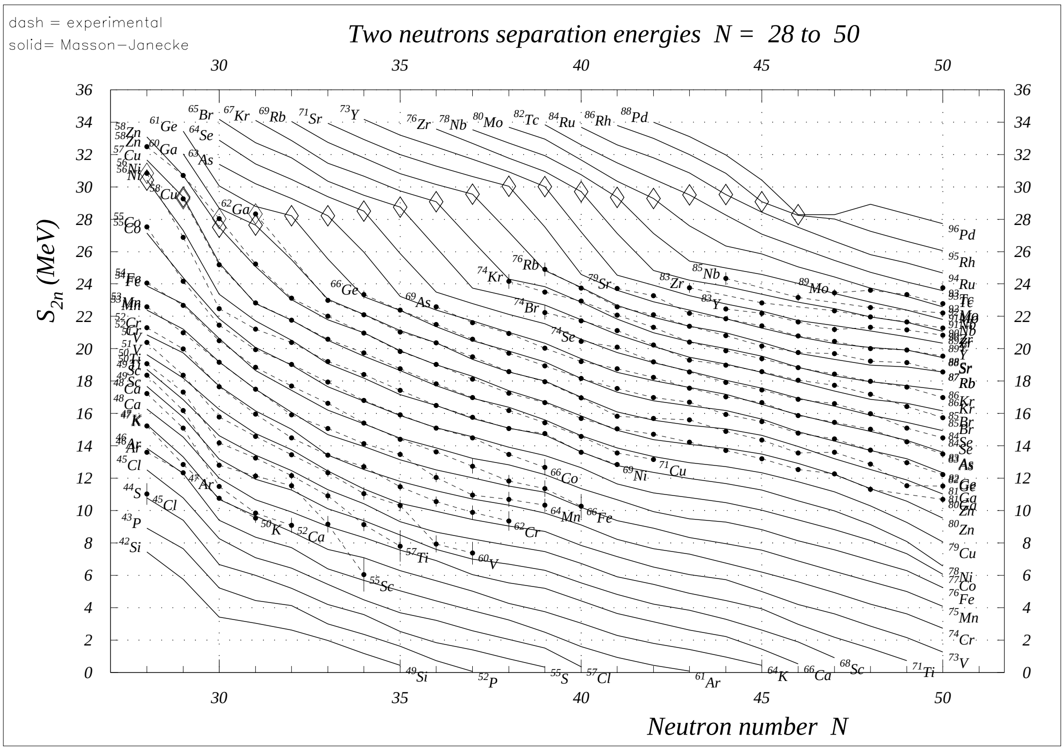

Figure 6: Same as in fig. 2 for the predictions of Masson and Jänecke [88Mas].

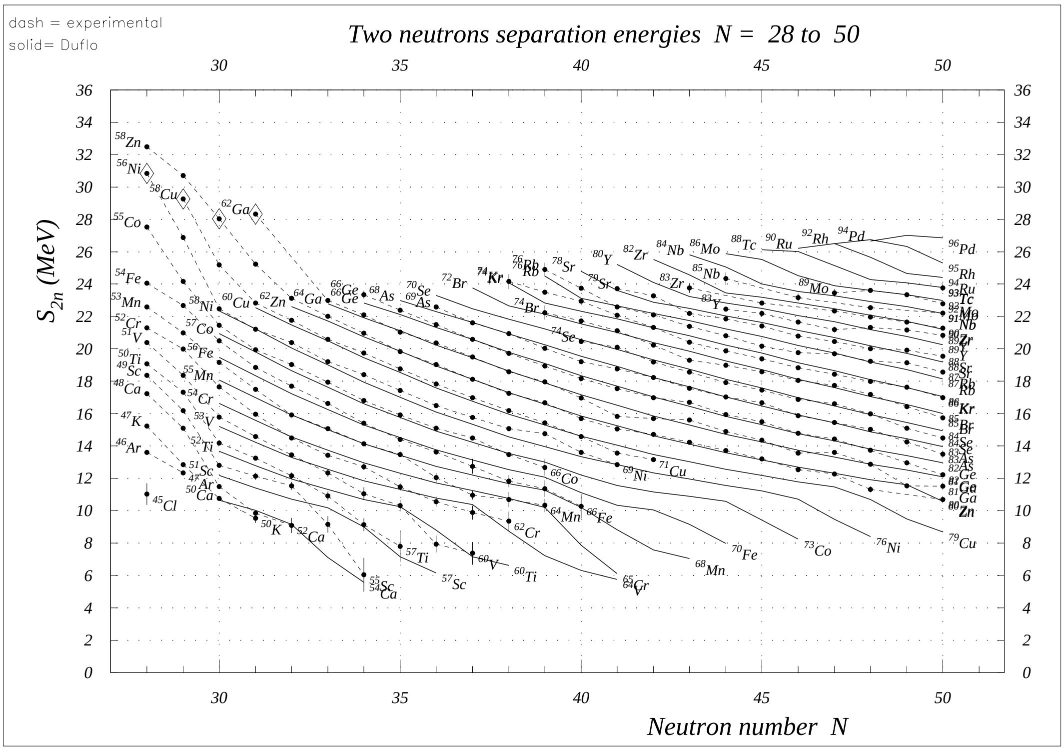

Figure 7: Same as in fig. 2 for the predictions of Duflo [92Duf].

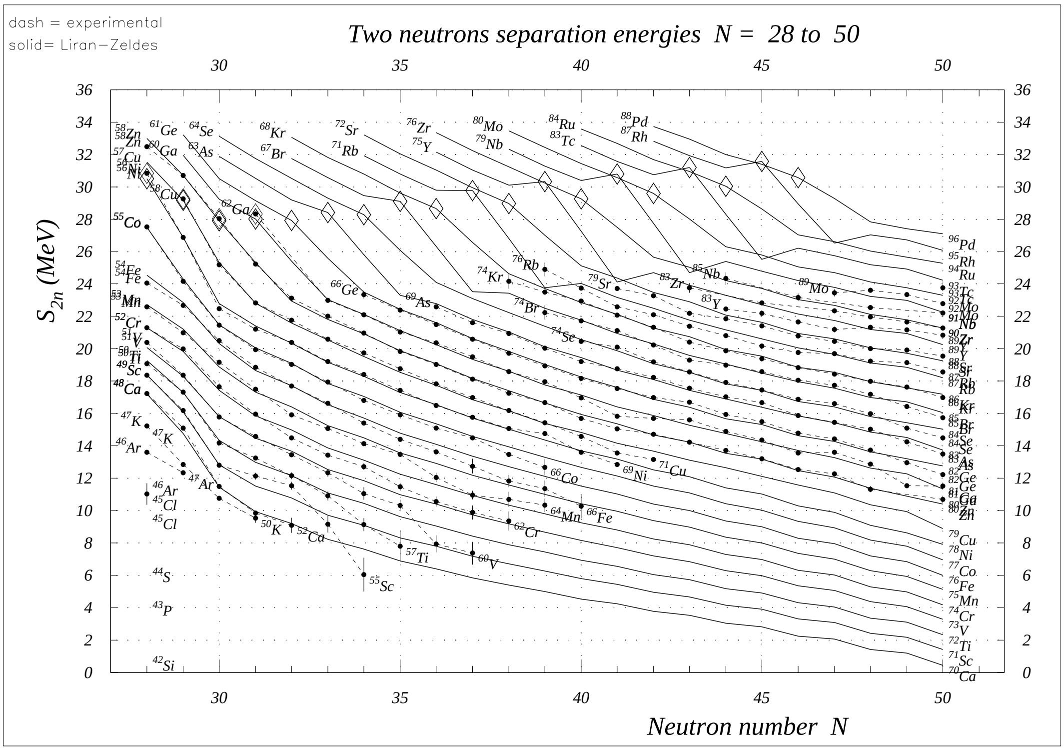

Figure 8: Same as in fig. 2 for the predictions of Liran and Zeldes [76Lir].

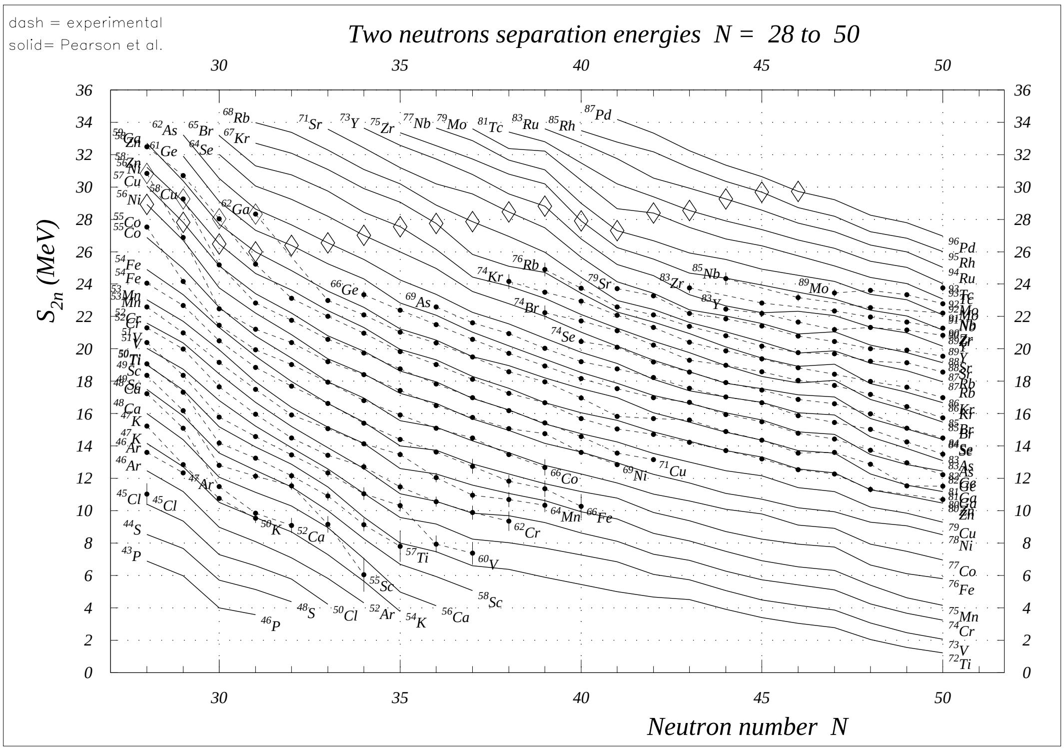

Figure 9: Same as in fig. 2 for the predictions of Pearson et al. [92Pea].

3 Extrapolating Further Out

A different situation is encountered when predictions are necessary for astrophysical calculations of r-process nucleosynthesis, i.e. towards the neutron drip line. With the exception of light masses, the neutron drip line lies far away from the last measured masses; therefore the extrapolations toward these regions will be called **far extrapolations**. One should note that the word far in this case does not refer to the distance from the bottom of the valley of stability, but rather from the limits presently defined by the nuclei with known masses, which are already "far from stability".

Although the method outlined in the previous section permits to go beyond the usual interpolations, the limits of the possibilities will be reached quite soon, as the reliability will decrease at each step. In order to go further out, some physical background is needed, which can be provided by the existing theories.

Before doing so, the predictive power of the models should be evaluated so as to serve eventually to assign weights in a weighted average. How far one can go this way will result from an analysis of the coherences and divergences among the predictions of different models.

One should distinguish two main types among the existing models for nuclear masses: the "local" models that use local relations and the "global" ones that start from a nuclear model calculation done with different ingredients of phenomenology. From this second type, the model of Pearson et al. [92Pea] has been strongly advocated for astrophysical purposes [92Gor]. A further distinction within the "global" models will be of use in the present work for those predictions having some of their parameters fitted separately for the different regions of the chart of nuclei; they will be called "global-regional" models. While for the "local" models, one may expect that a gradual deterioration might appear when getting to far outer masses, for the "global" ones, in principle, every physical premise is contained so that, even if fitted to a particular sample, one should expect not too large divergences. This should be especially true for the theories that use a relatively small number of parameters. In order to convince ourselves of this last statement, we tried an "overadjustment" for each of the models in a small region of the chart (the f-p shell for both N and Z). To the mass values $m_{th}(N, Z)$, predicted by a model, we added in a rather artificial way six terms: $m_{th}(N, Z) + aN^2 + bZ^2 + cNZ + dN + eZ + f$ and minimized the distance between the new theoretical *Surface of Masses* and the experimental one with respect to the parameters $a - f$. The results were spectacular in two respects: the surfaces of the theoretical masses near the bottom of the stability valley were too well adjusted to the experimental surface (an adjustment with a too low χ^2 is as unreliable as one with a too large χ^2), whereas outside the domain of known masses, but still inside the f-p region, the surfaces provided by the various "overadjusted" formulae displayed much stronger divergences from each other than did the original formulae. The first aspect is easily understandable since the authors carefully considered all possible terms that may contribute and which are physically meaningful. The second aspect calls for increased caution when going to far extrapolations. In fact, if the parameters were "pushed" in order to obtain an as small as possible rms (root-mean-square) deviation, calculated over a given sample of known nuclear masses, it is not surprising that beyond this sample the formula will start to diverge. Also, the problem of the stability of the

Table 1. Root Mean Square deviation (rms) and Compensated Linear Deviation (cld) of 13 mass models calculated for different sets of measured nuclear masses.

	MODEL	type	1 rms^a keV		2 cld^a keV	3 rms^b keV		4 cld^b keV	5 $rms^b/$ rms^a	6 $cld^b/$ cld^a	σ^{th} keV
1	Pape and Antony	local	269	(85)	154	195	(3)	72	(=**cpr**)		
2	Dussel,Caurier,Zuker	local	288	(1325)	176	288	(87)	135			
3	Möller and Nix	global	827	(1589)	595	1046	(168)	632	1.26	1.06	670
4	Möller et al.	global	766	(1589)	525	1048	(168)	593	1.37	1.13	671
5	Comay,Kelson,Zidon	local	269	(1628)	149	795	(174)	387	2.95	2.59	1004
6	Satpathy and Nayak	global	462	(1589)	298	1529	(171)	782	3.31	2.63	1500
7	Tachibana et al.	global	506	(1653)	321	834	(175)	448	1.65	1.39	625
8	Spanier and Johansson	global	697	(884)	495	1208	(63)	920	1.73	1.86	1500
9	Jänecke and Masson	local	162	(1629)	64	643	(174)	287	3.97	4.52	1299
10	Masson and Jänecke	local	232	(1578)	132	813	(168)	424	3.51	3.22	1365
11	Duflo	global	425	(1381)	288	912	(130)	450	2.15	1.57	706
12	Liran and Zeldes	global	401	(1582)	182	1305	(166)	755	3.25	4.14	1500
13	Pearson et al.	global	726	(1489)	543	951	(146)	586	1.31	1.08	632
	" average of theories"		405	(1583)	280	690	(168)	330	1.70	1.18	389

The figures in parentheses indicate the number of nuclei from the set of experimental masses for which a given theory makes predictions.
a) calculated for the set of 1655 masses of the 1986 atomic mass table [88Wap]
b) calculated for the set of 175 new masses measured since 1986

predictions with respect to small changes in the parameters is an important one. As shown recently [92Gor], relatively small changes in the parameters of a model may imply tens of MeV differences in masses near the drip lines and consequently orders of magnitude in calculated abundances if all other astrophysical premises are kept constant. Therefore it would be interesting, once a formula is finalized, to have its stability evaluated by its authors, for "medium-range" and for "far" extrapolated masses, by allowing each of the parameters to vary within its acceptable limits.

There are only a few facts that may give an idea about the predictive power of a nuclear mass formula. Since the 1986 atomic mass table [88Wap] some 175 new masses have been measured and this sample can be used to test the quality of predictions made by each model. To the 10 models already presented in the "1986-1987 Atomic Mass Predictions" of P. Haustein [88Hau] we added the ones of Duflo [92Duf], based on doublets and triplets, and the ETFSI model of Pearson et al. [92Pea], both adjusted also to the 1986 experimental masses. We also wished to include in our study the predictions of Liran and Zeldes [76Lir] published in 1976, since the 1986 compilation did not include any "semi-empirical shell model formula". The numbers that will be given below for [76Lir] will suffer comparatively from fitting to much older data, and the results of the present analysis for this model should be considered with special care. In selecting the models, we followed the same policy outlined by Haustein [90Hau] and did not therefore consider the models of Pape and Antony or of Dussel, Caurier and Zuker, which "do not contain predictions that cover large regions far from stability" [90Hau].

A commonly used quantity in evaluating the ability of a model to reproduce the mass

Table 2. Compensated Linear Deviation (*cld*) of 13 mass models calculated for different sets of measured nuclear masses.

	MODEL	type	7 cld^c keV	8 cld^d keV	9 $cld^d/$ cld^c
1	Pape and Antony	local	107 (2)	3 (1)	
2	Dussel,Caurier,Zuker	local	143 (63)	114 (24)	
3	Möller and Nix	global	540 (108)	796 (60)	1.47
4	Möller et al.	global	486 (108)	786 (60)	1.62
5	Comay,Kelson,Zidon	local	286 (109)	556 (65)	1.95
6	Satpathy and Nayak	global	547 (109)	1197 (62)	2.19
7	Tachibana et al.	global	384 (109)	554 (66)	1.44
8	Spanier and Johansson	global	812 (43)	1154 (20)	1.42
9	Jänecke and Masson	local	237 (109)	1371 (65)	1.56
10	Masson and Jänecke	local	297 (108)	652 (60)	2.19
11	Duflo	global	302 (84)	722 (46)	2.39
12	Liran and Zeldes	global	567 (107)	1095 (59)	1.93
13	Pearson et al.	global	541 (96)	672 (50)	1.24
	" average of theories"		271 (108)	435 (60)	1.60

The figures in parentheses indicate the number of nuclei from the set of experimental masses for which a given theory makes predictions.
c) calculated for the subset of 109 new masses inside the **"domain of known masses"**
d) calculated for the subset of 66 new masses outside the **"domain of known masses"**

surface is the root-mean-square rms deviation defined as:

$$rms = \sqrt{\frac{1}{n}\Sigma(m_{th} - m_{exp})^2} \tag{1}$$

where n is the number of nuclei in the sample. A more sophisticated method, based on the same principles, was developed by Möller and Nix [88Mol] to take into account both experimental and theoretical errors. However, what seems to us more important for the present analysis is to get a feeling of the degree of "adherence" of the theoretical mass surface to the real one. It is for this reason that we made the choice of a linear norm:

$$cld = \frac{1}{n}\Sigma \max\{(|m_{th} - m_{exp}| - \sigma_{exp}), 0\} \tag{2}$$

called **"compensated linear deviation"** (*cld*). Such a norm turned out to be very useful in a previous analysis of Rb, Cs and Fr masses [81Aud]. In a way, this *cld* represents the volume comprised between the theoretical and the experimental surfaces, divided by the number of nuclei in the sample. Obviously, the smaller that volume, the better the adherence of the two surfaces. The experimental errors were subtracted from $|m_{th} - m_{exp}|$ in (2) in order to account for the uncertainties on the actual masses. It is interesting to remark that, when fitting parameters, a minimization procedure using the *cld* gives results that are quite similar to those obtained when minimizing the rms deviation; only is the convergence with the *cld* slower and more delicate. Therefore, although the *cld* gives better information on the adherence between surfaces, we do not advocate for its use when minimizations are to be performed.

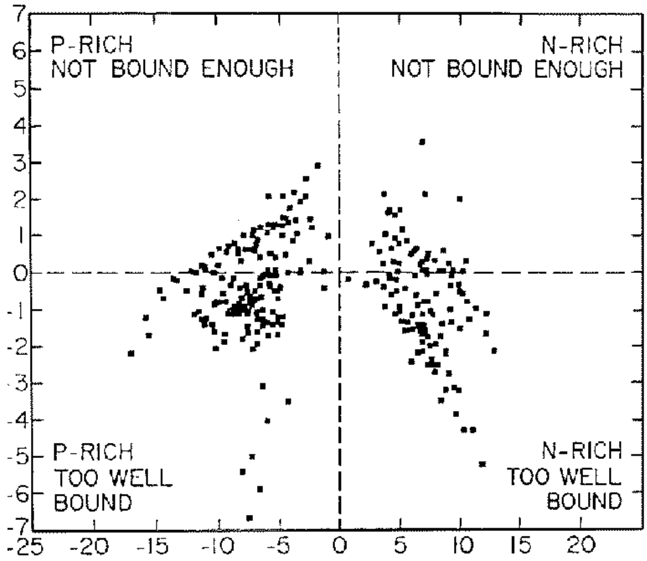

Figure 10: (taken from ref. [84Hau]) Differences between the predictions of the model of Myers [76Mye] and the 300 masses measured for the first time between 1976 and 1984 plotted as a function of neutrons from stability. [84Hau] mentions that "the droplet model generally places the predicted bottom of the valley of β-stability slightly above the measured masses and the curvature of the mass surface is generally greater than that predicted."

Table 1 presents the results of calculated *rms* and *cld* for various theories and experimental samples. The columns numbered 1 and 2 show the results for the set of 1655 masses of the 1986 table [88Wap] while the next two columns show the results for the extra set of the 175 new masses measured since 1986. Columns 5 and 6 give the ratios between the results for the two samples, respectively for *rms* and for *cld*. Both quantities grow when they are calculated over the sample of new masses. The values in column 4 reflect the close predictive ability of various models while those in column 6 may be called **"close predictive ratios"** or **cpr**. In fact, the distinction between different models, which will be very striking for far extrapolation, can already be observed for the close predictions: the **cpr** for the "global" models are systematically better.

At this point one should note that in the sample of 175 new masses, the big majority lie in the immediate vicinity of known masses. If we think of the limits between which the nuclei with known masses are placed on the chart of nuclides as a smooth envelope, defining the **"domain of known masses"**, the 175 new masses would be lying on both sides of the border of this domain. Therefore the predictions for some of these 175 masses can be considered as interpolations while the others can be included in the category of close predictions. This is only done to stress the idea of close predictions. The set of new masses is therefore split in two parts, those inside the "domain" (109 nuclei) and those outside (66 nuclei). The calculated *cld* for these two subsets are given in the columns 7 and 8 of table 2. Despite the small dimension of the samples, a tendency for the *cld* to increase is obvious as shown by the ratios in column 9.

We may draw attention to the fact that part of the increase of the deviations for the

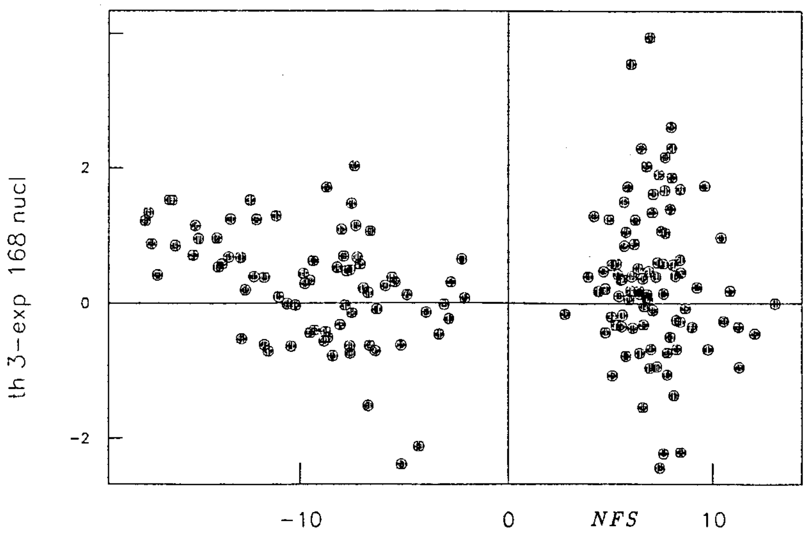

Figure 11: Similar to fig. 10 for the model of Möller and Nix [88Mol] and for the set of 175 new masses measured since 1986. The systematic feature observed in fig. 10 for the droplet model has disappeared here. Note that the vertical scale is much more dilated than in fig. 10.

set of 175 new masses (as displayed in columns 5 and 6) is due to irregularities of the experimental *Surface of Masses* for n-rich nuclei around N=35 (see fig. 2 for instance). The experimental values are due to one experiment only and these irregularities are not reproduced by any model. Remeasurement of these few masses and some others in the neighborhood would be highly desirable, either to resolve the present discrepancy, or to confirm a new structure which should be taken into account by future formulae. The *cld* is less affected than the *rms* since it takes into account the experimental uncertainties. Three formulae do not cover the area where these irregularities occur (rows 1, 2 and 8 in tables 1 and 2) and this fact is reflected, for the first two at least, in the *cld* and *rms* for the set of new masses.

Another interesting analysis is that proposed by Haustein [84Hau] in terms of a defined distance with respect to the stability line called "neutrons-from-stability":

$$NFS = N - Z - (0.4A^2)/(200 + A) \tag{3}$$

Figure 10 taken from this reference shows the result of such an analysis for the droplet model (version 1976) when compared with the sample of 300 masses measured for the first time between 1976 and 1984. In the same spirit we examined the 13 models. A typical example is presented in figure 11. While no systematic features of the kind mentioned in ref. [84Hau] were observed, this type of diagram (NFS-diagram) seems rather to suggest a limitation to 1.5 MeV of the maximal excursion of the predictions with respect to the experimental values and an enhanced dispersion for the n-rich nuclei. Due to the reduced size of the sample, these conclusions are only orientational. One should also remark, for example, that the droplet model, after improvements partly stimulated by Haustein's analysis [84Hau] no longer presents the tendency to overbind masses far from stability. A similar type of analysis was proposed by Goriely and Arnould [92Gor] who defined a

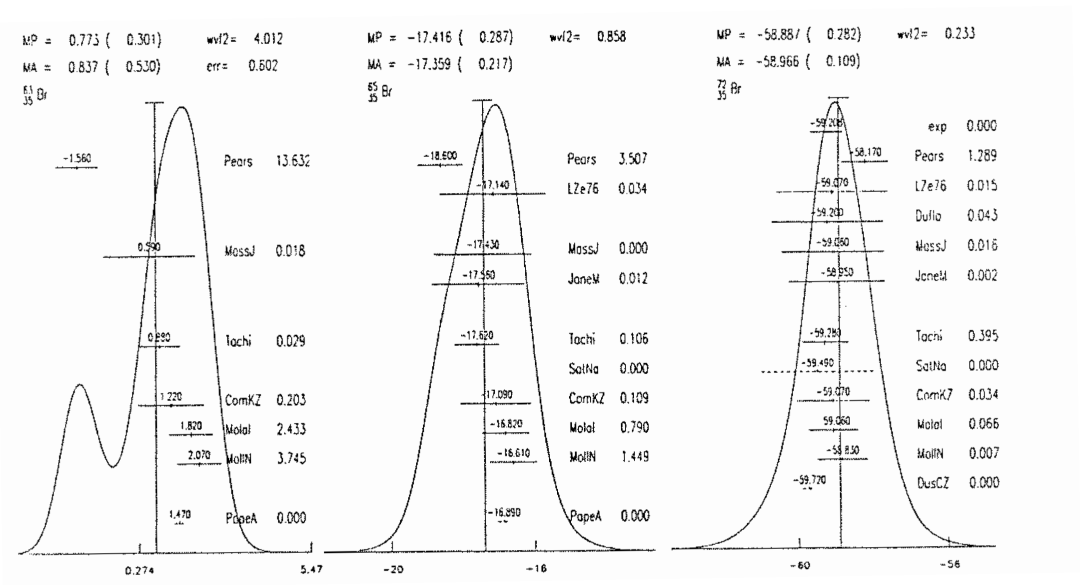

Figure 12: Ideograms [92Pdg] for three Br isotopes. The dispersion of theoretical predictions is observed to increase as one get close to the drip lines. Right: $^{72}_{35}\mathrm{Br}_{37}$ is the last measured mass. Center: $^{65}_{35}\mathrm{Br}_{30}$ is the last isotope in the medium-range extrapolation, i.e. where coherence of theories is acceptable. Left: $^{63}_{35}\mathrm{Br}_{28}$ is the last isotope to fulfill the conditions for averaging (i.e. 6 accepted predictions). MP is the weighted average of predicted masses with its error σ_p within brackets, in MeV, $wvf2$ is the normalized χ^2, denoted χ^2_ν in the text, err is the external error, i.e. $\sigma_p \times \chi_\nu$, in MeV.

"neutron surplus", which is more precise but less convenient than Haustein's NFS.

Now that the predictive powers of the models have been evaluated, the results of the above analysis could be exploited and one may consider making an adequate statistical treatment of the predictions, provided the divergences among models are within reasonable limits (see next paragraph). The same criteria for selection as in [90Hau] are adopted, namely at least six models should give predictions for a given nuclear mass and the first two rows in tables 1 and 2 should be excluded. A weighted average is being considered. The uncertainty associated with the predictions of each model in that average is taken as: $\sigma_i^{th} = cld_i \times cpr_i^2$, i.e. the compensated linear deviation for the set of measured masses (table 1 column 2) times the square of the corresponding close predictive ratio (column 6) with a limitation to 1.5 MeV due to the observation above. The σ^{th} are given in the last column of table 1. The parabolic increase with cpr_i has somewhat been suggested by the evolution in the columns 7 and 8 in table 2 and by the character of the extrapolations we are interested in, namely *beyond* the "close" ones. Ideograms [92Pdg] were produced for four complete isotopic series in the f-p region (Ni, Br, Mo and Sn) going then from the valley of stability to the drip lines. A few examples are shown in figure 12. Their examination showed that the dispersion among the predictions from different models increase drastically when one approaches the drip lines. However, for a relatively large region, this dispersion is within reasonable limits (the normalised χ^2, denoted χ^2_ν, close to unity). Under the conditions above, the averages of theoretical masses for 399 nuclei in the f-p shell region were calculated. Only three nuclei showed

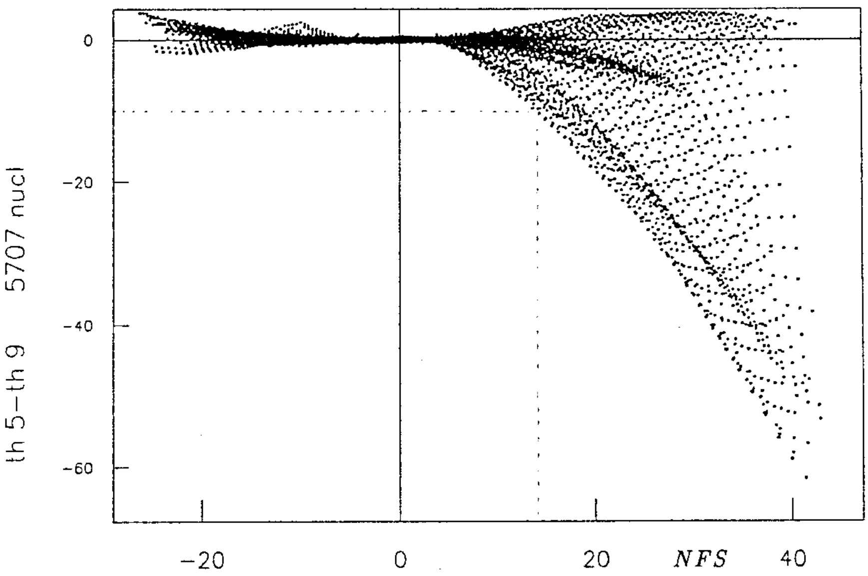

Figure 13: Differences between the predictions of two "local" relations: Comay, Kelson and Zidon [88Com] and Jänecke and Masson [88Jan] plotted as a function of "neutrons from stability" for 5707 nuclei. The strong coherence around stability ($NFS = 0$) is due to the very good agreement of both predictions with the known masses. Divergences exceeding 10 MeV start at $NFS = +14$ on the n-rich side.

more than two standard deviations. Though the present study was limited to the f-p region, this does not affect the generality of our conclusions. Averaging performed over the whole chart with the same conditions yielded results for 5018 nuclei of which 4227 had a dispersion below 1 MeV; they span values of NFS from -20 to +20. These crude results will require careful individual and global analysis.

To get an overall idea about the zone where different predictions are still coherent, one may apply the NFS diagrams to compare any two theoretical predictions, similarly to the analysis of [92Gor]. With these diagrams one could also observe the deviations of a given model from the calculated "average" of predictions. Though very efficient, this method suffers from the fact that the domains of predicted masses vary widely from one model to another. Many models do not venture into the vicinity of drip lines, i.e. where we are interested to compare their predictions. This fact is in turn reflected by the size of the above set of averaged values. Anyway, as a general trend one can mention a strong divergence of "local" models among themselves (fig. 13) and with respect to the "average" when departing from the measured masses. This tendency is also present but much attenuated for those "global" models that make predictions in a large enough domain (fig. 14). The NFS analysis confirmed the conclusion of the statistical treatment that going beyond $|NFS| = 20$ is hazardous for the time being. These numbers could define the frontier separating the "**medium-range extrapolations**" from the "**far extrapolations**".

Restricting ourselves to these orientational limits, the averaged values obtained for

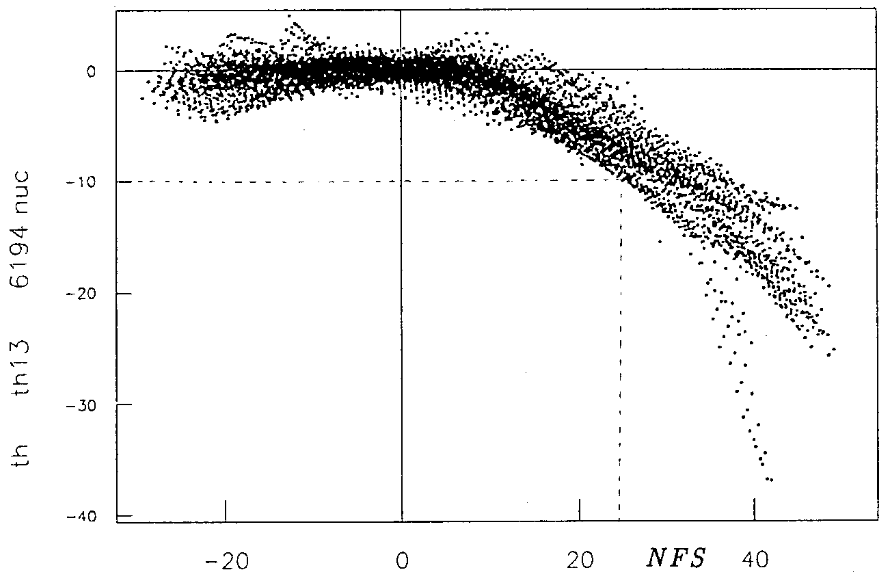

Figure 14: Same as in fig. 13 for two "global" predictions: Tachibana et al. [88Tac] and Pearson et al. [92Pea] for 6194 nuclei. Coherence around stability ($NFS = 0$) is naturally weaker than with local relations. Divergences exceeding 10 MeV start at $NFS = +25$ on the n-rich side. Only few differences exceed 20 MeV. Note that the "width" of the scattering plot is almost constant for the whole range of NFS.

the medium-range predictions can be compared with the experimental masses using the "Interactive Graphical" tool described at the beginning. An example is shown in fig. 15. The smooth character of the curves is preserved even for the region of the outer masses. Quantitative comparisons are given in tables 1 and 2 where last line in each table displays the deviations of such "average" of predictions from experimental values in the same way as was done for the 13 individual predictions.

Of course, the medium-range extrapolated values obtained in this way are far from being perfect, but they can be considered as the best estimate presently possible for these masses. Each theory contains some truth and certainly some approximations. Being constructed quite differently from each other, one may expect that the effects of these approximations do not coincide and would be random. If this is the case, then averaging would be licit and could be considered as the best meagre remedy until drastic improvements in theories is achieved. An indication that this procedure is acceptable is given by the last column of table 1, where the error associated to each model for the close extrapolations happens to be the nicest, and by far, for the "average" of theories.

Towards the neutron drip line, we have seen that the above procedure fails because of the important divergences among theories. This effect can be limited if the "local" models are not used in the far extrapolations, for many of the reasons outlined above. To compensate the resulting impoverishment in models, it would be highly desirable to have all "global" models extend their predictions up to the neutron drip line. Especially since, among the "global" models, the "global-regional" formulae, which require the existence of measured masses in every shell region (e.g. Liran and Zeldes or Tachibana et al.), will

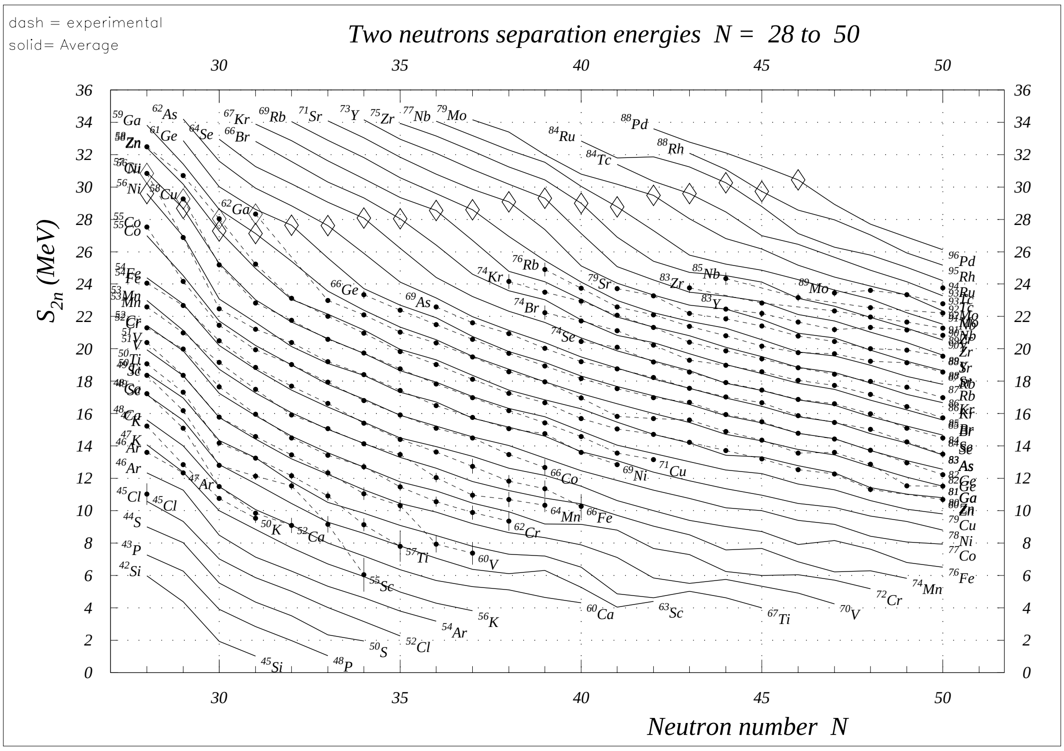

Figure 15: Average values of theories, obtained as described in the text, are compared to the experimental masses (points, error bars and dashed lines) in an S_{2n} representation for N=28-50. The smooth character of the curves is preserved even for outer masses. Further smoothing of these averages may be considered in the production phase of our work. Extension of these predictions on the n-rich side is limited, due to the small number of models making predictions far enough from stability.

not be able, in many places, to reach the neutron drip line, reducing thus their usefulness for far extrapolations.

4 Conclusions

An "Interactive Graphical" tool has been developed in order to better scrutinize the *Surface of Masses* and exploit its continuity property for making close extrapolations. By pointing at the qualities and the weaknesses of each mass prediction, the figures could directly suggest improvements to the formulae and could help their authors in questioning some parameters. An analysis of the predictive power of various models has been performed as well as of the coherence of the models in predicting masses far from stability. The result of this analysis showed that, based on existing models, the medium range extrapolations are reliable while extrapolations toward drip lines (far extrapolations) are to be taken with much care untill further improvements in the theories make them congruent in these regions.

The methods for close and medium-range extrapolations are now defined and tested.

Serial production of such predictions is the next step, but that will require much care and time. It would be interesting to have close extrapolations produced very soon in order to include them in the next table of experimental masses. Medium-range extrapolations will be evaluated and may enter a special table, if real interest is expressed. Special care will be taken to ensure the continuity property of the *Surface of Masses* at the border between the three regions: experimental masses, close extrapolations and medium-range extrapolations.

We wish to express our gratitude to A. H. Wapstra, M. Pearson and A. Pape for critical reading of this paper.

References

[63Bar] R. C. Barber, W. McLatchie, R. L. Bishop, P. Van Rookhuyzen and H. E. Duckworth, Can. J. Phys.**41** (1963) 1482.

[76Lir] S. Liran and N. Zeldes, 1975 Mass Predict., S. Maripuu ed., *ADNDT* **17** (1976) 431.

[76Mye] W. D. Myers, in 1975 Mass Predictions, S. Maripuu ed., *ADNDT* **17** (1976) 411.

[81Aud] G. Audi, PhD thesis, Université de Paris-Sud, Orsay (1981), unpublished.

[84Hau] P. E. Haustein, *Proc. 7-th Int. Conf. on Atomic Masses and Fundam. Const. AMCO-7*, Darmstadt 1984, O. Klepper ed., p.413.

[85Bos] K. Bos, G. Audi and A. H. Wapstra, Nucl. Phys.**A432** (1985) 140.

[85Wap] A. H. Wapstra, G. Audi and R. Hoekstra, Nucl. Phys.**A432** (1985) 185.

[88Com] E. Comay, I. Kelson and A. Zidon, in [88Hau] p.235.

[88Dus] G. Dussel, E. Caurier and A. P. Zuker, in [88Hau] p.205.

[88Hau] P. E. Haustein, 1986-1987 Atomic Mass Predictions, *ADNDT* **39** (1988) 185.

[88Jan] J. Jänecke and P. J. Masson, in [88Hau] p.265.

[88Mas] P. J. Masson and J. Jänecke, in [88Hau] p.273.

[88Mol] P. Möller and J. R. Nix, in [88Hau] p.213.

[88MMST] P. Möller, W. D. Myers, W. J. Swiatecki and J. Treiner, in [88Hau] p.225.

[88Pap] A. Pape and M. S. Antony, in [88Hau] p.201.

[88Sat] L. Satpathy and R. C. Nayak, in [88Hau] p.241.

[88Spa] L. Spanier and S. A. E. Johansson, in [88Hau] p.259.

[88Tac] T. Tachibana, M. Uno, M. Yamada and S. Yamada, in [88Hau] p.251.

[88Wap] A. H. Wapstra, G. Audi and R. Hoekstra, in [88Hau] p.281.

[90Hau] P. E. Haustein, *Exotic Nuclear Spectroscopy*, W.C. McHarris ed., Plenum, NY, 1990.

[92Duf] J. Duflo, submitted to Nucl. Phys., March 1992.

[92Gor] S. Goriely and M. Arnould, preprint submitted to *Astron. Astrophys.* , February 1992.

[92Pdg] Particle Data Group, "Review of Particle Properties", Phys. Rev.**D45** (1992).

[92Pea] M. Pearson and F. Tondeur, *private communication* and Y. Aboussir, J. M. Pearson, A. K. Dutta, F. Tondeur, preprint.